121. Investigation of rotational motion around a fixed axis with the aid of the Oberbeck's pendulum

Aim of the work: To check the main equation of dynamics of rotational motion using the Oberbeck's pendulum.

Instruments: cruciform Oberbeck's pendulum, electronic unit with photoelectric sensors, thread with plummet, ruler, calipers.

Experimental setup

The apparatus for studying rotational motion around a fixed axis is shown in Fig. 1. The column 1 is attached to the basis 2; the pendulum, lower fixed support 5 and upper movable support 6, lower and upper sleeves 7 and 8 are fastened to the column 1. The pendulum consists of the cross-piece 3 with detachable plummets 4 having the mass of m_0 and the two-level pulley 11, which can rotate freely in bearings around the fixed horizontal axis. A movable light block 9 is fastened to the upper sleeve 8. The thread 10 passes through the block 9; one end of the thread is fixed on the two-level pulley 11 having the radii of levels R_1 and R_2.

On the other end of the thread the plummet 12 of the mass m is fastened. The lower sleeve also holds a board 13 with an electromagnet 14 which is used to fix the cross-piece by a friction clutch. Both supports hold photoelectric sensors 15 separated by a distance h which can be changed by moving the support 6 along the column. This distance is measured from a gauge drawn on the column. At the moment when the plummet 12 passes by the upper sensor, the timer begins counting the time. Counting ends when the plummet passes by the lower photosensor; arresting electromagnet 14 is activated at the same moment. Thus, the time t necessary for the plummet 12 to go through the distance h between the photosensors 15 is measured.

Working with the experimental setup

1. Turn on the setup by pressing the button “Сеть”/“Power”. Lamps of the photosensors and the timer's display should start glowing.
2. Press the button “Пуск”/“Start” and wind the thread onto the pullet of the cross-piece to lift the plummet m to the position where the plummet's lower plane is at the level of the upper photosensor. The plummet must not cross the light ray in the sensor. In this position the plummet has the height h with respect to the lower sensor.
3. Push up the button “Пуск”/“Start”; the electromagnet of the friction clutch should switch on and hold the cross-piece in its initial position.
4. Press the button “Сброс”/“Clear” to prepare the timer.
5. Press the button “Пуск”/“Start”. The magnet is switched off releasing the cross-piece, and the plummet m begins to move downward. The duration of the descent is measured by the electronic timer which turns on and off when the plummet crosses the photosensors' light rays.

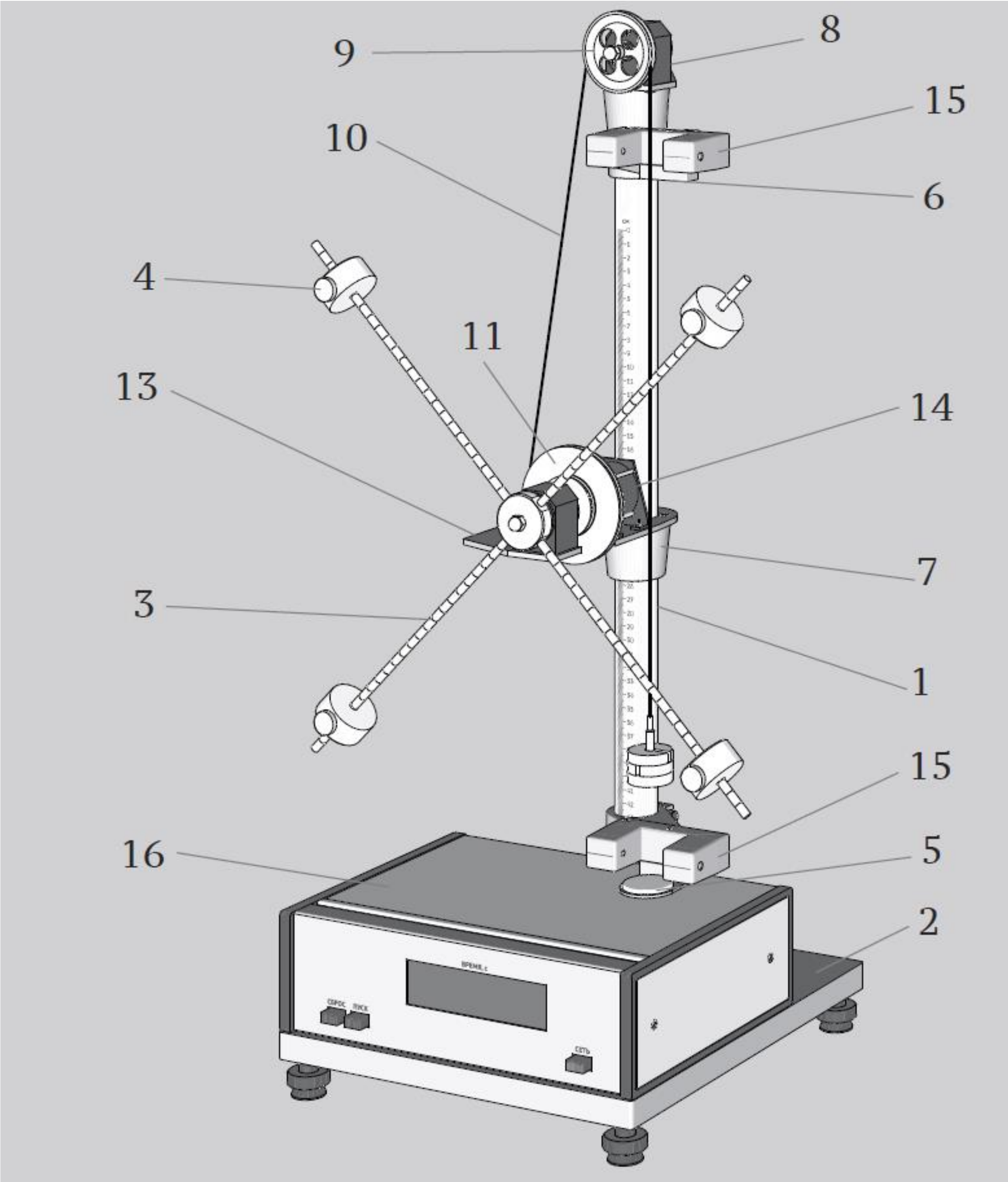

Figure 1. Experimental setup

Exercise 1. Proving the main equation of dynamics of rotational motion around a fixed axis at a constant moment of inertia

Experimental

The main equation of dynamics of rotational motion around a fixed axis is written as

$$M_z = I_z\beta, \qquad (1)$$

where M_z is the projection of the resultant moment of force to the rotation axis; I_z is the moment of inertia of the object with respect to that axis; β is the angular acceleration.

If the moment of inertia of the studied system (the cross-piece with fastened plummets and the pulley) is constant, the change in the resultant moment of force, which causes the motion, should lead to a proportional change in the angular acceleration of the system:

$$\frac{M_{z1}}{M_{z2}} = \frac{\beta_1}{\beta_2} \text{ if } I_z = \text{const.} \qquad (2)$$

Some remarks should be made regarding the movement of the plummet, pulley (around the axis Z_p), and the cross-piece (around the axis Z; see Fig. 2). Friction in the support of the cross-peace and the pulley can be neglected in further analysis (the friction coefficient in rolling-contact bearings treated with industrial lubricants is small, ~0.001). In our experiment the mass of the movable pulley, the mass and deformation of the thread are also very small, so that the pulley can be considered weightless, and the thread can be considered weightless and non-stretched (inextensible). This means that the strain is the same along the whole thread's length. Hence, the cross-piece is rotating under the action of the moment of the thread strain force **T**.

Since the falling plummet *m* experiences constant forces – the gravity force *m***g** and the thread strain force **T**, the acceleration of the plummet is also constant (not changing with time). Then, we can conclude using the second Newton's law that

$$ma = mg - T \qquad (3)$$

and

$$T = m(g - a). \qquad (4)$$

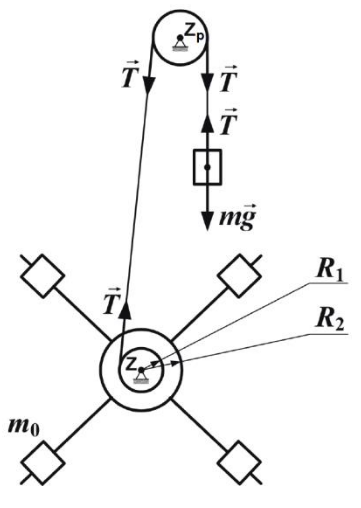

Figure 2. Forces in the system

If a body falls with a constant acceleration from the height h during the time t with the zero initial velocity, then its acceleration can be found as

$$a = \frac{2h}{t^2}. \qquad (5)$$

Therefore, the thread strain force is

$$T = m\left(g - \frac{2h}{t^2}\right). \qquad (6)$$

Note that if the thread does not slip, the acceleration of the plummet is the same (by its absolute value) as the tangential acceleration of the points at the pulley's rim on which the thread is wound. Hence, for the cross-piece rotating with the angular acceleration β we can conclude that $a = \beta R$, where R is the radius of the pulley level at which the thread is wound. Using equation (5), we get

$$\beta = \frac{2h}{t^2 R}. \qquad (7)$$

The projection of the strain force **T** on the rotation axis Z is defined as follows:

$$M_z = TR = mR\left(g - \frac{2h}{t^2}\right). \qquad (8)$$

By substituting the parameters calculated using Eqs. (7) and (8) in the initial Eq. (2) we can make sure that the main equation of dynamics of rotational motion around a fixed axis is fulfilled.

Algorithm of measurements for exercise 1

1. Fasten plummets m_0 on the rods of the cross-piece at identical distances r from the rotation axis (choose yourself the value of r).
2. Measure the radii of the pulley's levels R_1 and R_2 using calipers.
3. Measure the height h (the distance between the sensors 15) with the aid of the gauge on the column 1.
4. Fasten the thread with the plummet m (the mass m is told by the teacher) on the pulley's level R_1. Release the plummet and measure the falling time t_1 from the height h. Repeat the measurements 3–5 times and find the average value $\langle t_1 \rangle$.
5. Fasten the thread with the plummet m (the mass m is told by the teacher) on the pulley's level R_2. As in step 4, measure the falling time t_2 and find the mean value $\langle t_2 \rangle$.
6. Substitute obtained values in Eq. (7) to calculate the angular accelerations of the cross-piece β_1 and β_2, and in Eq. (8) to calculate the projections of the resultant moments of forces M_{z1} and M_{z2}.
7. To check the main equation of dynamics of rotational motion around a fixed axis, substitute the values β_1 and β_2, M_{z1} and M_{z2} in the correlation (2).
8. Estimate the inaccuracy of one of the obtained parameters (at the teacher's option).
9. Make conclusions.

Exercise 1. Proving the main equation of dynamics of rotational motion around a fixed axis at a constant moment of inertia

Experimental

According to Eq. (2), changing the moment of inertia of the cross-piece should lead to a change in the angular acceleration of the system if the resultant moment remains constant, i.e.,

$$\frac{I_{z1}}{I_{z2}} = \frac{\beta_2}{\beta_1} \text{ if } M_z = \text{const.} \qquad (9)$$

Note that in this experiment the work of the friction forces is negligible. The masses of the pulley and the thread are also little, and hence we may neglect their mechanical energy in our measurements. Therefore, to determine the moment of inertia of the cross-piece with plummets with respect to the rotation axis Z, we can use the law of conservation of mechanical energy for the system of bodies including the cross-piece with plummets m_0, the light movable pulley, inextensible weightless thread, and the plummet *m*.

Wind the thread onto the pullet of the cross-piece to lift the plummet *m* to the position where the plummet's lower plane is at the level of the upper photosensor. Since the distance between the sensors is *h*, the plummet *m* gains the potential energy $U = mgh$ with respect to the lower sensor's level. If the plummet is released, it will move downward with the acceleration *a*, and the cross-peace will rotate with the angular acceleration β. The plummet's (*m*) potential energy *U* will be transformed to the kinetic energy of translational motion $mv^2/2$ and to the kinetic energy of rotation of the cross-piece with plummets m_0 and the two-level pulley $I_z\omega^2/2$ (ω is the angular speed). The law of conservation of energy requires that

$$mgh = \frac{mv^2}{2} + \frac{I_z\omega^2}{2}, \qquad (10)$$

where the falling plummet's speed *v* and the angular (rotational) speed ω correspond to the time moment when the plummet *m* is at the level of the lower photosensor.

From Eq. (5) and knowing that $v = at$ we get the correlation

$$v = \frac{2h}{t}. \qquad (11)$$

The angular speed of the cross-piece ω is related to the plummets speed of falling as

$$\omega = \frac{v}{R} = \frac{2h}{Rt}, \qquad (12)$$

where *R* is the radius of the pulley's level at which the thread is wound.

Combining the formulas (11), (12), and (10), we can derive the expression for determination of the moment of inertia of the cross-piece with the plummets and the pulley:

$$I = mR^2\left(\frac{gt^2}{2h} - 1\right). \qquad (13)$$

By substituting the values found from Eqs. (7) and (13) to formula (9) we can make sure that the main equation of dynamics of rotational motion around a fixed axis is fulfilled.

Algorithm of measurements for exercise 2

1. Fasten plummets m_0 on the rods of the cross-piece at identical distances r_1 from the rotation axis (ask the teacher for the value of r_1).
2. Measure the radii of one of the pulley's levels (use the same level in all further measurements!).
3. Measure the height h (the distance between the sensors 15.
4. Fasten the thread with the plummet m on the pulley's level R_1. Release the plummet and measure the falling time t_1 from the height h. Repeat the measurements 3–5 times and find the average value $\langle t_1 \rangle$.
5. Move the plummets m_0 on the rods to a new position r_2 (at the teacher's option).
6. Measure the time t_2 three-five times as in step 4 and calculate the mean time $\langle t_2 \rangle$.
7. Substitute obtained values in Eq. (7) to calculate the angular accelerations of the cross-piece β_1 and β_2, and in Eq. (13) to calculate the moments of inertia I_{z1} and I_{z2}.
8. To check the main equation of dynamics of rotational motion around a fixed axis, substitute the values β_1 and β_2, I_{z1} and I_{z2} in the correlation (9).
9. Estimate the inaccuracy of one of the obtained parameters (at the teacher's option).
10. Make conclusions.