311. Determination of specific resistance of a metallic conductor

Purpose: measuring the resistance of a conductor using ammeter and voltmeter; determining the resistivity of the wire's material.

Equipment: laboratory apparatus FRM-01 for measuring the resistance, containing a nichrome wire, power supply, ammeter, voltmeter, and current regulator.

Brief theory

Ohm's law for a circuit segment without EMF sources

If there is a potential difference $\Delta\varphi = \varphi_2 - \varphi_1$ at the ends of a simple circuit made of conductors, then electric current will arise in this circuit. The current I flowing through a certain segment is proportional to the potential difference $\Delta\varphi$ at its ends and change inversely with the resistance R of this circuit segment (or the conductor):

$$I = \frac{\Delta\varphi}{R}. \qquad (1)$$

The value $U = I \cdot R$ is called the *voltage drop* at the conductor. If the considered segment does not contain any elements providing electromotive force (EMF), then the potential difference at the ends of this segment is equal to the voltage drop at it, i.e. $\Delta\varphi = U$.
If a voltmeter (whose arrow declines due to the current flowing through the voltmeter's coil) is attached to the points *1* and *2* of a circuit, it will show the potential difference between these points. It is equal to the voltage drop U at the voltmeter:

$$U = I_V \cdot R_V, \qquad (2)$$

where R_V is the voltmeter's resistance and I_V is the current flowing through it.

Resistance of conductors

If a circuit segment is a wire with the length l and a constant cross-section S, its chemical composition is homogeneous, then the resistance R of this conductor is defined as

$$R = \frac{\rho \cdot l}{S}, \qquad (3)$$

where ρ is the specific resistance (also called resistivity) of the material.
Resistivity is numerically equal to the resistance of a conductor having a unit length and unit cross-section. It depends on the chemical composition and temperature. In System International it is measured in Ω·m; often the units of $\Omega \cdot mm^2/m$ are used.
At the room temperature, conductors made of chemically pure metals possess the smallest resistivity. Alloys have a larger resistivity, which allows using them for making resistors with great resistance values (rheostats, heating elements, shunting and additional resistors). Specific resistances of some materials are given in Table 1.

Table 1.

Material	Resistivity at 20°C, $\Omega \cdot mm^2/m$
Silver	0.016
Copper	0.017
Aluminium	0.028
Iron	0.093
Constantan (Cu 58.8%, Ni 40%, Mn 1.2%)	0.44 – 0.52
Nichrome (Ni + Cr)	1.0 – 1.1
Graphite	8.0

Methods of measuring the resistance

The main methods of measuring the resistance to a direct current are the indirect method, the method of direct evaluation, and the bridge method. The choice of the approach depends on the expected value of the resistance and required accuracy. The most universal among indirect methods is the method of ammeter and voltmeter, which relies on the Ohm's law for a circuit segment. Equations (1) and (2) lead us to the consequence

$$R = U/I. \tag{4}$$

Thus, by measuring the potential difference *U at the ends of a conductor* and the current *I flowing through it*, we can determine its resistance *R*.
The bridge scheme is considered in another work. This method does not rely on measurements of current and voltage, and hence it is more accurate.
The direct evaluation method involves measurements with the aid of an ohmmeter. This approach is relatively rough and used mostly for preliminary estimation and testing the commutation circuits.
In the present work the method of ammeter and voltmeter is studied.

Technical method with exact measurement of the current

This method always has some inaccuracy due to involvement of measuring instruments, which depends on the manner of connecting of these instruments in the circuit.
In scheme shown in Fig. 1a the voltmeter PV measures the potential difference $U = \varphi_2 - \varphi_1$ at the sequentially connected conductor with the resistance *R and* ammeter PA with the resistance R_A. Therefore, the potential difference between nodes *1* and *2* shown by the voltmeter is equal to the sum of the voltages at the conductor and the ammeter:

$$U = IR + IR_A. \tag{5}$$

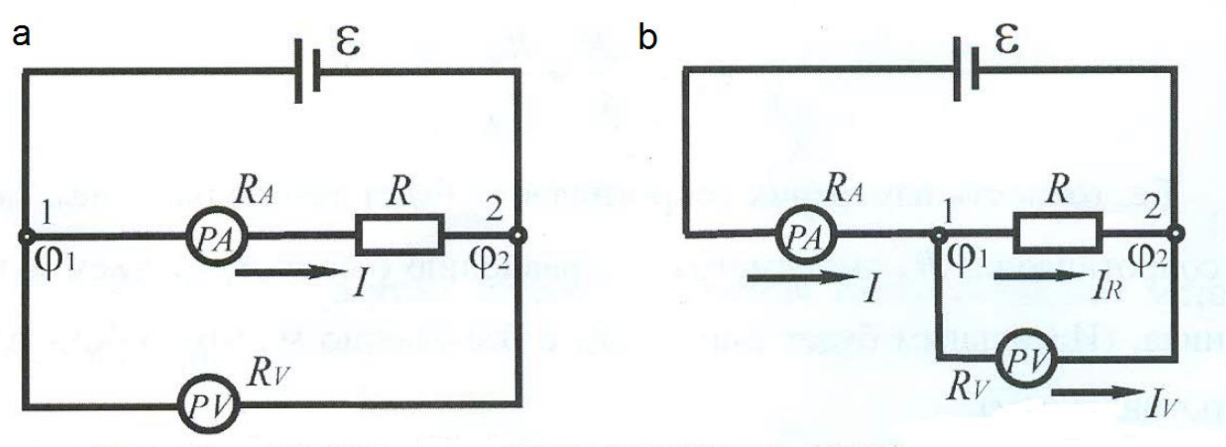

Fig. 1. Connection schemes for indirect measurement of the resistance.

Calculation made by Eq. (4) contains the systematic inaccuracy, since here we obtain actually an "experimental" (observed) resistance R_e, including both the conductor and the ammeter. The true resistance of the resistor (e.g., a wire), according to Eq. (5), can be derived as

$$R = \frac{U}{I} - R_A, \tag{6}$$

where *U* stands for the potential difference measured by the voltmeter.

The methodical error is thus the difference $\Delta R = R_e - R = R_A$. The internal resistance of an ammeter is typically small (and hence we can assume $R \approx R_e$), and the smaller it is compared to the resistance to measure (R), the better is the accuracy. The relative inaccuracy of the method is

$$\varepsilon_{m1} = \frac{\Delta R}{R} \approx \frac{R_A}{R_e}. \qquad (7)$$

An ideal ammeter has zero internal resistance.

Technical method with exact measurement of the voltage

In the scheme on Fig. 1b the total current flowing through the resistor R and voltmeter PV (with an internal resistance of R_V) is measured. The potential difference in this case is the same for both conductor and voltmeter; let us denote it U (voltmeter's reading). The Ohm's law in this case predicts the following currents I_R and I_V flowing through the resistor and voltmeter, respectively: $I_R = U/R$, $I_V = U/R_V$. The measured current is $I = I_R + I_V$, and hence

$$I_R = I - I_V = I - \frac{U}{R_V}. \qquad (8)$$

The resistance of the conductor R_e can be found using Eq. (4) from the reading of the measuring gauges, if the current flowing through the voltmeter can be neglected. The proper resistance R is

$$R = \frac{U}{I_R} = \frac{U/I}{1 - U/(IR_V)} = \frac{R_e}{1 - R_e/R_V}. \qquad (9)$$

Internal resistance of a voltmeter is usually relatively high, so we can assume that $R \approx R_e$ and $R_V - R_e \approx R_V$. Then

$$\varepsilon_{m2} = \frac{\Delta R}{R} = \frac{R_e - R}{R} = \frac{R_e^2}{R(R_V - R_e)} \approx \frac{R_e}{R_V}. \qquad (10)$$

Equation (10) shows that the higher is the internal resistance of a voltmeter, the higher is the accuracy. Analogue electronic and digital voltmeters are high-resistance devices.
Methodical inaccuracy appears if Eq. (4) is used. It can be eliminated if internal resistances of the instruments are known. The instrumental errors depending on the construction of the devices and their accuracy rating yield another source of the systematic error; these are not studied in the present laboratory work.

Experimental setup

The apparatus FRM-01 is shown in Fig. 2. The basis 1 holds the column 2 with the millimetre gauge 3. Two fixed supports 4 and one movable support 5 are attached to the column; the support 5 can be moved and fixed in any position. A nichrome wire 6 is strung between the upper and bottom supports 4.
A contact on the movable support provides a good galvanic junction with the wire. A line on the support 5 is used to determine the length of the measured segment of the nichrome wire. The top, bottom, and middle (movable) contacts with the wire are connected to the measuring unit 7 by low-resistance wires.
The face of the block 7 contains the ammeter PA, voltmeter PV, knob W1 for switching the apparatus on (into the 220 V mains), switches W2 and W3, and the rheostat handwheel R1. When the knob W3 is eased up, the wire can be used in measurement employing the bridge scheme; when it is pushed down, the active resistance of the wire is measured using the ammeter and voltmeter.

It the knob W2 is eased up, the method with exact measuring of current is used (as shown in Fig. 1a); when it is pushed down, the scheme in Fig. 1b works (with exact measurement of the voltage).

Algorithm of measurements

1. Prepare the table for the measurement results.
   Table 2.

№	I_i, mA	U_i, V	$R_{e\,i}$, Ω	R_i, Ω
…				

2. Set the current regulator handwheel R1 to the minimal value (turn it anti-clockwise as far as it will go). Moving the support 5, set an arbitrary wire length *l*. Write down this length and the wire's diameter *d* (from the table or measuring it by a micrometre). Internal resistances of the measuring gauges R_A and R_I are shown on the apparatus.
3. Turn the apparatus on. Make five measurement of the resistance with different current values by the method with exact measurement of the current.
4. Prepare another table analogous to Table 2. Make a series of measurements using the method with exact measurement of the voltage.
5. Turn the apparatus off.

Processing of the results

1. Calculate the uncorrected ($R_{e\,i}$) and corrected (R_i) values of the wire's resistance using Eqs. (4), (6), and (9), and also their mean values $\langle R_e \rangle$ and $\langle R \rangle$ for each method of measuring.
2. Determine the inaccuracies ε_{m1} and ε_{m2} from Eqs. (7) and (10) for each method. Decide which connection scheme is preferable.
3. Find the resistivity ρ of the nichrome wire using Eq. (3). To do this, take the value $\langle R \rangle$ from the measuring approach which turned out to have the smaller error.

Questions

1. Give the definition of the *potential difference* and *voltage drop* at a circuit segment. When they are equal to each other?
2. What is the physical meaning of the resistance of a conductor? On which factors does it depend?
3. How the measurement of the resistance with exact measuring of the current is performed? What is the source of inaccuracy in this method, and how can it be allowed for?
4. How the measurement of the resistance with exact measuring of the voltage is performed? What is the source of inaccuracy in this method, and how can it be allowed for?