Compensation Method of Measuring Electromotive Force

Kirchhoff's rules are useful when calculating strains and currents in branched electric circuits.
Current law: At every branch point the sum of the current strengths of the incoming current is the same as the sum of outgoing currents.
Typically, incoming currents are considered positive; e.g., in the point A in Figure 1 $I_1 > 0$ and $I_2 > 0$, but $I < 0$.
Voltage law: In every closed electrical circuit (loop) of a network the sum of the partial voltages on the lines (resistors, consumers) is the same as the sum of the voltages ("electromotive forces") of the power sources connected.
It is necessary to determine a sense of rotation for the loop. Currents that flow in this rotational direction, and voltages that cause currents which flow in the same direction (from + to –), are considered positive and those opposite of this direction are considered negative. For example, clockwise direction may be assumed to be positive. Let us choose two contours in the circuit shown in Figure 1: contour I will be AEBCA, and contour II will be ACe$_1$A. Then I_2 and I turn out to be "positive" in loop II, while I_1 and I are "negative" in contour II.
Note that true directions of current may be unknown in the beginning of calculations. It may happen then that we'll get a negative value for some current strength; it will simply mean that its direction is opposite to one that we have chosen arbitrary in the beginning.
Mathematical representation of Kirchhoff's rules is as follows:

$$\sum I_k = 0,$$

$$\sum I_k R_k = \sum E_k .$$

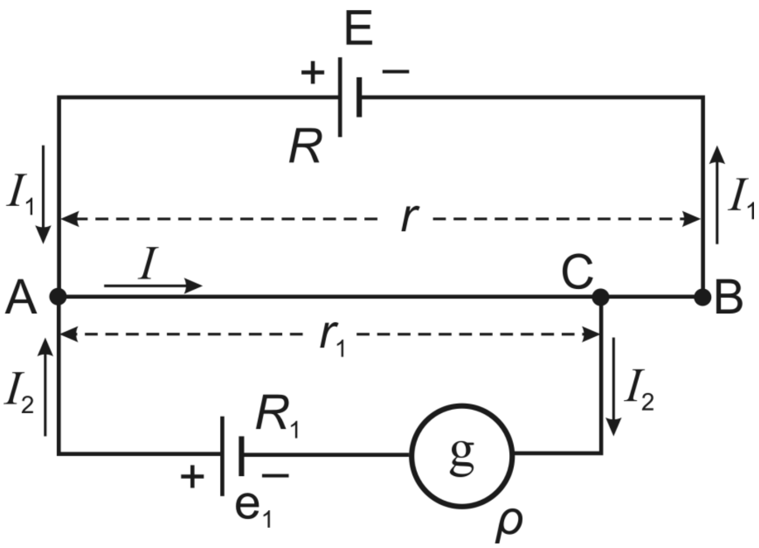

Figure 1.

In the particular case shown in Figure 1, these equations can be rewritten as

$I_1 + I_2 - I = 0,$ (1) (point A)

$-E = -I_1 \cdot (R + (r - r_1)) - Ir_1,$ (2) (loop I)

$e_1 = I_2 \cdot (R_1 + \rho) + Ir_1.$ (3) (loop II)

Here R is the resistance of the circuit part AEB (in general case; but provided that wires have a very low resistance, this may be regarded as the inner resistance of the current source E), R_1 is the resistance of the segment Ae_1C excluding the resistance of the galvanometer (which is denoted ρ), r is the resistance of the measuring wire AB, and r_1 is the resistance of the wire's segment AC.

The present work is dedicated to measuring the electromotive force (EMF) of a current source by the compensation method. Electromotive force is the value equal to the work imparted by non-electrostatic sources (e.g., chemical sources) to move a unit charge over the whole electric circuit. It is measured in the same units as the electric strain – volts (V) in System International.

The measuring scheme is shown in Figure 1. Two elements having the EMFs of E and e_1 are connected to the point A of the slide wire (rheochord) with their contacts of the same polarity. If the galvanometer in the loop containing element e_1 does not detect current, i.e., $I_2 = 0$, then $I_1 = I$, and equations (2) and (3) can be rewritten as $E = I(R + r)$ and $e_1 = Ir_1$. From these two relations we can derive that

$$e_1 = \frac{Er_1}{R + r}. \quad (4)$$

If the source e_1 is replaced by another element e_2, a new resistance of the segment AC should be adjusted to return the galvanometer reading I_2 to zero. If this new resistance is r_2, then $e_2 = \frac{Er_2}{R + r}$. By dividing two last relations by each other, we obtain a proportion

$$\frac{e_1}{e_2} = \frac{r_1}{r_2}. \quad (5)$$

Resistances r_1 and r_2 are proportional to corresponding lengths l_1 and l_2 of the segments of the rheochord AB which is made from a uniform wire with a constant cross-section. Finally, we get the working formula

$$e_1 = e_2 \frac{l_1}{l_2}.$$

A standard source is used as the element e_2, whose EMF changes negligibly with time.

The most popular is the cadmium Weston normal cell. Its EMF at 20°C is 1.0183 V and is almost temperature-independent at the room temperature: when it increases by 1°C, the decrease in the EMF is less than 10^{-4} V.

The compensation method has the following advantages. (1) The currents flowing in the compared cells (e_1 and e_2) is close to zero, hence the voltage drop inside the element (which decreases the difference of potentials at the element's contacts) is negligible. Sensitive galvanometers allow decreasing the current to the order of $10^{-9} – 10^{-10}$ A. Voltage drop in the wires does not play any role either in this case. (2) The galvanometer shows the zero value and needs no calibration. Resistances r_1 and r_2 in Eq. (5) can be determined with the relative accuracy level of 10^{-2}%. (3) The EMF of the auxiliary source E does not influence the result. The only requirement to the source is that the value of E is constant during the measurements; in addition, E should be larger than e_1 and e_2.

Experimental setup

The circuit for determining the EMF of a cell is shown in Figure 2. E is the auxiliary current source, AB is the sliding wire, C is the slider of the rheochord, e_2 is the normal cell (e_2 = 1.0183 V). g is the galvanometer working as a null-indicator, K_1 and K_2 are the single- and double-current keys, K_3 is the knob for short-time turning the galvanometer on. The resistance box R_m is used to prevent the normal cell from giving unpermitted (large) power.

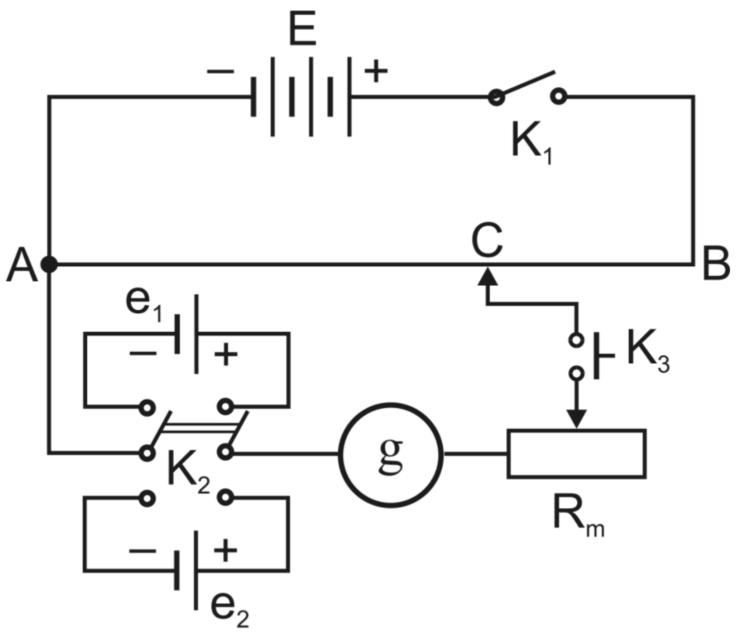

Figure 2.

Algorithm of measurements

1. If necessary, assemble the circuit according to the scheme in Figure 2. Choose the maximal resistance R_m in the box. Put the slider C in the middle position of the rheochord, and connect the normal cell e_2 with the key K_2.
2. Turn the key K_1 on.
3. Decrease the resistance R_m in the box and press the knob K_3 quickly in order to find the position of the slider C which compensates the normal element. In this case the galvanometer should show the zero current when R_m = 0. Measure the length AC = l_2 using the gauge the apparatus.

4. Return R_m to the maximal value and connect the element e_1 using the key K_2. Like on stage 3, compensate the EMF e_1 and measure the corresponding length of the rheochord segment AC = l_1.
5. Calculate the EMF of the investigated source using Eq. (5).

Note

All measurements should be done quickly to obtain accurate results (don't hold the knob K_3 for too long). When assembling the circuit, check the polarity of the cells!

Questions

1. External (non-electrostatic) forces. Electromotive force. Ohm' s law in general form.
2. The first and second Kirchhoff's rules. Choice of signs.
3. Sources of constant electrical current. Normal cell.
4. Methods of measuring EMF. Advantages of the compensation method.
5. Scheme of the experimental circuit. Deriving the working formula.