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Parametrically excited microelectromechanical system in the problems of orientation of moving objects

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Abstract. A mathematical model of the microelectromechanical system (MEMS) is obtained, which performs the task of orienting mobile objects. A distinctive feature of the construction of such MEMS is its parametric excitation by modulating the static stiffness of the suspension of the sensitive element. Based on the analysis of model equations, the possibility of determining the direction of the true meridian with the required accuracy is shown. The results of numerical calculations are given that allow not only to illustrate the operation of the device as a high-speed gyrocompass, but also to determine the conditions that ensure the stable operation of the device in the incoherent mode of its excitation.

1. Introduction

One of the important directions of creating modern orientation instruments is their construction based on micromechanical gyroscopes (MMG) of the planar group of the RR-type (rotary-rotary), which is caused by their high technological ability and good prospects for reducing weight, dimensions and cost. However, the methods of orienting mobile objects using MMG [1] known to date do not fully satisfy the requirements of speed and accuracy. The proposed article is a continuation of a series of works [2], in which theoretical bases for the improvement of gyroscopic devices are developed. A new approach to the enhancement of MMG functionality is considered, based on the parametric excitation of the static stiffness of the suspension of its sensitive element.

2. The operating principle of MMG

The Figure 1 shows a kinematic MMG of the RR type, including a rotor 6 in a torsion suspension 5 with an electrostatic drive consisting of an anchor 3 on the substrate associated with the base 1. The drive provides an oscillatory mode of the rotor movement relative to the axis *OZ* (primary oscillations of the rotor). In the presence of a portable angular velocity of the base motion, the resulting gyroscopic moment generates secondary oscillations of the rotor around the axis *OX*, whose amplitude is proportional to the measured angular velocity. The oscillations of the rotor are converted into an electrical signal by an angle sensor 7. The torque sensor 4 is used to create a compensation mode for measuring the angular velocity, and the reference voltage generator 2 is used to obtain information on the motion of the base in the coordinate system of the device body (demodulation).
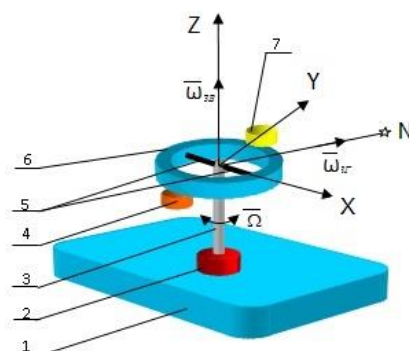
Figure 1. Kinematic scheme MMG RR-type: 1 - base; 2 - reference voltage generator; 3 - anchor; 4 - the gauge of the moment; 5 - torsion suspension; 6 - rotor; 7 - angle sensor; N - the direction of the true meridian; $\omega_{3\text{B}}$, $\omega_{3\Gamma}$ - respectively, the horizontal and vertical components of the angular velocity of Earth's rotation; Ω is the angular velocity of rotation of the anchor.

In the incoherent mode of parametric excitation, there is a "strong resonance" (the amplitude of the oscillator oscillations increases in comparison with its value at resonance) and a "weak resonance" (the amplitude of the oscillations decreases in comparison with the resonance value), which is associated with a change in the value coefficient of damping of the gyroscope. In this mode, along with the primary oscillations and oscillations with a combination frequency that occurs with parametric excitation, the rotor oscillates with a beat frequency. In this case, a change in the phase of the oscillations of the rotor creates a "swing" of the sensitivity axis of the device with a beat frequency relative to the position corresponding to the absence of excitation.
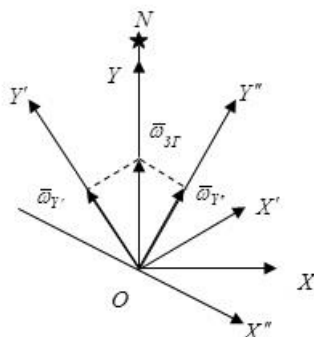

Figure 2. The change in the position of the MMG sensitivity axis when its static stiffness is modulated (incoherent mode).

The Figure 2 shows the position of the sensitivity axis (the OX axis) of the device with respect to the true meridian for the case of its resonant tuning (the OY axis is directed to the north) and its position change with incoherent excitation mode. For the case of "strong resonance" - the position of the axes $OX'Y'$, and for the case of "weak resonance" - the position of the axes $OX''Y''$. It can be seen that the projection $\omega_{3\Gamma}$ to the sensitivity axis OX of the tuned device is zero (the value of the information signal, that is the output voltage of the sensitivity channel $U_x = 0$), while the axis OY coincides with the direction of the true meridian. With parametric excitation, a periodic change in the signal U_x is observed with the same amplitude relative to the "zero position" (corresponding to the direction along the "West-East" line).

In the case of deviation of the device body (sensitivity axis) by some angle ψ, the "center of oscillations" will shift from the "zero position" to the value corresponding to this angle. Turning the device body around the axis OZ so as to combine the "center of oscillations" with the previously indicated "zero position", i.e. achieving a situation where the amplitude of periodic deviations of the

axis in the positive and negative directions is the same, you can achieve alignment of the axis with the direction of the true meridian. The measured rotation angle of the device body will correspond to its desired azimuth.

It should be noted that the considered method for determining the direction of a true meridian with the help of a parametrically excited MMG, despite the apparent analogy with the well-known classical gyrocompassing principle based on a three-stage "heavy" gyroscope, has qualitative differences in the measurement principle itself. This leads to an increase in the accuracy of determining the direction of the true meridian while increasing the speed of the device.

This unconventional use of MMG, connected with its parametric excitation, makes special demands on the dynamics of the device, which, in turn, leads to the need for a numerical analysis of the corresponding gyroscopic system based on a rigorous mathematical model.

2. The mathematical model of the parametrically excited MMG

The specificity of the work of MMG allows, based on the use of circuitry solutions, without changing the design of the mechanical circuit, to increase the sensitivity of the device to the measured angular velocity by its parametric excitation [3].

The excitation of the MMG, as an oscillatory system, is carried out by modulating the static stiffness of the suspension by changing within a small range the alternating current applied to the additional winding of the moment sensor by law, which leads to the creation of a moment

$$M = \Delta k \sin(\omega_{\mathrm{m}} t - \varphi), \tag{1}$$

where where ω_{m} is the modulation frequency, Δk is the amplitude of static stiffness oscillations of the rotor suspension, φ is the initial phase, and t is the time.

To describe the motion of MMG mounted on the moving base, we introduce a coordinate system $0\xi^*\eta^*\zeta^*$ with the origin at the center of mass of the gyro rotor.
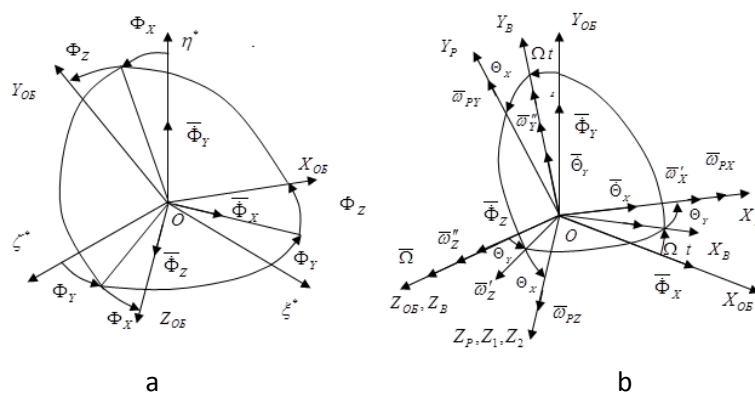

Figure 3. Coordinate systems.

With a movable base we can tie the coordinate system $OX_{06}Y_{06}Z_{06}$, the axis Z_{06} of which coincides with the axis of rotation of the rotor MMG (Figure 3a). The motion of the base of the device will considered as known, i.e. in each instant of time is known the orientation of the coordinate system $OX_{06}Y_{06}Z_{06}$ relative to the inertial $0\xi^*\eta^*\zeta^*$, and the projection $\dot\Phi_x, \dot\Phi_y, \dot\Phi_z$ of the vector of absolute angular velocity on the base axis system $0\xi^*\eta^*\zeta^*$ are given functions of time. In addition to the above systems are required two systems of axes $OX_{\mathrm{B}}Y_{\mathrm{B}}Z_{\mathrm{B}}$, $OX_pY_pZ_p$ associated respectively with the shaft of a drive motor and with the principal axes of inertia of the rotor (Figure 3b).

The point of origin of the system $OX_{\mathrm{B}}Y_{\mathrm{B}}Z_{\mathrm{B}}$, $OX_pY_pZ_p$ lies in the center of mass of the MMG and its position relative to the system $OX_{06}Y_{06}Z_{06}$ is successive turns in the positive direction on appropriate angles θ_z for $OX_{\mathrm{B}}Y_{\mathrm{B}}Z_{\mathrm{B}}$ and θ_x, θ_y for $OX_pY_pZ_p$.

For deriving the equations of motion of a gyroscopic system will use the variational principle of Ostrogradsky – Hamilton (see for example [4]), from which follow equations

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_\iota} - \frac{\partial T}{\partial q_i} = Q_i - \frac{\partial \Pi_i}{\partial q_i} - \frac{\partial \Phi_i}{\partial \dot{q}_i}, \tag{2}$$

where T is the kinetic energy of the considered gyroscopic system; q_i – generalized coordinates defining the position of all points of the system in space; Q_i – generalized forces acting on the system; $\Pi_i = 0.5k_i q_i^2$ – the potential energy of the system under consideration; $\Phi_i = 0.5\mu_i \dot{q}_i^2$ is the dissipative function of the system; i – number of generalized coordinates describing the degrees of freedom of the considered electromechanical system.

We choose as the generalized coordinates the rotation angles of the rotor $\theta_x = \beta$, $\theta_y = \alpha$, $\theta_z = \gamma$, which uniquely determine its position in the coordinate system associated with the anchor, and as the generalized forces, the moments of damping of the rotor and the moments of elasticity of the torsions. Suppose that the rotor is an absolutely rigid body, and the suspension has rigidity, which excludes the displacement of the center of mass of the rotor relative to the center of the suspension. With such limitations on the design, it is possible not to take into account the effect of portable accelerations on the dynamics of the MMG.

The differential equations of motion for the case of a constant angular velocity of rotation of the base, after the factorization procedure [5] and subsequent linearization with the help of the Jacobi matrix, taking into account the generalized forces, will have the form

$$A\ddot{\alpha} + \mu_\alpha\dot{\alpha} + \left(k_\alpha + (C - B)\dot{\gamma}(\dot{\gamma} + 2\dot{\Phi}_z) + \Delta k \sin(\omega_m t - \varphi)\right)\alpha$$
$$= -A\ddot{\Phi}_x - (C + A - B)\dot{\gamma}\dot{\Phi}_y + (A + B - C)\dot{\beta}\dot{\Phi}_z,$$
$$B\ddot{\beta} + \mu_\beta\dot{\beta} + \left(k_\beta + (C - A)\dot{\gamma}(\dot{\gamma} + 2\dot{\Phi}_z)\right)\beta = -B\ddot{\Phi}_y - (C + B - A)\dot{\gamma}\dot{\Phi}_x + (A + B - C)\dot{\alpha}\dot{\Phi}_z,$$
$$C\ddot{\gamma} + \mu_\gamma\dot{\gamma} + k_\gamma\gamma = -C\ddot{\Phi}_z + (C + A - B)\dot{\alpha}\dot{\Phi}_y - (C + B - A)\dot{\beta}\dot{\Phi}_x + M_0 \sin\Omega t, \tag{3}$$

where A, B, C are the principal moments of inertia of the rotor; $\mu_\alpha, \mu_\beta, \mu_\gamma$ are the coefficients of viscous friction with respect to the corresponding coordinates; $k_\alpha, k_\beta, k_\gamma$- rigidity of the elastic suspension elements relative to the axes of the secondary and primary gyro oscillations; M_0 is the amplitude of the torque developed by the drive relative to the axis of the primary oscillations.

3. Numerical study
Notice, that $k_\beta \gg k_\alpha$, $M_0 \gg (C + A - B)\dot{\alpha}\dot{\Phi}_y$, and in the system of differential equations (3) we can put $\beta = \dot{\beta} = 0$, then for the case of motion of a base with constant angular velocity we obtain simplified equations

$$A\ddot{\alpha} + \mu_\alpha\dot{\alpha} + (k_\alpha + \Delta k \sin(\omega_m t - \varphi)) = -(C + A - B)\dot{\gamma}\dot{\Phi}_y, \tag{4}$$
$$C\ddot{\gamma} + \mu_\gamma\dot{\gamma} + k_\gamma\gamma = M_0 \sin\Omega t.$$

The solution of the second equation of system (4) has the form

$$\gamma(t) = \frac{M_0(1 - e^{-a_\gamma t})\cos\Omega t}{2a_\gamma C\Omega}, \qquad a_\gamma = \frac{\mu_\gamma}{2C}. \tag{5}$$

In the steady-state oscillation mode we have

$$\gamma(t) = \frac{M_0(1 - e^{-a_\gamma t})\cos\Omega t}{2a_\gamma C\Omega}, \qquad a_\gamma = \frac{\mu_\gamma}{2C}. \tag{6}$$

Substituting expression (6) into the first equation of system (4), we obtain

$$\ddot{\alpha} + 2a_\alpha\dot{\alpha} + \omega_0^2(1 + m\sin(\omega_m t - \varphi))\alpha = K\dot{\Phi}_y \sin\Omega t, \tag{7}$$

where $a_\alpha = \frac{\mu_\alpha}{2A}$, $\omega_0 = \sqrt{\frac{k_\alpha}{A}}$, $K = \frac{(C+B-A)\gamma_0\,\Omega}{A}$, $m = \frac{\Delta k}{k_\alpha}$ – the modulation factor, $\dot{\Phi}_y = \omega_{3\Gamma}\cos\psi$, ψ – the angular deviation of the axis from the direction of the true meridian.

A feature of the differential equation (7) is the presence of a term associated with a periodic change in the positional moment $\omega_0^2\, m\sin(\omega_m t - \varphi)\,\alpha$. The presence of such periodically varying energy parameters MMG as gyroscopic moments and static stiffness included in the respective nonhomogeneous differential equations, provides favorable conditions for the parametric excitation of the mechanical contour of the considered gyro [6].

Numerical calculations were performed using the Maple 9 with $\omega_m \approx 2\Omega$ (incoherent mode). The calculation algorithm consisted of the following operations:

- calculation of the value $\alpha_\text{в}(t)$ of the angular motion of the gyroscope rotor in the shaft coordinate system;
- determination of the value $\alpha_\text{к}(t)$ of the angular motion of the rotor in the coordinate system associated with the body and, accordingly, determination of the output signal U_x of the demodulator by multiplying the solution by a periodic function $\sin\Omega t$ (demodulation process);
- filtering the output signal U_x of the demodulator; t.i. screening of periodic components of the output signal with a frequency equal to 2Ω;
- smoothing out the resulting graphical solutions based on regression analysis in a Maple 9 environment.
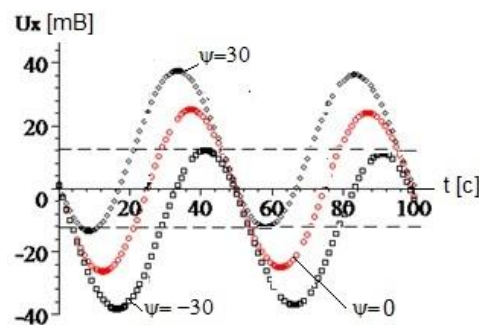

Figure 4. The change of output signal U_x.

In Figure 4 graphs of the output signal $U_x(t)$ are shown for three cases: 1) the MMG axis OY coincides with the direction of the true meridian N ("the center of oscillations" coincides with the position of the line "west-east"); 2) the axis OY is deviated from the direction of the true meridian by an angle $\psi = 30^0$; 3) the axis is deviated from the direction of the true meridian by an angle $\psi = -30^0$.

These solutions confirm that in the case of deviation of the body of the device by a certain angle, the "center of oscillations" (the dashed line in Figure 4) is displaced from the "zero position" by an amount proportional to this angle.

Thus, the results of numerical studies of the presented mathematical model describing the dynamics of motion of a parametrically excited MMG showed the possibility of its use as a ground-based precision device for orienting mobile objects.

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