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Use of long-term models for analysis of comet Encke's motion

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Abstract

This paper is focused on the problem of the reactive non-gravitational effects change in the cometary motion caused by the comet's activity fading. The modern researches have been reviewed. The necessity of a new model compatible with Marsden's model of non-gravitational forces is shown. Modifications of Marsden's model for the cases in which the main factors of the parameters change are the deposition of the significant non-volatile mass and the growth of the superficial crust are developed. They contain besides the usual Marsden's parameters two and three additional values correspondingly. The developed equations have been used to explain the known change of Marsden's non-gravitational parameters for the comet Encke during 225 years of its observational interval. Although the model with significant non-volatile mass contains less free values, it explains the characteristic features of the comet's non-gravitational parameters behavior better. The model can be further applied for the comet's observed astrometric positions fitting.
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1. Introduction

According to modern theory, non-gravitational effects in comet motion are the result of reactive force action from cometary ice sublimation (Sekanina and Kracht, 2015). Therefore, it is quite natural that comet Encke, which has an exceptional combination of a small perihelion distance and a small orbital period, is the first comet for which non-gravitational effects have been observed. It is also the first comet for which secular changes of non-gravitational effects have been detected. However, the reasons for these changes are still the subject of discussion, impeding the construction of a unified comet motion theory (Maquet et al., 2012).

At the present time, for a mathematical description of the cometary non-gravitational forces, the "Style II" Marsden's model (Marsden et al., 1973) is the most widely used; this model describes sublimation change depending on heliocentric distance, but non-gravitational parameters, according to the model, characterizing the amplitude of this change, are constant. Usually 3–5 apparitions of comets can be linked by Marsden's model. The comet's motion representation is based on this partition in the JPL NASA "HORIZONS" service (Giorgini et al., 1996). Integrating the orbit that was obtained according to modern observations to the comet's discovery in 1786 via this service, the difference between the observed and calculated moment of the perihelion passage is 58.2 days. Integrating the orbit from 1786 to the present day, the error is up to 264 days. For the 1898 orbit, the deviations in 1786 and 2010 are 49.3 and 18.5 days, respectively. This is 3 orders greater than the inconsistencies, which can be explained by an orbital period determination error.

Initially, Marsden assumed that the parameters introduced by him were supposed to decrease exponentially with time (Marsden, 1969). The solution for comet Encke according to the 1947–1967 observations leaved 1.77″ root

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mean square error; however, by 1931, the deviation from the observations had reached 560″. Linking the 1927–1967 apparitions, the root mean square error increased to 2.58″ and systematic trends of up to 6″ were revealed. By 1786 the solution had deviated 158 days from the observations. Later it had become clear that the character of the observed changes of non-gravitational parameters did not correspond to this assumption (Marsden and Sekanina, 1974).

At present, Marsden's model modifications have been mostly developed where the changes in the non-gravitational parameters are explained by precession of the rotation axis, and by the comet nucleus spottiness. The idea of the precession model as well as its version with spottiness belongs to Sekanina (Whipple and Sekanina, 1979; Sekanina, 1988). He applied these models to approximate Marsden's parameters changes, but not the comet's observed positions. From the first model it followed that in about 1990, comet Encke was supposed to change the sign of the transversal non-gravitational parameter, something that has not actually happened until now. For astrometric observations fitting, both models were adapted by Szutowicz (Sitarski, 1992; Szutowicz, 2000). Szutowicz never attempted Sekanina's forced precession model for comet Encke; in the spotty models she linked, for example, 3 apparitions of 1996–2003 with a root mean square error of 0.98″, noting that, in general it is more difficult to find parameters of the spotty models for this comet than for others (Jockers et al., 2011). Independently, a precession model was adapted by Chernetenko (1992). Besides the orbital elements and 7 non-gravitational parameters of this model, Chernetenko determined comet Encke photocentre's offset from centre of mass in single apparitions or in pairs of apparitions (Chernetenko, 1992). Thus the continuous theory for motion of comet Encke from 1901 to 1987 with a root mean square error of unit weight 2.96″ was constructed, but the theory contained 17 additional parameters of the photocentre offset.

Sekanina's precession models do not consider a decrease of the comet's nucleus mass and sublimating area that necessarily accompany reactive force, so these models are not self-consistent. Makover considered as undoubted that the change of the comet's secular acceleration and systematic decrease of its brightness found by Vsekhsvyatskii are caused by the same reasons: namely, depletion of frozen gas depositions in nucleus (Makover, 1955). Rickman considered the change in comets' non-gravitational parameters along with secular change of their brightness as a verification of their physical evolution (Rickman, 1988; Rickman et al., 1991). Comet Encke's dust trail observations at infrared wavelengths have showed that its mass loss rate due to dust is much higher than was expected (Reach et al., 2000).

There are few works about cometary non-gravitational effects, taking into account the fading. However, none contains a comparison of theory with astrometric observations of any real comet.

Before Sekanina developed his precession models, he had presented some formulae to estimate the lower limit of mass loss rate on the basis of non-gravitational parameters, and had calculated the lower limit for several short-period comets (Sekanina, 1969, 1970, 1971). He also noted that only in the case of comet Encke, dynamical data are sufficient to derive the law of mass loss rate change with time. A theoretical consideration of a purely icy comet nucleus gave a non-gravitational acceleration increase with time as a result of its mass decrease, which sharply contradicts the observational data. To overcome this problem, it was assumed that there were some non-volatile substances in the nucleus. Sekanina also arbitrarily assumed that these substances form a porous monolith. The restriction on the porous substance's possible density forced an assumption that the non-volatile substances are situated only near the nucleus centre, and that the mantle consists of pure ice. Few versions of evaporating matter transfer through the pores were considered. Finally, the secular acceleration progress of comet Encke was adequately represented by this model, though the final formulae were mathematically simplified for conformity with the computing possibilities of that period. Extrapolating different versions of the model predicted a complete comet's activity termination between 2022 and 2036. Excessive addition to these works about the model of the cometary nucleus structure as the "core-mantle", which has not yet been confirmed, hindered their development.

In the works of Lebedinets, change of mass and surface area of the comet's nucleus was taken into consideration (Lebedinets et al., 1983, 1990; Lebedinets, 1985; Pivnenko, 1989). At first the spherical nucleus had been considered, then a change in its shape was simulated. However, there was an assumption that the cometary nucleus is almost ice and a relatively small part of the solid admixture does not affect evaporation rate. Under these conditions, non-gravitational effects can only increase with time, which contradicts the tendency observed in a number of comets. As this model was only applied to explain the structure of meteor swarms, not comet motion, an error went unnoticed.

Kozlov and Medvedev considered the effect of dust content on the shape and dynamic of a mass-losing cometary nucleus (Kozlov and Medvedev, 2000). The equations derived from thermophysical and mechanical considerations were applied to the cometary nucleus model moving in the orbit of comet Halley (orbital motion was calculated by unperturbed formulae) with arbitrarily assumed values of radius, density, dust content, micropore diameter in the dust matrix, strength of the dust matrix and rotation axis orientation angles. It was noted that the size of the model nucleus is significantly reduced, compared to real comet nuclei, to cut down the computing time. As a result of sublimation, the nucleus takes a dumb-bell like shape; the ratio of the semiaxes of the ellipsoid enveloping the nucleus is 2: 1 for the case of small dust content and 1.1: 1 for large dust content. It was also noted that change of the comet's orbital period due to non-gravitational acceleration first gradually increases, then reaches its maximum, afterwards decreasing (a similar pattern is actually observed for comet Encke). For the case of small dust content, the maximum is pronounced; for large dust content, the decrease of non-gravitational effects starts at almost the very beginning of the simulated period.

So it is necessary to elucidate how the parameters of the most widely used Marsden's model are supposed to change during the fading of comet activity, and then to build a corresponding numerical model on the basis of comet Encke's astrometric observations. It is clear that under the conditions of our current poor knowledge of cometary nuclei axial rotation, the size, shape, structure, composition and stochastic nature of non-gravitational forces, such a model would be very schematic and averaged. We should therefore not yet expect that the precision of positions calculated via this model be as high as the precision of the astrometric observations. As new data about the physical characteristics of comets appear, there will be opportunities to make the model more precise.

2. Mathematical models of non-gravitational acceleration

From the aforementioned previous works, it follows that the observed decrease of non-gravitational parameters cannot be explained by homogeneous nucleus sublimation, during which all the surface substance is removed from the comet, as it requires a decrease of the ratio of effective evaporating area to mass. This heterogeneity can be achieved by two fundamentally different options: deposition of significant non-volatile mass, or formation of a crust on the nucleus surface that reduces the effective area. It is clear that, actually, there is a combination of these options: a non-volatile substance of any nature makes up some additional mass, and also somehow changes the sublimation rate. Nevertheless, if the deposited mass is a significant part of total nucleus mass, we may conclude that during the deposition process the deposited mass almost does not impede the sublimation. If the crust reduces the sublimation quickly enough, then the crust's mass should not form a significant part of the total nucleus mass. That is why it is reasonable to consider these two cases separately, which significantly reduces the number of free parameters.

In the second case it is necessary to concretise the ratio of effective evaporating area to geometrical area (i.e. the extinction coefficient of sublimation) as a function of the crust thickness. There have been some attempts to find this important function in cometary astronomy both in theoretical (Shul'man, 1972; Skorov et al., 2002) and experimental (Ibadinov, 1999) ways. These studies produced completely different results. Moreover, the inversely proportional dependence announced by Ibadinov (1999) results in an infinitely fast sublimation in the case of pure ice. In this paper, two elementary functions will be considered: linear (equal to Shul'man's assumption that crust formation finishes when the volume, containing a coating area equal to nucleus area, evaporates), and exponentially decreasing.

Geometrical area depends on size and shape. It is known that the shape of comet nuclei moving in elliptical orbits remains nearly constant (Medvedev, 1996). It also allows us to have a smaller number of model parameters.

As the mass ejection by the comet is not strictly directed, the Meshchersky equation gives the reactive force $\vec{F}_r$ acting on comet of mass m:

$$\vec{F}_r = \lambda \vec{u} \frac{dm}{dt}, \tag{1}$$

where as in Marsden's model (Marsden et al., 1973) the anisotropy coefficient λ and vector of escaping matter velocity relative to the comet in orbital coordinate system $\vec{u}$ are constant. A number of particles

$$N = \frac{mN_A}{M} \tag{2}$$

(where N_A is Avogadro's number, M is molecular mass), ejected per unit time from a unit pure area, is given in Marsden's model as

$$-\frac{1}{\beta S}\frac{dN}{dt} \equiv Z = Z_0 g(r(t)), \tag{3}$$

where S is geometrical area, $0 \leqslant \beta \leqslant 1$ is a ratio of effective area to geometrical area, $r(t)$ is the heliocentric distance of the comet (in AU),

$$g(r) = 0.111262 \cdot 10^{-8}(r/2.808)^{-2.15}\left(1+(r/2.808)^{5.093}\right)^{-4.6142}. \tag{4}$$

During the preliminary calculation we can replace $g(r)$ with the mean value that depends on the size and shape of the orbit: $\langle g(r(t))\rangle = g(a,e)$. So we have the components of reactive acceleration in an orbital coordinate system ($i = 1,\ 2,\ 3$ are for the radial, transverse and normal directions):

$$w_i = -\frac{\lambda u_i M Z_0 \beta S}{N_A m} g(r). \tag{5}$$

We can derive Marsden's parameters by definition (in AU/(10^4 days)^2) :

$$A_i = -\frac{\lambda u_i M Z_0 \beta S}{N_A m}. \tag{6}$$

2.1. Model 1

Let there be a deposition of significant non-volatile mass that does not impede the sublimation. Then in Eqs. (1)–(6) we take $\beta = 1$. If the nucleus does not change its shape, then

$$\frac{S}{m_{ice}} = \frac{\varphi}{\rho_{ice} R}, \tag{7}$$

where φ depends on the shape ($\varphi = 3$ for a sphere), ρ_{ice} and m_{ice} are density and mass of ice, R is its mean radius, defined by:

$$R = \sqrt[3]{\frac{3m_{ice}}{4\pi\rho_{ice}}}. \tag{8}$$

Analogously using the mass of a non-volatile substance $m_{end} = m - m_{ice}$ we can define:

$$R_{end} = \sqrt[3]{\frac{3m_{end}}{4\pi\rho_{end}}}. \tag{9}$$

From this we can write the Marsden's parameters:

$$A_i = -\frac{\lambda u_i M Z_0 \varphi}{N_A \rho_{ice}} \frac{R^2}{R^3 + (\rho_{end} R_{end}^3/\rho_{ice})}, \tag{10}$$

and denoting A_i and R values at some time t_0 by A_{i0} and R_0, we obtain the following ratio:

$$\frac{A_i}{A_{i0}} = \frac{R^2}{R_0^2} \frac{R_0^3 + (\rho_{end} R_{end}^3/\rho_{ice})}{R^3 + (\rho_{end} R_{end}^3/\rho_{ice})}. \tag{11}$$

Combining Eqs. (2), (3), (7) and (8) when $\beta = 1$ we obtain:

$$\frac{dR}{dt} = -\frac{\varphi M Z_0}{3N_A \rho_{ice}} g(r). \tag{12}$$

Denoting in Eqs. (11) and (12) $\sqrt[3]{\frac{\rho_{ice}}{\rho_{end} R_{end}^3}} R = \chi \geqslant 0$ and $\frac{\varphi M Z_0}{3N_A \rho_{ice}^{2/3} \rho_{end}^{1/3} R_{end}} = \alpha \geqslant 0$ we can write:

$$\begin{cases} A_i = A_{i0} \frac{\chi^2(\chi_0^3+1)}{\chi_0^2(\chi^3+1)}, \\ \frac{d\chi}{dt} = -\alpha g(r(t)), \\ \alpha = const. \end{cases} \tag{13}$$

It is natural that while using these equations we should take into consideration that χ negative values make no sense. When $\chi = 0$ (and therefore $A_i = 0$), χ (and therefore A_i) stops changing.

2.2. Model 2

Let there be a growth of the low-mass crust that reduces the effective area linearly in thickness. Then Eqs. (1)–(6) are valid. Analogously to Eq. (7):

$$\frac{S}{m} = \frac{\varphi}{\rho R}, \tag{14}$$

where ρ is density of the nucleus (which is supposed as initially homogeneous), and its mean radius is

$$R = \sqrt[3]{\frac{3m}{4\pi\rho}}. \tag{15}$$

From this we can write Marsden's parameters:

$$A_i = -\frac{\lambda u_i M Z_0 \varphi \beta}{N_A \rho R}. \tag{16}$$

Denoting A_i, R and β at the moment t_0 by A_{i0}, R_0, β_0 we make a combination:

$$\frac{A_i}{A_{i0}} = \frac{R_0 \beta}{\beta_0 R}. \tag{17}$$

According to our assumption,

$$\beta = 1 - h/H, \tag{18}$$

where H is crust thickness that terminates the sublimation,

$$h = \frac{(m_{begin} - m) f/\rho}{S} \tag{19}$$

is its current thickness, m_{begin} is initial mass, f is the bulk part of the non-volatile substance. Taking into consideration Eqs. (14) and (15) in Eq. (19), we obtain:

$$h = \frac{f}{\varphi} \frac{R_{begin}^3 - R^3}{R^2}. \tag{20}$$

Substituting to Eq. (18) into Eq. (20) and R_{begin} expressed from these equations at $t = t_0$ we obtain:

$$\beta = 1 + \frac{f}{\varphi H} \frac{R^3 - R_0^3}{R^2} - \frac{R_0^2(1-\beta_0)}{R^2}. \tag{21}$$

Combining Eqs. (2), (3), (14) and (15) we have:

$$\frac{dR}{dt} = -\frac{\varphi M Z_0}{3N_A \rho} \beta g(r). \tag{22}$$

Denoting $\frac{3f}{\varphi H} R = \chi$ and $\frac{M Z_0 f}{N_A \rho H} = \alpha$ in Eqs. (17), (21) and (22) we obtain a set:

$$\begin{cases} A_i = A_{i0} \frac{\chi_0 \beta}{\beta_0 \chi}, \\ \beta = 1 + \frac{\chi^3 - \chi_0^3}{3\chi^2} - \frac{\chi_0^2(1-\beta_0)}{\chi^2}, \\ \frac{d\chi}{dt} = -\alpha \beta g(r(t)), \\ \alpha = const. \end{cases} \tag{23}$$

Using these equations we should take into consideration that $\beta > 1$ values make no sense. If the model is developed correctly, then by the time the comet reaches a short-period orbit with perihelion inside the asteroid belt, it should have $\beta \leqslant 1$, and before that ice sublimation is small and change of β is insignificant. However, these sharp changes of orbit happen during the close approach to a large planet, when motion is unstable and it is complicated to determine whence the comet came. Negative values of β and χ make no sense as well.

2.3. Model 3

Let us suppose that there is a growth of the low-mass crust, which reduces the effective area exponentially in thickness:

$$\beta = \exp(-h/H), \tag{24}$$

where H is the characteristic thickness of sublimation reduction. Eqs. (1)–(6) are valid. Eqs. (14)–(17) and (19)–(20) from Model 2 are valid as well. Analogous to Eqs. (21)–(23), we have:

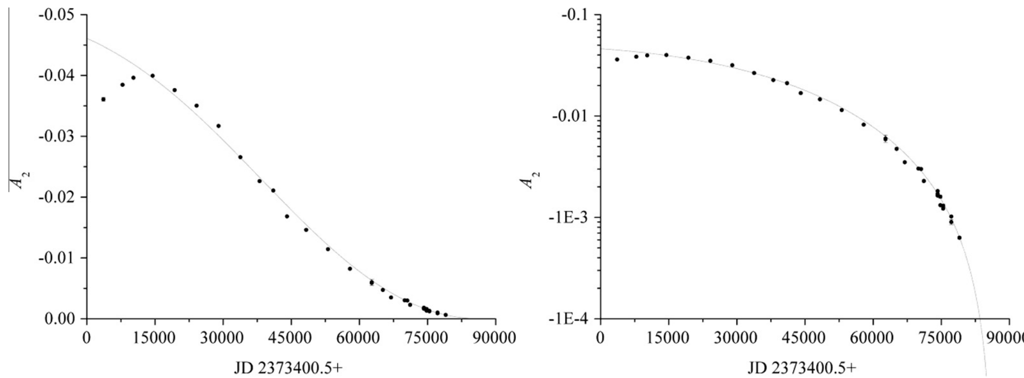

Fig. 1. The formal solution of the set (13) for the comet Encke in linear and logarithmic scales.

Table 1
Parameters of the solution in Fig. 1 for the start and end dates of the model.

Year	A_2	α, day^{-1}	χ
1786	−0.0461	$3.24\cdot10^{-5}$	0.945
2032	0		0

Table 2
Parameters of the solution in Fig. 2 for the start and end dates of the model.

Year	A_2	α, day^{-1}	β	χ
1786	−0.0372	$3.32\cdot10^{-5}$	0.99	0.637
2032	−0.000455		0.00547	0.288

$$\begin{cases} A_i = A_{i0}\frac{\chi_0\beta}{\beta_0\chi}, \\ \beta = \exp\left(\frac{\chi^3-\chi_0^3+3\chi_0^2\ln\beta_0}{3\chi^2}\right), \\ \frac{d\chi}{dt} = -\alpha\beta g(r(t)), \\ \alpha = const. \end{cases} \tag{25}$$

For values β and χ there are the same restrictions imposed as in Model 2.

3. Formal solutions (representation of non-gravitational parameters) for comet Encke

As it is known, when combining 3–5 comet apparitions, A_2 is the most reliably determined non-gravitational parameter (though for physical reasons the most stable parameter is supposed to be A_1, which represents the main component of non-gravitational acceleration). For preliminary estimating parameters of the considered models, applied to comet Encke, values of A_2, calculated by the observations in different groups of comet apparitions together with their root mean square errors, from different sources (e.g. Marsden and Sekanina, 1974) were collected. If a root mean square error was not explicitly present in a source, then it is considered as a half of the last decimal. As Sekanina, from whose works (Marsden and Sekanina, 1974) half of the A_2 values used here are taken, noticed: the true uncertainty may be several times larger than formal accuracy (Whipple and Sekanina, 1979). The values of Marsden's parameters are supposed to be referred to the middle of the observation periods from which they were determined (Sekanina, 1969; Marsden and Sekanina, 1974).

Finding formal solutions of the equations derived above is fitting values of α, β, χ parameters that minimise the sum of the squares of ratios of $A_2(t)$ deviations from the literature values to their root mean square errors. The task can be simplified by taking into consideration that perturbations of comet Encke's orbit, though strongly influencing

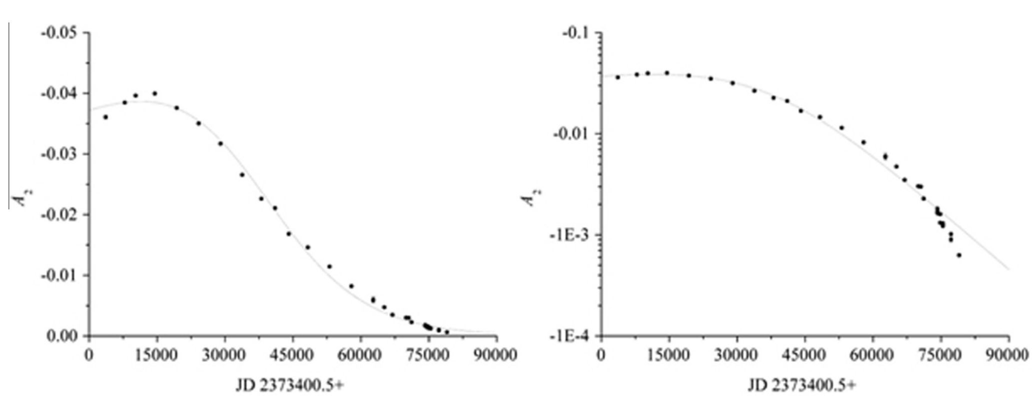

Fig. 2. The formal solution of the set (23) for the comet Encke in linear and logarithmic scales.

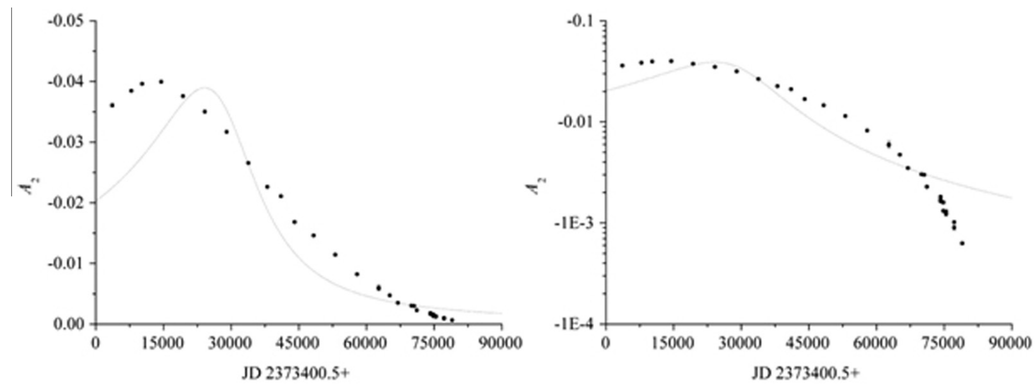

Fig. 3. The formal solution of the set (25) for the comet Encke in linear and logarithmic scales.

Table 3
Parameters of the solution in Fig. 3 for the start and end dates of the model.

Year	A_2	α, day^{-1}	β	χ
1786	−0.0203	$1.21{\cdot}10^{-5}$	0.99	0.127
2032	−0.00177		0.00910	0.0134

its apparent positions, were not significant for thermal conditions. Then we can substitute orbital elements a, e with some mean values (in this case 2.21723 AU and 0.84772), and calculate the time average function value $g(r)$ along the ellipse, replacing the function during the calculations. Finding these solutions, the date January 18, 1786 was taken as the reference point. Historically the accuracy of the observations has increased, so the accuracy of A_2 determination has increased as well. Comet Encke during this period had a decrease of the absolute value of A_2. With such a combination, a more correct representation of the precision of the model gives the $A_2(t)$ curve on a logarithmic scale.

3.1. Model 1

The solution of Eq. (13) obtained using this approach is shown in Fig. 1; its parameters are in Table 1. As can be seen, it adequately represents A_2 during a period of 170 years to the present. Extrapolation shows that at about 2022 A_2 will have reached zero, meaning that the comet will have become extinct.

3.2. Model 2

The solution of Eq. (23) is shown in Fig. 2; its parameters are in Table 2. It was necessary to use $\beta \leqslant 1$ restriction when we were finding this solution. The model adequately represents A_2 for more than 150 years. Significant deviations take place in the modern epoch and the switch to a faster change of parameter is sharp. This can be explained, for example, by the confinement of smaller than hitherto particles in the growing crust (Shul'man, 1972). In the initial period, the values of β were extremely large, i.e. in this model the nucleus surface appeared to be initially almost pure.

3.3. Model 3

The solution of Eq. (25) is shown in Fig. 3; its parameters are in Table 3. It can be seen that the model only shows the change of A_2 qualitatively (i.e. a pronounced peak and slow decline), so this model can be excluded from further consideration. This shows the importance of correctly choosing the coefficient of sublimation as a function of crust thickness.

4. Summary and conclusions

The problem of constructing a periodic comets' motion theory that would connect all their apparitions is not yet solved. The reasons for that are non-gravitational effects of reactive nature. Firstly, that refers to comet Encke, which has a small perihelion distance and a long observation history.

In this article the equations, representing a modification of Marsden's standard model for the case of Marsden's parameters change caused by the comet's activity fading, are derived. It is shown that in case of comet Encke the erosion cannot be homogeneous. 2 scenarios are considered: the accumulation of significant non-volatile mass and the surface crust formation. A formal solution of the derived equations shows that the first scenario better explains the characteristic features of comet Encke's transversal non-gravitational parameter change with time. It also predicts the upcoming completion of the comet's activity. However the second scenario cannot be excluded either. We have found in this case the sublimation weakening with increasing crust's thickness might be close to linear rather than exponential. Further we can apply the equations of ours to simulate astrometric positions of a comet.

The considering that works on studying the motion of comets and meteors are conventional in Kazan Federal University (KFU) (Rizvanov and Nefedjev, 2005; Rizvanov et al., 2007; Sokolova et al., 2013, 2014), this work was funded by the subsidy allocated to KFU for the state assignment in the sphere of scientific activities and grant RFBR 15-02-01638-a.

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