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Analytical Solution of Direct and Inverse Problems in the Internal Gravitational Waves Studies by the Doppler Frequency Shift Method

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Abstract—An analytical solution of direct and inverse problems arising in the study of the internal gravitational waves (IGWs) dynamic via recording of the Doppler frequency shift, is presented. The direct problem is to determine the response of the Doppler shift to IGWs in the region of the radio wave reflection point; the inverse problem is the determination of IGW parameters from data on the Doppler frequency shift. Solutions were obtained in an approximation of the isothermal ionosphere for the heights of the *F*-region. They are presented in a form convenient for their practical use and can have a wide range of applications, including the detection of soliton-like wave structures in the *F*-region of the ionosphere.

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1. INTRODUCTION

During experimental studies of the ionospheric dynamic processes by various sounding methods, one of the main conceptual problems is an adequate interpretation of the variations in the detected signal. In particular, since the signal in most cases reflects fluctuations in the electron concentration, which are associated with the dynamics of ionospheric heterogeneities (moving ionospheric disturbances, MIDs), the source of the generation of such MIDs must be determined, and the role of that source can be taken on by internal gravitational waves (IGWs) (Bryunelli and Namgaladze, 1988; Hocke and Schlegel, 1996) and acoustic gravitational waves (AGWs) (Grigor'ev, 1999; Nagorskii, 1999;) in the neutral component, which, in turn, are excited by impulse type sources of different natures (Belashov, 1989; Belashov, 1990; Pertsev and Shalimov, 1996; Drobzheva and Krasnov, 2003; see also Belashov and Vladimirov, 2005). For example, theoretical studies (Belashov, 1992, 2006; Belashov et al., 2006; Belashova et al., 2007) first predicted phenomena such as the generation of two-dimensional IGWs solitons in the regions of steep gradients of the main ionospheric parameters (on the fronts of solar terminator and solar eclipse spot). These received qualitative confirmation in the experiments on ionospheric sounding (Belashov and Poddelsky, 1992; Galushko et al., 2007; Nasyrov et al., 2016; Nasyrov et al., 2017). However, it is well known that there are no direct methods for the measurement of neutral component dynamic parameters, i.e. IGW characteristics, on ionospheric heights. Thus, the calculation of quantitative dynamic IGW characteristics from the measured data is definitely still relevant at present.

Let us consider here the problem set above on the example of one of the most effective (from the point of view of the dynamic process study method) of the ionosphere-sounding method of recording the Doppler frequency shift (DFS) of the reflected main pulse. Please note that there are two types of problems in IGW studies by the DFS method that coexist and compliment each other:

—study of the DFS response to IGWs with a priori known parameters (e.g., under a known or modeled mechanism of the IGW excitation), i.e., the direct problem, and

—interpretation of the recorded IGWs in terms of wave disturbances of the neutral component, i.e., the inverse problem.

In the current research, problems of both types are considered analytically, and their solutions are obtained in a form that is convenient for practical use. Despite of the work in which the problem of DFS reconstruction of the neutral plasma component velocity field was raised for the first time (Savel'ev, 1987), we will not introduce here restrictions on the spatial dimensions of the disturbances and consider the DFS variations for IGWs moving at a low-angle to the horizontal.

2. DIRECT AND INVERSE PROBLEMS

In the isotropic case, disturbance of the radio wave phase $\delta\varphi = -(\omega/c)\int_L \delta n dl$ leads to a Doppler frequency shift

$$\omega_D = -\frac{\omega}{c}\int_L \frac{dn}{dN}\frac{\partial}{\partial t}\delta N dl, \tag{1}$$

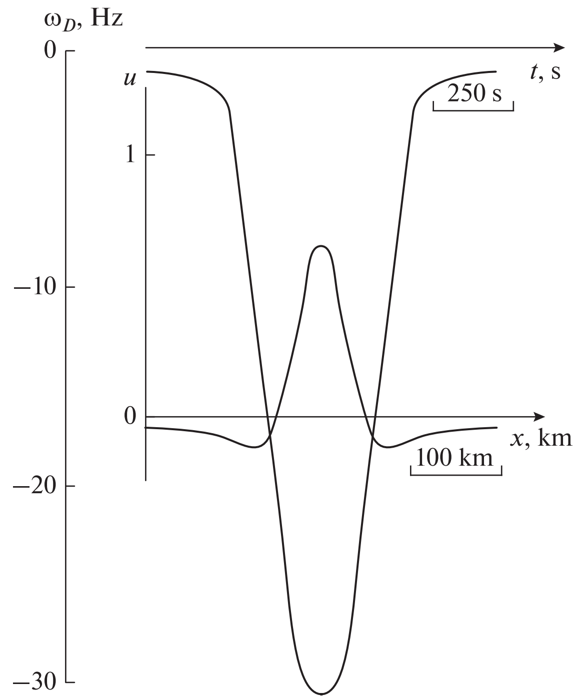

Fig. 1. Profile $u = u_z/\sqrt{gH}$ for the soliton IGW KP equation for the upper atmosphere and corresponding DFS variation.

where $n = \sqrt{1 - N/N_\omega}$ is the refraction index and N and δN are the electron concentration and its disturbance. Considering region F of the ionosphere, where oxygen atom and ion are dominant in the neutral and ionic compound respectively, let us write an equation of continuity for N in the form (Belashov, 1990)

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial z}\left[\left(\frac{\partial N}{\partial z} + \frac{N}{2H_i}\right) D_0 e^{z/H_i} - u_z\left(1 - e^{-\nu t'}\right) N \sin I \cos I\right] - \beta N + Q, \tag{2}$$

where u_z is vertical component of the neutral particle velocity; ν is the delay constant in the disturbance of the ionized component relating to the neutral one; $t' = t - t_0$, t_0 is the time of disturbance start in the neutral component; H_i is the height of the homogeneous atmosphere for ions; I is the magnetic inclination; $D_0 \exp(z/H_i) = D_\alpha \sin^2 I$, D_α is the coefficient of ambipolar diffusion; $\beta = \beta_0 \exp(-Pz/H_i)$, and Q is the coefficient of recombination and velocity of ion formation, respectively.

Approximating the concentration profile of charge particles about a reflection point by the exponent $N = N_0 \exp(z/H_i)$ and integrating the right side of the equation (1), taking into account (2) within the limits of $z = H_i$ to $z = 0$, we obtain for the Doppler frequency shift

$$\omega_D = \left[\beta H_i - \left(3\frac{D_0}{H_i} + \frac{H_i}{N_0} Q\right) e^{-z/H_i}\right] \Delta k_z + H_i\left(1 - e^{-\nu t'}\right) \frac{\sin 2I}{2}\left(\frac{1}{2H} + \frac{1}{H_i}\right) \int_L u_z dk_z, \tag{3}$$

where $z = h - h_0$; $\Delta k_z = -2(\omega/c)\cos\theta\sqrt{1 - N(z)/N_0}$, θ is the angle between wavevector and the vertical at z level. The integral in the right side of (3) can be calculated by asymptotic Taylor expansion with $H_i \to 0$:

$$\int_L u_z dk_z \approx \int_{-H_i}^{0} \left[u_z(\mathbf{r}_0) + (\mathbf{r} - \mathbf{r}_0)\frac{\partial u_z}{\partial \mathbf{r}_0}\right] dk_z(z) = u_z(0)\Delta k_z\left(1 + \frac{\langle z \rangle}{2H}\right),$$

where $\langle z \rangle \approx -0.258H_i$; $\Delta k_z \approx -1.59(\omega/c)\cos\theta$; $u_z(0) = u_z(x, y, t)\big|_{z=0}$. Then, applying $q = Q/\beta N$, from (3) at the level of $z = 0$, which corresponds to a reflection point, we obtain

$$\omega_D = H_i \Delta k_z\left[\beta(1 - q) - 3\frac{D_0}{H_i^2} + \frac{\sin 2I}{2}\left(\frac{1}{2H} + \frac{1}{H_i}\right)\left(1 - 0.129\frac{H_i}{H}\right) u_z(0)\left(1 - e^{-\nu t'}\right)\right]. \tag{4}$$

Equation (4) is the solution of the direct problem formed above. Reversing equations (3), (4), we easily find u_z as a function of ω_D for the inverse problem solution.

Let us write out solutions of the direct and inverse problems in a case of $T_e = T_i$:

$$\omega_D = 2\Delta k_z\left[\beta H(1 - q) - 0.75\frac{D_0}{H} + 0.371 \sin 2I \times u_z\left(1 - e^{-\nu t'}\right)\right],$$
$$u_z = \frac{0.5\omega_D/\Delta k_z + 0.75 D_0/H - \beta H(1 - q)}{0.371 \sin 2I\,[1 - \exp(-\nu t')]}. \tag{5}$$

Let us remark that, at about noon time, when processes of ionization and recombination in ionosphere relatively counterbalance each other, i.e. $q \cong 1$, the equations in (5) become even simpler. During nighttime conditions, ion formation is almost absent, and $q = 0$.

The results of calculations according to formula (4) for conditions in the F-region of the ionosphere correspond to the real conditions when $T_e = T_i$ (solutions (5) are valid), $\Delta k_z = -5.3 \times 10^{-2}$ m^{-1}, $\theta = 0$ (vertical sounding), $I = 63.4°$ (region of the magnetic latitude $\varphi_m = 45°$), $\beta_0 = 3.1 \times 10^{-4}$ s^{-1}, $D_0 = 3.1 \times 10^5$ m^2 s^{-1}, $q = 0$ (night), and when u_z is a 2D soliton solution of the generalized Kadomtsev–Petviashvili equation (KPE) for the upper atmosphere (obtained by Belashov, 1989; see also Belashov and Vladimirov, 2005) are shown in Fig. 1.

Please note that ω_D variations of this type have been repeatedly observed in numerous experiments of Doppler sounding of the ionosphere, which, when the results of this work are taken into account, can be interpreted as an effect caused by moving solitary structures in the *F*-region—2D IGW solitons.

3. CONCLUSIONS

The obtained solutions of the direct and inverse problems are clear and quite convenient for practical use in problems related to the identification of IGW disturbances in the records of the indicated DFS signal during ionospheric Doppler sounding of the ionosphere. This representation of the link between DFS and the vertical component of the neutral particles velocity is convenient, because a preliminary reconstruction of the electron density profile over the disturbance velocity field is not needed to solve the direct problem of computation of the Doppler shift. When solving the inverse problem, we can immediately reconstruct the velocity field of the neutral component from the Doppler shift records—directly from the dopplerogram. The presented particular example clearly illustrates the practical significance of the obtained results in studies of IGWs by the DFS method.

The obtained results are currently of particular importance for the detection of soliton-like wave “forerunners” in the *F*-region, which are generated in regions of steep gradients of the basic ionospheric parameters during the motion of the solar eclipse spot and solar terminator (Belashova et al., 2007; Belashov and Belashova, 2015) and are observed in numerous ionospheric sounding experiments (Belashov and Poddelsky, 1992; Galushko et al., 2007; Nasyrov et al., 2016; Nasyrov et al, 2017). The results can also be useful in for problems related to the impulse impact on the ionosphere of sources such as seismic events (Pertsev and Shalimov, 1996) and land-surface artificial explosions (Drobzheva and Krasnov, 2003).

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