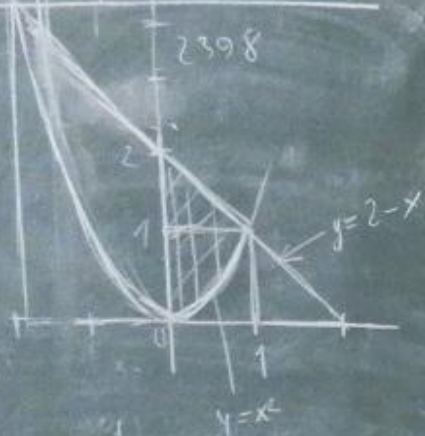


13 марта 2018 г., вт,



$$S = \int_{-2}^1 [2-x-x^2] dx = 2x - \frac{x^2}{2} - \frac{x^3}{3} \Big|_{-2}^1 = \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right) = \frac{7}{6} - \left(-6 + \frac{8}{3}\right) = \frac{7}{6} + 6 - \frac{8}{3} = \frac{7}{6} + \frac{12}{2} - \frac{8}{3} = \frac{7}{6} + \frac{12}{2} - \frac{16}{3} = \frac{7}{6} + \frac{36}{6} - \frac{32}{6} = \frac{11}{6}$$

15^{го}, 1008, МА, р. 09-722(2), Казанск АВ.

$$x^2 = 2 - x$$

$$x^2 + x - 2 = 0$$

$$x = 1$$

$$x = -2$$

2431. $y = x^{\frac{3}{2}}$, $0 \leq x \leq 4$.

$$S = \int_0^4 \sqrt{1 + y'(x)^2} dx = \int_0^4 \sqrt{1 + \frac{9}{4}x} dx$$

$$y'(x) = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$$
$$= \frac{3}{2}\sqrt{t} \quad t = x$$

$$\frac{x^2}{2} - \frac{x^3}{3} \Big|_{-2}^1 = \left(2 - \frac{1}{2} - \frac{1}{3} = \frac{12-3-2}{6} = \frac{7}{6}\right) - \left(-4 - 2 + \frac{8}{3}\right) = \frac{7}{6} - \left(-6 + \frac{8}{3}\right) = \frac{7}{6} + 6 - \frac{8}{3} = \frac{7}{6} + \frac{12}{2} - \frac{8}{3} = \frac{7}{6} + \frac{36}{6} - \frac{32}{6} = \frac{11}{6}$$

15⁷⁰, 1008, МА, р. 09-722(2), Kazanul AB.

$$x^2 = 2 - x$$

$$x^2 + x - 2 = 0$$

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2431. $y = x^{\frac{3}{2}}$, $0 \leq x \leq 4$.

$$S = \int_0^4 \sqrt{1 + y'(x)^2} dx = \int_0^4 \sqrt{1 + \frac{9}{4}x} dx$$

$$y'(x) = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$$
$$= \frac{3}{2}\sqrt{\frac{4}{9}t} = \frac{2}{3}\sqrt{t}$$

$$1 + \frac{9}{4}x = t$$

$$= \frac{4}{9} \int_0^4 \sqrt{1 + \frac{9}{4}x} d(1 + \frac{9}{4}x) =$$
$$= \frac{4}{9} \cdot \frac{2}{3} \cdot t^{\frac{3}{2}} \Big|_1^{10} = \frac{8}{27} (10^{\frac{3}{2}} - 1)$$

$$\frac{x^2}{2} - \frac{x^3}{3} \Big|_2^1 = \left(2 - \frac{1}{2} - \frac{1}{3} = \frac{12 - 3 - 2}{6} = \frac{7}{6} \right) - \left(-4 - 2 + \frac{8}{3} \right) =$$
$$= \frac{7}{6} + 6 - \frac{8}{3} = \frac{7 \cdot 10}{6} + 6 = 6 - \frac{2}{6} =$$
$$= 6 - \frac{3}{2} = 6 - 1 - \frac{1}{2} =$$
$$= 5 - \frac{1}{2} = 4\frac{1}{2}$$

$$\frac{7}{6} + 6 - \frac{8}{3} = \frac{7 \cdot 10}{6} + 6 = 6 - \frac{2}{6} =$$
$$= 6 - \frac{3}{2} = 6 - 1 - \frac{1}{2} =$$
$$= 5 - \frac{1}{2} = 4\frac{1}{2}$$

13 марта 2018 г., бм,

2440. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$



$$\begin{cases} x = a \cos^3 t \\ y = a \sin^3 t \end{cases}$$

$$\frac{1}{4} s = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{x'(t)^2 + y'(t)^2} dt$$

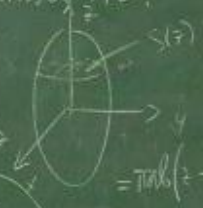
$$\begin{aligned} x'(t) &= -3a \cos^2 t \sin t \\ y'(t) &= 3a \sin^2 t \cos t \end{aligned}$$

15^{го}, 1008, МА, р. 09-722(2), Kazanul AB.

(акупова)

2463. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$$V = \int_{-c}^c S(z) dz = \pi ab \int_{-c}^c \left(1 - \frac{z^2}{c^2}\right) dz = \pi ab \left(z - \frac{z^3}{3c^2}\right) \Big|_{-c}^c = \frac{4\pi abc}{3}$$



$$dt = 3a \int_0^{\pi/2} \sin t \cos t dt = \frac{3a}{2} \sin^2 t \Big|_0^{\pi/2} = \frac{3a}{2}$$

$$\begin{aligned} x'(t)^2 &= 9a^2 \cos^4 t \sin^2 t \\ y'(t)^2 &= 9a^2 \sin^4 t \cos^2 t \end{aligned}$$

s = 6a

$$9a^2 \sin^2 t \cos^2 t (\cos^2 t + \sin^2 t) = 9a^2 \sin^2 t \cos^2 t$$

$$|z| = \text{max} \in (-c, c)$$

$$\frac{x^2}{a^2} + \frac{y^2}{c^2} = 1 - \frac{z^2}{c^2}$$

$$\frac{x^2}{(a\sqrt{1-\frac{z^2}{c^2}})^2} + \frac{y^2}{(c\sqrt{1-\frac{z^2}{c^2}})^2} = 1$$

$$\begin{aligned} S(z) &= \pi AB = \\ &= \pi a c \left(1 - \frac{z^2}{c^2}\right) \end{aligned}$$

$$z_c = \frac{4\pi abc}{3}$$

13 марта 2018 г., бм,

2440. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$



$$\begin{cases} x = a \cos^3 t \\ y = a \sin^3 t \end{cases}$$

$$\frac{1}{4}s = \int_0^{\frac{\pi}{2}} \sqrt{x'(t)^2 + y'(t)^2} dt$$

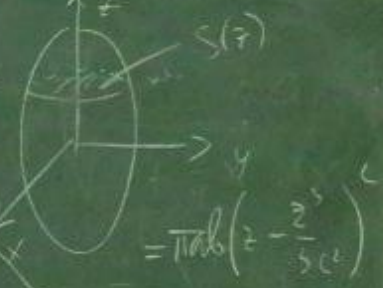
$$\begin{aligned} x'(t) &= 3a \cos^2 t (-\sin t) \\ y'(t) &= 3a \sin^2 t (\cos t) \end{aligned}$$

15^{го}, 1008, МА, р. 09-722(2), Kazanuseb AB.

(ауыпанда)

2463. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$$V = \int_{-c}^c S(z) dz = \pi ab \int_{-c}^c \left(1 - \frac{z^2}{c^2}\right) dz$$



$$dt = 3a \int_0^{\frac{\pi}{2}} \sin t \cos t dt = \frac{3a}{2} \sin^2 t \Big|_0^{\frac{\pi}{2}} = \frac{3a}{2}$$

$$\begin{aligned} x'(t)^2 &= 9a^2 \cos^4 t \sin^2 t \\ y'(t)^2 &= 9a^2 \sin^4 t \cos^2 t \end{aligned}$$

s = 6a

$$9a^2 \sin^2 t \cos^2 t (\underbrace{\cos^2 t + \sin^2 t}_1) = 9a^2 \sin^2 t \cos^2 t$$

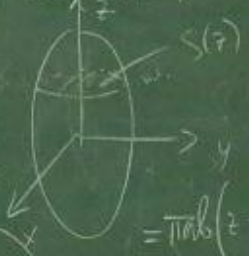
$$\begin{aligned} &= \pi ab \left(z - \frac{z^3}{3c^2} \right) \Big|_{-c}^c \\ &= 2\pi ab \left(c - \frac{c^3}{3c^2} \right) \\ &= \frac{2\pi ab}{3} \end{aligned}$$

15⁷⁰; 1008, MA, sp. 09-722(2), Kazanul AB.

(демпонда)

21463. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$V = \int_{-c}^c S(z) dz = \pi ab \int_{-c}^c \left(1 - \frac{z^2}{c^2}\right) dz = \pi ab \left(z - \frac{z^3}{3c^2}\right) \Big|_{-c}^c = \frac{4\pi abc^2}{3}$



$dt = 3a \int_0^{\pi/2} \sin t \cos t dt = \frac{3a}{2} \sin^2 t \Big|_0^{\pi/2} = \frac{3a}{2}$

$x'(t)^2 = 9a^2 \cos^4 t \sin^2 t$
 $y'(t)^2 = 9a^2 \sin^4 t \cos^2 t$

$\frac{9a^2 \sin^2 t \cos^2 t (\cos^2 t + \sin^2 t)}{9a^2 \sin^2 t \cos^2 t} = 9a^2 \sin^2 t \cos^2 t$

s = 6a

$\pi ab \left(c - \frac{c^3}{3c^2}\right) = 2\pi ab \left(c - \frac{c}{3}\right) = \frac{2\pi abc}{3}$

|z-фукс (-c, c)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{z^2}{c^2}$

$\frac{x^2}{\left(a\sqrt{1-\frac{z^2}{c^2}}\right)^2} + \frac{y^2}{\left(b\sqrt{1-\frac{z^2}{c^2}}\right)^2} = 1$

$S(z) = \pi AB = \pi ab \left(1 - \frac{z^2}{c^2}\right)$
 $\frac{4\pi abc}{3}$

13 марта 2018 г. бм

2404. $y^2 = x^2(a^2 - x^2)$

2488. $y = \tan x, 0 \leq x \leq \frac{\pi}{4}$

$P = 2 \int_0^{\frac{\pi}{4}} |\tan x| ds$

$ds = \sqrt{1 + y'(x)^2} dx$

15^{го}, 1008, МА, кв. 09-722(2), Kazanuch AB

- площадь лоб.

$\frac{\pi}{4}$, $\tan x = 1$



$y(x)^2 dx = \sqrt{1 + \frac{1}{\cos^2 x}} dx$

$d' = \frac{1}{\cos x}$

$P = 2\pi \int_0^{\frac{\pi}{4}} \tan x \cdot \frac{\sqrt{1 + \frac{1}{\cos^2 x}}}{\cos^2 x} dx$

$1 + \tan^2 x = \frac{1}{\cos^2 x} = \sec^2 x$

$\cos^2 x = \frac{1}{1 + \tan^2 x}$

$\cos x = \frac{1}{\sqrt{1 + \tan^2 x}}$

$= \pi \int_0^{\frac{\pi}{4}} \frac{1}{\sqrt{1 + \tan^2 x}} dx$

$= \pi \int_0^1 \frac{1}{\sqrt{1 + t^2}} dt$

- гиперболический синус:

$= 2\pi \int_0^{\frac{\pi}{4}} \tan x \sqrt{1 + \frac{1}{(1 + \tan^2 x)^2}} dt x$

$= 2\pi \int_0^1 t \sqrt{1 + \frac{1}{(1+t^2)^2}} dt =$

$= \pi \int_0^1 \sqrt{1 + \frac{1}{(1+t^2)^2}} d(1+t^2) =$

$= \pi \int_1^2 \sqrt{1 + \frac{1}{u^2}} du =$

$= \pi \int_1^2 \frac{\sqrt{1+u^2}}{u} du$

$= \pi \left(\frac{1}{2} \ln \frac{1+\sqrt{1+u^2}}{1-\sqrt{1+u^2}} + \sqrt{1+u^2} \right) \Big|_1^2$

13 марта 2018 г., вт,

2404. $y^2 = x^2(a^2 - x^2)$

2488. $y = \tan x, 0 \leq x \leq \frac{\pi}{4}$

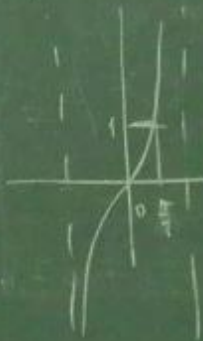
$$P = 2 \int_0^{\frac{\pi}{4}} |\tan x| ds$$

$$ds = \sqrt{1 +$$

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→ расколоть.

$\frac{\pi}{4}$, $\cos^2 x$ 0x



$$y'(x)^2 dx = \sqrt{1 + \frac{1}{\cos^4 x}} dx$$

$$y' = \frac{1}{\cos^2 x}$$

$$P = 2\pi \int_0^{\frac{\pi}{4}} \tan x \cdot \frac{\sqrt{1 + \cos^4 x}}{\cos^2 x} dx$$

$$1 + \cos^4 x = \frac{1}{\cos^2 x} = \frac{1}{1 + \tan^2 x}$$

$$\cos^2 x = \frac{1}{1 + \tan^2 x}$$

$$\cos^4 x = \frac{1}{(1 + \tan^2 x)^2}$$

$$= \pi \int_0^{\frac{\pi}{4}} \frac{1}{1 + \tan^2 x} dx$$

$$= \pi \int_0^{\frac{\pi}{4}} \frac{\cos^2 x}{1 + \tan^2 x} dx = \pi \int_0^{\frac{\pi}{4}} \frac{\cos^2 x}{\sec^2 x} dx = \pi \int_0^{\frac{\pi}{4}} \cos^4 x dx$$

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+ рассогласов.

$\frac{\pi}{4}$, $\cos^2 x$



$$y'(x)^2 dx = \sqrt{1 + \frac{1}{\cos^4 x}} dx$$

$$y' = \frac{1}{\cos^2 x}$$

$$P = 2\pi \int_0^{\frac{\pi}{4}} \frac{\sqrt{1 + \cos^4 x}}{\cos^2 x} dx$$

$$1 + \cos^4 x = \frac{1}{\cos^2 x}$$

$$\cos^2 x = \frac{1}{1 + \tan^2 x}$$

$$\cos^4 x = \frac{1}{(1 + \tan^2 x)^2}$$

$$= \pi \int_0^2 t^{-1} (1+t^2)^{-2} dt$$

$$= \pi \int_0^2 \frac{1}{t^2} \frac{1}{(1+t^2)^2} dt = \pi \int_0^2 \frac{1}{t^2(1+t^2)^2} dt$$

$$= 2\pi \int_0^{\frac{\pi}{4}} \frac{\sin x \sqrt{1 + \cos^4 x}}{\cos^2 x} dx$$

гиперболическая замена

$$= \pi \int_0^{\frac{\pi}{4}} \sqrt{1 + \frac{1}{\cos^4 x}} d\left(\frac{1}{\cos^2 x}\right)$$

$$= \pi \int_0^2 \sqrt{1 + \frac{1}{t^2}} dt$$

+ гиперболическая замена:

$$= 2\pi \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x} \sqrt{1 + \frac{1}{(1 + \tan^2 x)^2}} dt =$$

$$= 2\pi \int_0^1 t \sqrt{1 + \frac{1}{(1+t^2)^2}} dt =$$

$$= \pi \int_0^1 \sqrt{1 + \frac{1}{(1+t^2)^2}} d(1+t^2) =$$

$$= \pi \int_1^2 \sqrt{1 + \frac{1}{u^2}} du =$$

$$= \pi \int_1^2 \frac{\sqrt{1+u^2}}{u} du = \pi \int_1^2 \frac{d\sqrt{1+u^2}}{\sqrt{1+u^2}}$$

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$$\pi \int_1^2 \frac{(1+t^2)^{\frac{1}{2}}}{t} dt = \quad t = \operatorname{tg} v$$

$$= \pi \int_{\frac{\pi}{4}}^{\operatorname{arctg} 2} \frac{1}{\sin v \cos v} dv$$

$t = \operatorname{tg} v$
 $1 = \operatorname{tg} \frac{\pi}{4}$
 $2 = \operatorname{tg}(\operatorname{arctg} 2)$

Ответ:

$$\pi \left[\sqrt{5} - \sqrt{2} + \ln \frac{(\sqrt{2}+1)(\sqrt{5}-1)}{2} \right]$$

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$$\pi \int_{\frac{\pi}{4}}^{\operatorname{arctg} 2} \frac{1}{\sin v \cos v} dv = \quad u = \cos v$$

$$= -\pi \int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{5}}} \frac{d \cos v}{(1-\cos^2 v) \cos v} = -\pi \int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{5}}} \frac{1}{(1-u^2)u} du = \pi \int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{5}}} \left(\frac{1}{1-u^2} + \frac{1}{u} \right) du$$

$$1 + \operatorname{tg}^2 v = \frac{1}{\cos^2 v} \quad \cos^2 = \frac{1}{5}$$

$$= \pi \int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{5}}} \frac{1}{1-t^2} dt = \pi \int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{5}}} \frac{1}{2(1-t)(1+t)} dt = \pi \int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{5}}} \frac{1}{2} \left(\frac{1}{1-t} - \frac{1}{1+t} \right) dt$$

$$\frac{1}{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \frac{1}{2} \ln \frac{\sqrt{5}+1}{\sqrt{5}-1} = \ln \frac{(\sqrt{2}+1)(\sqrt{5}-1)}{2}$$

$$= \frac{1}{2} \ln \frac{(\sqrt{2}+1)(\sqrt{5}-1)}{2} = \ln \frac{(\sqrt{2}+1)(\sqrt{5}-1)}{2}$$

$$= \pi \left(\frac{1}{2} \ln \frac{1+t}{1-t} - \frac{1}{u} \right) \Big|_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{5}}} = \pi \left(\frac{1}{2} \ln \frac{1+\frac{1}{\sqrt{5}}}{1-\frac{1}{\sqrt{5}}} - \frac{1}{\sqrt{5}} - \left(\frac{1}{2} \ln \frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}} - \frac{1}{\sqrt{2}} \right) \right)$$

13, марта 2018 г., бм,

$$\pi \int_1^2 \frac{(1+t^2)^{\frac{1}{2}}}{t} dt = \quad t = \operatorname{tg} v$$

$$t = \operatorname{tg} v$$

$$1 = \operatorname{tg} \frac{\pi}{4}$$

$$2 = \operatorname{tg}(\operatorname{arctg} 2)$$

$$= \pi \int_{\frac{\pi}{4}}^{\operatorname{arctg} 2} \frac{\operatorname{arctg} 2}{\frac{1}{\cos^2 v}} dv$$

Ответ:

$$\pi \left[\sqrt{5} - \sqrt{2} + \ln \frac{(\sqrt{2}+1)(\sqrt{5}-1)}{2} \right]$$

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$$\pi \int_{\frac{\pi}{4}}^{\operatorname{arctg} 2} \frac{\frac{1}{\cos v}}{\frac{\sin v}{\cos v}} \cdot \frac{1}{\cos^2 v} dv =$$

$$\frac{\operatorname{arctg} 2}{\sin v \cos^2 v} = -\pi \int_{\frac{\pi}{4}}^{\operatorname{arctg} 2} \frac{d \cos v}{(1-\cos^2 v) \cos^2 v} = -\pi \int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{5}}} \frac{\frac{1}{\sqrt{c}}}{(1-u^2) u^2} du = \pi \int_{\frac{1}{\sqrt{5}}}^{\frac{1}{\sqrt{2}}} \left(\frac{1}{1-u^2} + \frac{1}{u^2} \right) du$$

$$= \pi \left(\frac{1}{2} \ln \dots \right)$$

$$1 + \frac{t^2}{4} = \frac{1}{\cos^2 v} \quad \cos^2 = \frac{1}{4}$$

$$= \pi \int_1^2 \frac{1}{t^2} (1+t^2)^{\frac{1}{2}} dt$$

$$= \pi \int_{\frac{\pi}{4}}^{\operatorname{arctg} 2} \frac{\frac{1}{\cos v}}{\frac{\sin v}{\cos v}} \cdot \frac{1}{\cos^2 v} dv = \pi \int_{\frac{\pi}{4}}^{\operatorname{arctg} 2} \frac{1}{\sin v \cos^2 v} dv$$

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$$\pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\cos v} \cdot \frac{1}{\cos^2 v} dv = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\sin^2 v \cos^2 v} dv$$

$\xrightarrow{u = \cos v}$ $\xrightarrow{du = -\sin v dv}$ $\xrightarrow{u = \frac{1}{\sqrt{2}}}$

$$= -\pi \int_{\frac{1}{\sqrt{2}}}^0 \frac{d \cos v}{(1 - \cos^2 v) \cos^2 v} = -\pi \int_{\frac{1}{\sqrt{2}}}^0 \frac{du}{(1-u^2)u^2} = \pi \int_{\frac{1}{\sqrt{2}}}^0 \left(\frac{1}{1-u^2} + \frac{1}{u^2} \right) du$$

$$1 + \frac{1}{4}v = \frac{1}{\cos^2 v} \quad \cos^2 = \frac{1}{4}$$

$$= \pi \int_1^2 \frac{1}{t^2(1+t^2)^2} dt$$

$$= \pi \int_1^2 \frac{1}{t^2} - \frac{2}{1+t^2} + \frac{1}{1+t^2} dt$$

$$\frac{1}{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \frac{1}{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} =$$

$$= \frac{1}{2} \ln \frac{(\sqrt{2}+1)^2 (\sqrt{2}-1)^2}{(\sqrt{2}-1)(\sqrt{2}+1) (\sqrt{2}+1)(\sqrt{2}-1)} = \ln \frac{[(\sqrt{2}+1)(\sqrt{2}-1)]^2}{4}$$

$$= \ln \frac{1}{4} = -\ln 4$$

$$= \pi \left(\frac{1}{2} \ln \frac{1+u}{1-u} - \frac{1}{u} \right) \Big|_{\frac{1}{\sqrt{2}}}^0 =$$

$$= \pi \left(\frac{1}{2} \ln \frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}} - \sqrt{2} \right) - \pi \left(\frac{1}{2} \ln \frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}} - \sqrt{2} \right)$$

$$= \pi \int_1^2 \frac{1}{t^2} - \frac{2}{1+t^2} + \frac{1}{1+t^2} dt$$

$$= \pi \left(-\frac{1}{t} - 2 \arctan t + \arctan t \right) \Big|_1^2$$

$$= \pi \left(-\frac{1}{2} - \arctan 2 + \arctan 1 + 1 + \arctan 1 - \arctan 2 \right)$$