Improving the Calculation of Variable Cross Section Compressed Wooden Bars Stability

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Abstract. The article presents a method for determining critical loads for centrally compressed wooden bars with a cross-sectional stiffness variable along the length. Rectangular cross-section bars are considered, the section height of which varies according to a linear law, and the width is constant. The solution is carried out in the elastic formulation. To obtain a solution that is valid for an arbitrary bar geometry, dimensionless parameters are introduced. When solving the stability problem, the basis is the static Euler criterion. The solution of the main resolving equation is performed numerically by the finite difference method. As a result, the task is reduced to the problem of matrix eigenvalues. The implementation of the calculation is performed in the MATLAB environment. Comparisons are made with the current Russian standards for the design of wooden structures. In Russian design standards, a controversial provision is that the variable stiffness of the bars is estimated by the same calculation formula, regardless of the type of fastening. It has been established that the error of Russian design codes for a bar hinged at the ends can in some cases exceed 5%. Corrected calculation formulas suitable for engineering calculations are proposed. In addition, to confirm the reliability of the results, a finite element analysis is performed in the LIRA-SAPR software package.

Keywords: Wood · Bar · Stability · Buckling · Variable Stiffness · Finite Difference Method · Finite Element Method

1 Introduction

Wood has been one of the main structural materials in construction for many centuries, due to its high physical, mechanical and technical qualities. At the present stage of the construction industry development, there has been significant progress in the field of design and construction of buildings and structures using wood due to the use of glued wooden structures [1–3].

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To improve design solutions and reduce the material consumption of construction, it is necessary to develop scientifically based methods for calculating and optimizing building structures. In many designs, elements with a constant cross-sectional geometry along the length are used, however, for reasons of reducing material consumption, in some cases it is advisable to use elements of variable stiffness [4–9].

A lot of works are devoted to the solution of problems for compressed elements stability with a section variable along the length, including [10–14]. In the current Russian standards for the design of wooden structures (SP 64.13330.2017), for compressed elements with a cross-sectional height that varies along the length, the variable stiffness is taken into account by the coefficient k_{zhN}, which for various fastening options is proposed to be determined using the same calculation formula, with which one cannot agree.

The purpose of this work is to improve the normative methods for calculating the stability of compressed structural elements made of wood with a section variable in length.

2 Methods

We consider a centrally compressed rack hinged at the ends with a rectangular cross section (Fig. 1), the height of which varies according to a linear law. The law of height change can be written as:

$$h(x) = h_0\left(\beta + \frac{(1-\beta)x}{l}\right). \tag{1}$$

We assume that the rack material is elastic. To determine the critical load, we use the differential equation for the buckling of the bar:

$$EI(x)\frac{d^2w}{dx^2} + Fw = 0 \tag{2}$$

with the boundary conditions $w(0) = w(l) = 0$.

To make the solution valid for an arbitrary bar geometry, we introduce a dimensionless coordinate $\xi = x/l$, $\xi \in [0;1]$. Then expression (1) will take the form:

$$h(x) = h_0(\beta + (1-\beta)\xi) = h_0\varphi(\xi). \tag{3}$$

In case of stability loss in the xz plane, in Eq. (2) as the axial moment of inertia, the moment of inertia I_{y} should be substituted, which is determined by the formula:

$$I_y(\xi) = \frac{bh^3}{12} = \frac{bh_0^3}{12}\varphi^3(\xi) = I_y^0\varphi^3(\xi). \tag{4}$$

The transition from the derivative with respect to x to the derivative with respect to ξ in Eq. (2) is performed as follows:

$$\begin{gathered} \frac{dw}{dx} = \frac{dw}{d\xi}\frac{d\xi}{dx} = \frac{1}{l}\frac{dw}{d\xi}; \\ \frac{d^2w}{dx^2} = \frac{1}{l^2}\frac{d^2w}{d\xi^2}. \end{gathered} \tag{5}$$

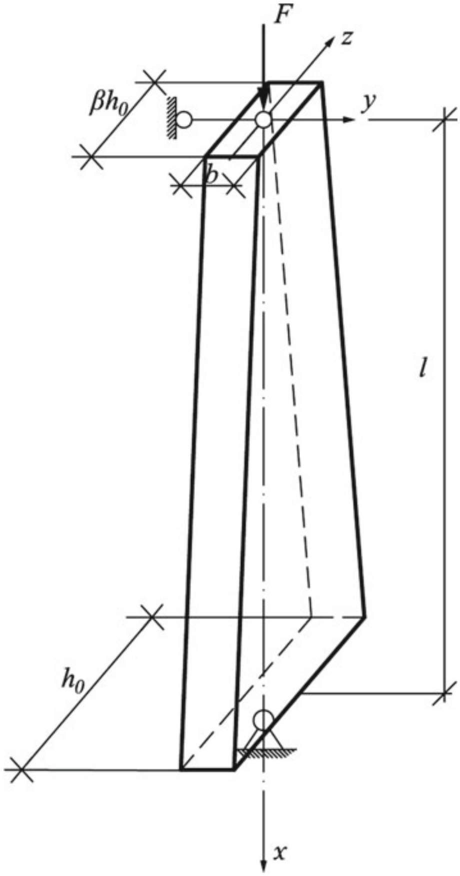

Fig. 1 Design scheme

Substituting (5) and (4) into (2), we get:

$$EI_y^0 \varphi^3(\xi) \frac{1}{l^2} \frac{d^2 w}{d\xi^2} + Fw = 0 \tag{6}$$

Or

$$\varphi^3(\xi) \frac{d^2 w}{d\xi^2} + \lambda w = 0, \tag{7}$$

where $\lambda = \frac{Fl^2}{EI_y^0}$.

The critical force is expressed in terms of the dimensionless parameter λ as:

$$F_{cr} = \frac{\lambda EI_y^0}{l^2}. \tag{8}$$

This formula coincides in structure with the Euler formula. For $EI(x) = \text{const}$, i.e. $\beta = 1$: $\lambda = \pi^2$.

In the current Russian standards for the design of wooden structures, the variable stiffness of the bar is taken into account by the coefficient k_{zhN}, which depends on the parameter β. This coefficient for a bar hinged at the ends is expressed in terms of the parameter λ as follows:

$$k_{zhN}(\beta) = \frac{\lambda(\beta)}{\pi^2}. \tag{9}$$

To solve Eq. (7), we use the finite difference method. On the interval [0; 1], a uniform grid with step Δξ is introduced. The finite difference approximation of Eq. (7) for the *i*-th node is written as:

$$\varphi^3(\xi_i)\frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta\xi^2} + \lambda w_i = 0. \tag{10}$$

Compiling this equation for all grid nodes, except for the extreme ones, in which $w_0 = w_\mathrm{n} = 0$, we obtain a system of linear algebraic equations:

$$([A] + \lambda[E])\{X\} = 0, \tag{11}$$

where $\{X\} = \{w_1 w_2 \ldots w_{n-1}\}^T$, $[E]$ is the identity matrix,

$$[A]\Delta\xi^2 = \begin{bmatrix} -2\varphi^3(\xi_1) & \varphi^3(\xi_1) & 0 & 0 & \ldots & 0 & 0 & 0 \\ \varphi^3(\xi_2) & -2\varphi^3(\xi_2) & \varphi^3(\xi_2) & 0 & \ldots & 0 & 0 & 0 \\ 0 & \varphi^3(\xi_3) & -2\varphi^3(\xi_3) & \varphi^3(\xi_3) & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & 0 & \varphi^3(\xi_{n-1}) & -2\varphi^3(\xi_{n-1}) \end{bmatrix}.$$

The system of Eqs. (11) is homogeneous and has a nonzero solution only if its determinant is equal to zero:

$$|[A] + \lambda[E]| = 0. \tag{12}$$

The parameter λ corresponding to the critical load is the minimum eigenvalue of the matrix [*A*], taken with a minus sign:

$$\lambda = \min(-eig(A)), \tag{13}$$

where eig(*A*) is a function returning the eigenvalues of the matrix [*A*].

3 Results and Discussion

The calculation was implemented in the MATLAB environment. The first step to estimate the required number of intervals in ξ was to solve the problem for a bar of constant cross section. Table 1 shows the values of the parameter λ for a different number of intervals n in ξ, as well as the deviation from the exact result equal to π^2.

In further calculations, the number of intervals *n* was assumed to be 100.

Table 1 Dependence of the parameter λ for a bar of constant cross section on the number of intervals n in ξ

n	2	3	4	5	6	7	8	9	10
λ	8	9	9.37	9.55	9.65	9.71	9.74	9.77	9.79
δ, %	18.9	8.8	5.1	3.2	2.2	1.6	1.3	1.0	0.8

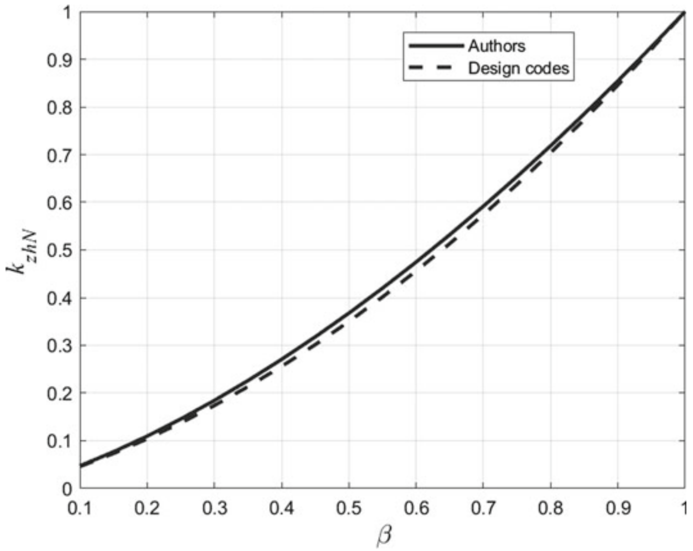

Fig. 2 The dependence of the coefficient k_{zhN} on the parameter β at loss of stability in the *xz* plane

Figure 2 shows the dependence of the coefficient k_{zhN} on the parameter β obtained as a result of the calculation. The dashed line corresponds to the formula presented in Russian standards:

$$k_{zhN} = (0.4 + 0.6\beta)\beta. \tag{14}$$

The greatest discrepancy between the results is 5.7% at β = 0.3. For the dependence $k_{zhN}(\beta)$ shown in Fig. 2 we have selected a refined approximating formula:

$$k_{zhN}(\beta) = 0.5116\beta^2 + 0.5004\beta - 0.0103. \tag{15}$$

We consider next the case of stability loss in the *xy* plane. In this case, the moment of inertia I_z should be substituted into Eq. (2) as $I(x)$, determined by the formula:

$$I_z(x) = \frac{b^3 h(x)}{12} = \frac{b^3 h_0}{12}\varphi(\xi) = I_z^0 \varphi(\xi). \tag{16}$$

The differential equation for buckling takes the form:

$$\varphi(\xi)\frac{d^2w}{d\xi^2} + \lambda w = 0, \tag{17}$$

where $\lambda = \frac{Fl^2}{EI_z^0}$.

This equation is solved similarly to Eq. (7). In Russian design standards for wooden structures with buckling in the xy plane, the coefficient k_{zhN} is determined by the formula:

$$k_{zhN} = 0.4 + 0.6\beta. \tag{18}$$

Figure 3 shows the graph of the coefficient k_{zhN} dependence on β obtained as a result of the calculation. The dashed line corresponds to a straight line constructed according to formula (18).

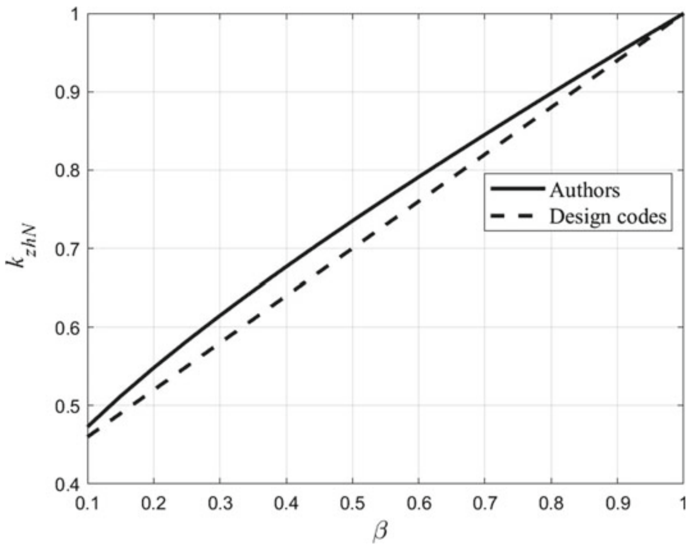

Fig. 3 The dependence of the coefficient k_{zhN} on the parameter β at loss of stability in the xy plane

The maximum deviation of the author's solution from the normative values is also observed at β = 0.3 and is, as before, 5.7%. We propose the following refined formula for the coefficient k_{zhN}:

$$k_{zhN} = -0.1305\beta^2 + 0.7193\beta + 0.4079. \tag{19}$$

To confirm the reliability of the results, a calculation was made for the stability of the bar with variable cross section in the LIRA-SAPR software package at β = 0.5, h_0 = 15 cm, b = 10 cm, l = 3 m. The critical force was 100.79 kN. The Euler force for a bar of constant section $b \times h_0$ is 137 kN. The actual coefficient k_{zhN} = 0.735. The normative value of the coefficient calculated by formula (18) was 0.7. According to the refined formula (19) proposed by us it is equal to 0.735. The model in LIRA-SAPR and the form of buckling are shown in Fig. 4.

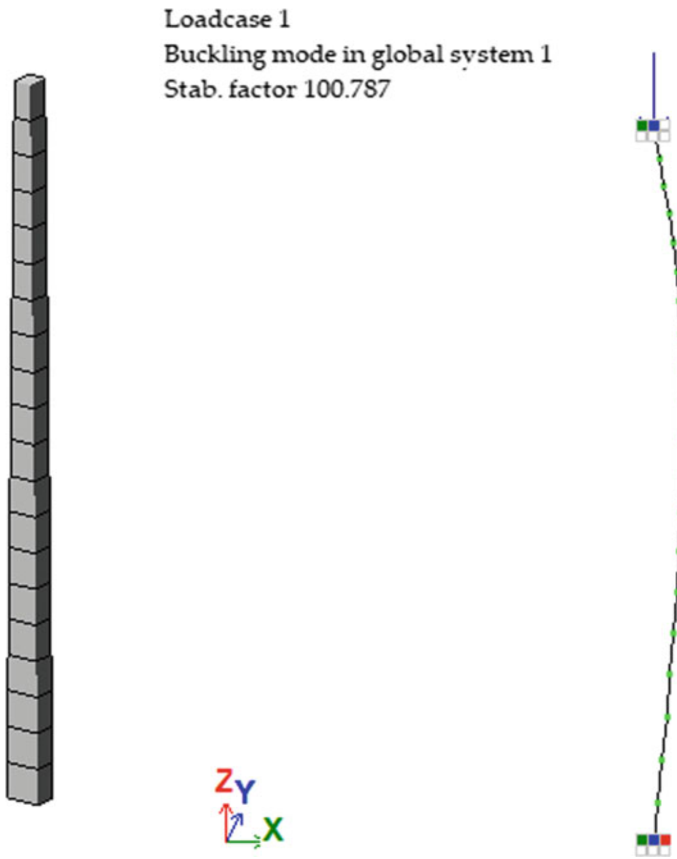

Fig. 4 Model of a bar with variable cross section and the form of buckling in LIRA-SAPR

4 Conclusion

A technique has been developed for determining the critical load for compressed wooden bars with a linearly varying cross-sectional height based on the finite difference method. A comparison was made with the calculation dependencies presented in Russian design codes SP 64.13330.2017. It is established that the error of normative formulas exceeds 5%. Using the least squares method, refined formulas are proposed. The reliability of the results obtained by the authors is confirmed by finite element modeling in the LIRA-SAPR software package.

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