421. Determining the refractive index and dispersion of a prism using a goniometer

Purpose

Getting acquainted with the method of finding the refractive index of a material by the minimal deviation angle in a prism; studying the dependence of the refractive index on the light wavelength.

Tasks to solve

- Learn the operation principle of a goniometer
- Determine refractive indices for different wavelengths

Theory

Dispersion is the phenomenon of dependence of the refractive index on the light frequency (or wavelength). By measuring the refractive index n at different wavelength, the dependence $n = f(\lambda)$ can be obtained, which is called a dispersion curve.

The index n of colourless transparent material in the visible light band decreases as the wavelength grows ($dn/d\lambda < 0$). This behaviour is referred to as the *normal dispersion*. Long before the electron theory of dispersion was known, Cauchy had derived a theoretical formula for the dependence $n = f(\lambda)$:

$$n = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4},$$

where a, b, and c are constants for a given material which are found experimentally.

In many cases we can limit ourselves to the first two terms:

$$n = a + \frac{b}{\lambda^2}. \qquad (1)$$

In this work the method of finding the refractive index of glass by measuring the minimal deviation angle of a parallel monochromatic light beam propagating through a prism is described.

Let a beam falls onto one side of a prism made of glass with the refractive index n; the angle of incidence is α (Fig. 1). The angle between the refracting prism sides is denoted A.

The ray leaving the prism makes an angle β with the rear side; the angle between this ray and the continuation of the incident ray is φ. The refraction angle depends on the angle of incidence. It can be proved that if the ray path in the prism is symmetric (that is, $\alpha = \beta$), then the angle φ has the smallest possible value. Denote it φ_{min}. The values n, A, and φ_{min} are related to each other by the following expression:

$$n = \frac{\sin\left(\frac{A+\varphi_{\min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}. \qquad (2)$$

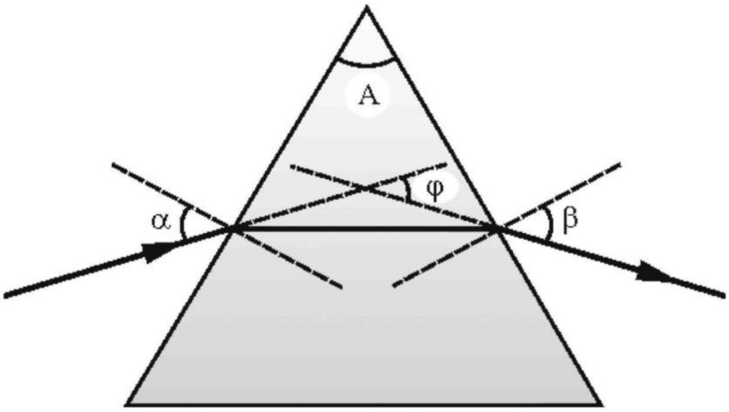

Fig. 1. Ray path in the prism.

Thus, finding the refractive index requires measuring the angles A and φ_{min}. This measurement can be carried out with the aid of a goniometer (a device for accurate measurement refraction angles).

Goniometer

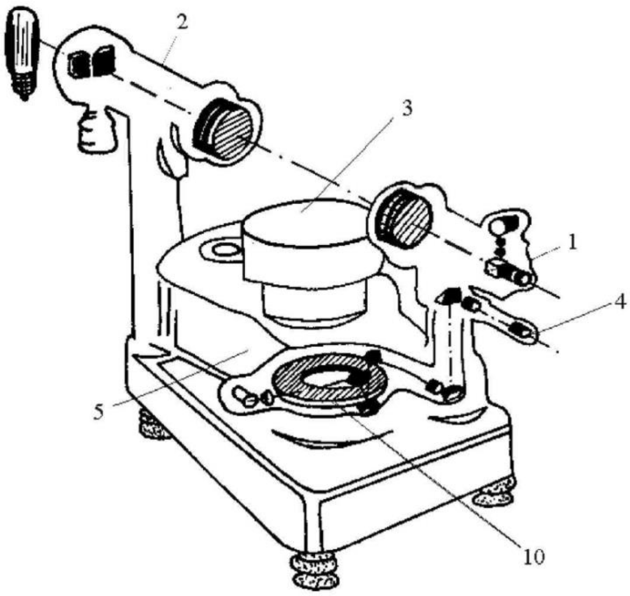

Fig. 2. Goniometer (GS-5).

Goniometer (Figs. 2 and 3) consists of the following main units: tube 1 for observing refracted or reflected rays; collimator 2 which makes a parallel light beam; table 3 for locating the object to study; measuring gauge for determining rotation angles of the observing tube (this part includes limb 10, optical microscope 4, and optical micrometer).

The observing tube is a telescope with a long focal length of the objective lens and a short-focus ocular. The ocular is equipped with a vertical thread used for selecting a certain spectral line.

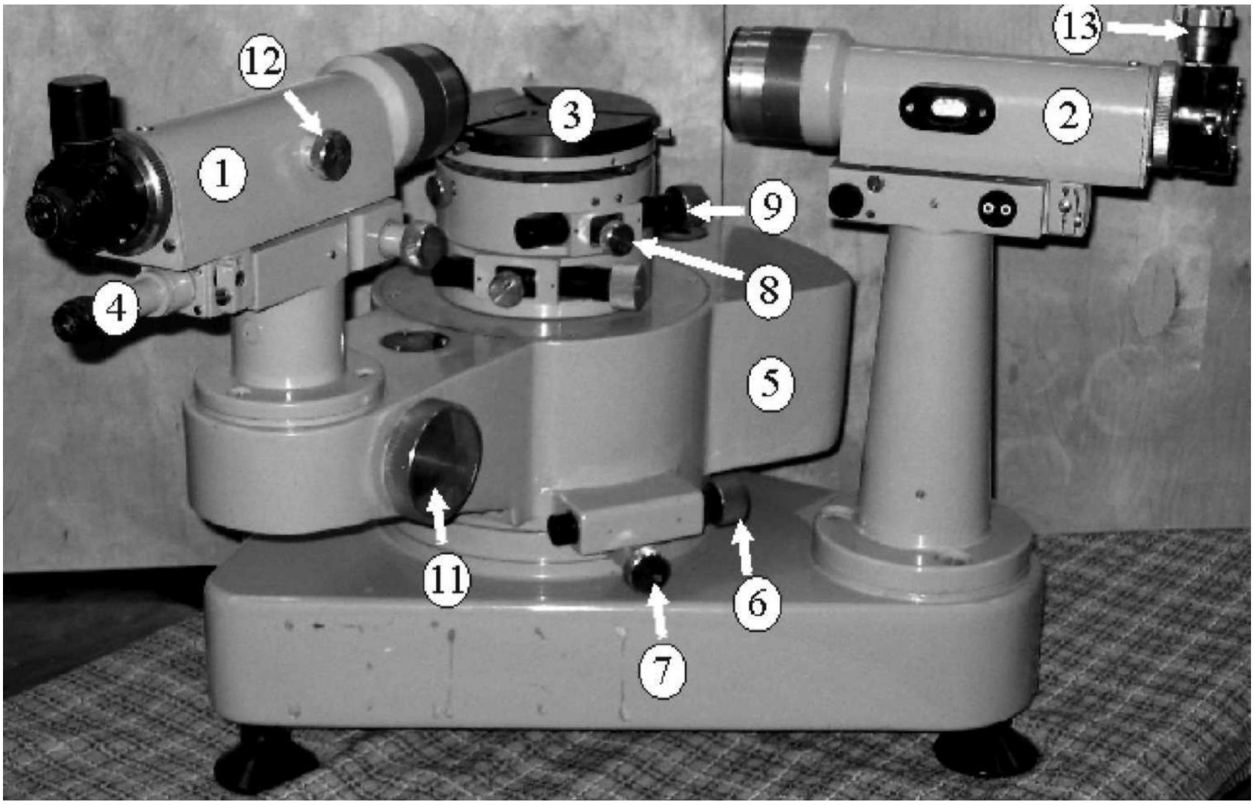

Fig. 3. Exterior of the goniometer GS-5.

For spectral studies, the prism is put on the table 3 and illuminated with a parallel light beam, which is formed by the collimator 2. The latter is a tube fixed on the base of the apparatus. There is an objective lens on one end of the collimator and a slit on the other end, in the focal plane of the objective lens. The slit is illuminated with a mercury lamp, which produces a discrete (line) spectrum (the spectrum of Hg is printed on the working place). Parallel rays going out of the collimator, are refracted by the objective lens of the observing tube and make an image of the slit in its focal plane. If the observing tube is focused on the infinite distance, a fine image of the slit will be seen through it. Observing tube 1 and microscope 4 are attached to the movable mounting 5. If the screw 7 is loosened, the mounting can be rotated by a hand. If the screw 7 is tightened up, the mounting 5 can be precisely rotated by a small angle with the micrometric screw 6. When adjusting the goniometer, the observing tube is rotated with respect to the motionless table 3. The table is fixed with the screw 8. Precise rotation of the table is achieved by the micrometric screw 9 with the screw 8 being tightened. *During the measurements, the table should be fixed, and only the observing tube should be rotated.*

To measure the rotation angle of the tube, the gauge consisting of the limb 10, optical micrometer, and microscope 4 is used. The field of vision of the microscope is shown in Fig. 4.

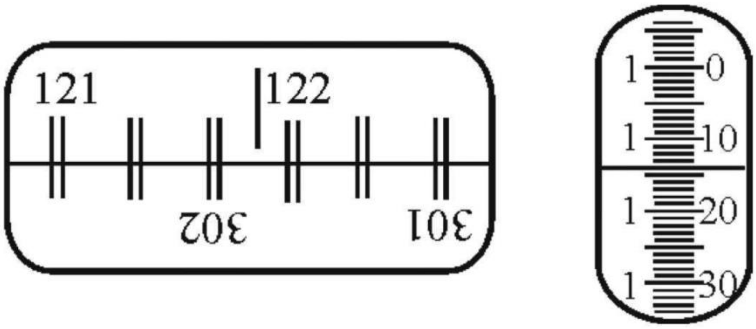

Fig. 4. Field of vision of the reading microscope.

The left window shows diametrically opposite segments of the limb and the vertical index. The scale interval of the limb is 20′. The limb 10 is fixed on the motionless basis, and hence the reading from the limb changes when the observing tube is rotated.
The right window shows the image of the scale of the optical micrometer. Shifting the scale by 600 intervals corresponds to the shift of the upper limb marks with respect to the lower scale by 10′. In other words, the scale interval of the right gauge is 10′/600 = 1″.
To take the reading from the limb, you need to turn the knob 11 of the optical micrometer so that the marks of the upper and lower limb images in the left window coincide (like it is shown in Fig. 4).
The degree count is given by *the closest number to the left* of the vertical index (in the example above it is 121°).
The number of tens of minutes is equal to the *number of intervals* fitting in the space between the upper double mark, which was selected on the previous step (integer degree count), and the lower double mark which differs from the upper by 180°. In our case it is 301° (121 + 180 = 301), there are five intervals between these two marks, and so the count of tens of minutes is 5·10′ = 50′.
Integer angular minutes are counted on the left vertical scale in the right window; the right part of the same scale gives the number of seconds. All in all, the angle corresponding to the picture shown in Fig. 4, is 121°51′14″.

Preparation of the goniometer to measurements

1. *Adjusting the observing tube.* The tube should be focused to the infinite distance. To do this, look at some distance object through the observing tube (e.g., at a building in the street) and attain a fine image by rotating the screw 12.
2. *Focusing the collimator.* Place the lamp in front of the collimator slit, and switch it on. Illuminate the slit and look at its image through the observing tube. Make the fine image of the slit by rotating the screw (the collimator’s screw is not shown in the figures, but its location is analogous to that of the screw 12 on the observing tube).
3. *Choose an appropriate slit width.* By rotating the screw 13 adjust the slit width so that the emission lines in the spectra are narrow and fine, but

bright enough at the same time. When you decrease the width of the gap, the lines are first narrowing with their brightness level constant, and then the line width becomes constants and their brightness decreases abruptly. You should find this threshold (so-called “normal slit width”) and continue measurements at this value.

Show the results of the above procedure to the teacher or engineer before you continue the work.

Exercise 1. Determination of the refracting angle of the prism

1. Put the prism on the table 3 so that its edge is vertical, and the light beam from the collimator fall on both sides forming the refracting angle (see Fig. 5). Fix the table with the screw 8.
2. Release the screw 7 and turn the optical tube along a beam reflected from one of the sides of the prism (position I in Fig. 5).
3. By slightly rotating the tube with your hand, obtain the image of the collimator’s slit in reflected light. Focus this image using the screw 12. Superpose the image of the slit with the vertical line in the eye-glass and take the reading of the angle.

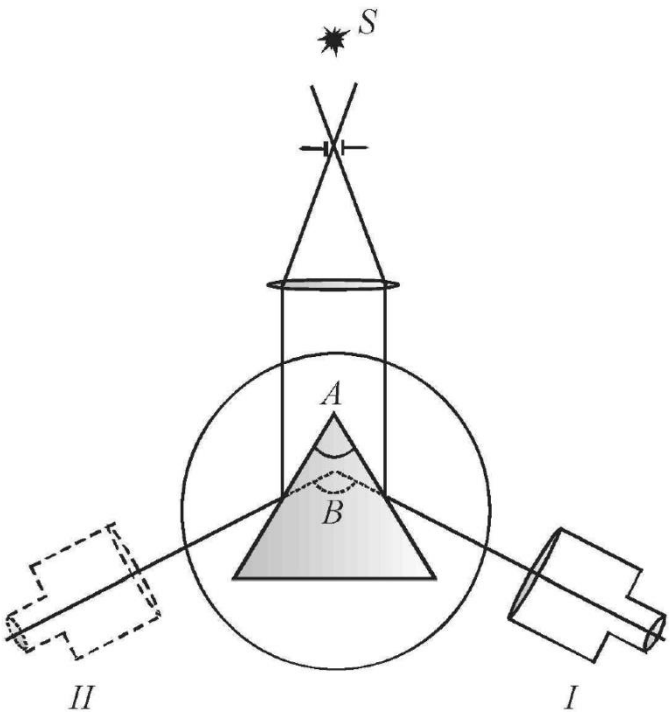

Fig. 5.

4. Release the screw 7 and turn the tube to find the image of the slit reflected from the other side of the prism (position II in Fig. 5). Tighten up the screw 7, superpose the slit’s image with the vertical line and read the angle. The difference between the angles measured in steps 3 and 4 is equal to the angle B, which is twice as large as the refracting angle A.

Exercise 2. Determination of the refractive index and dispersion of the material of the prism

To determine the refractive index, the minimal deviation (refraction) angle of light should be known. It is measured as described below.

1. Take off the prism, turn the observing tube to see the image of the collimator's slit, superpose it with the vertical thread, and read the angle α_0 (it is the direction of the incident ray).
2. Put the prism on the table so that the beam exiting the collimator passes through the prism (Fig. 6). Set approximately the bigger angle of incidence; rotate the tube and catch the spectrum in the field of vision.
3. Release the screw 8. Rotate slowly with your hand the table (decreasing the angle of incidence) and observe the manner of shifting of the spectrum in the observing tube. First it will move to one direction, and then it will shift to the other side, though the rotation direction of the prism (table) remains the same. The moment when the movement direction changes corresponds to the prism position providing the minimal deviation (refraction) angle of the ray.

<u>Algorithm of measurements</u>

1. Superpose the vertical thread with the spectral line for which the refractive index of the prism (of glass) has to be found. Find the table position α corresponding to the minimal deviation angle φ_{min} and calculate the latter as $\varphi_{min} = \alpha - \alpha_0$. Knowing this value and the refracting angle A of the prism, calculate the refractive index using Eq. (2) for the wavelength corresponding to the chosen spectral line.
2. Repeat these measurements for all observed spectral lines of mercury and calculate corresponding refractive indices for them.
3. Built the plot of the dependency $n = f(\lambda)$. Define its slope (tangent $dn/d\lambda$, which is actually the dispersion of glass) in the long-wave and short-wave parts of the spectrum.

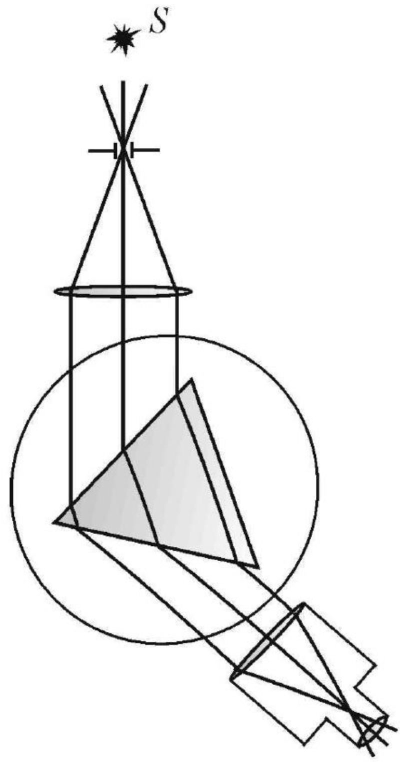

Fig. 6.

4. Check is the Cauchy formula (1) is fulfilled is the wave range in which the investigation was conducted. To do this, use the linearization method. Choose variables so that the studied function becomes linear. In our case, if new variables $Y = n$ and $X = 1/\lambda^2$ are introduced, then the Eq, (1) will be written as $Y = a + bX$. If the Cauchy formula is applicable,

the plot $Y(X)$ will be a straight line with the slope b and crossing the vertical axis at the point with coordinate a.

Questions

1. Normal and anomalous dispersion.
2. Classical (electronic) theory of dispersion. Complex refractive index.
3. Correlation of the refractive index with concentration. The Lorentz–Lorenz equation.
4. Ray path in the prism. The minimal deviation angle.
5. Prove that the angles A and B are related to each other as A = B/2.
6. What is the physical meaning of the notion "dispersion of a material"?
7. Formulate the purpose of the work, describe the experimental part, and discuss your results.