$\langle\varphi^2\rangle$ of a quantized scalar field in a nonzero temperature thermal state at a long throat

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An analytical approximation of $\langle\varphi^2\rangle$ for a quantized scalar field in a thermal state at arbitrary temperature is obtained. The scalar field is assumed to be both massive and massless, with an arbitrary coupling ξ to the scalar curvature, and in a thermal state at (an) arbitrary temperature T. The gravitational background is assumed to be static spherically symmetric

$$ds^2 = -f(\rho)dt^2 + d\rho^2 + r^2(\rho)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{1}$$

and slowly varying

$$\varepsilon_{_{WKB}} = L_\star/L \ll 1, \tag{2}$$

where

$$L_\star(\rho) = \frac{r(\rho)}{\sqrt{2\xi + m^2 r^2(\rho)}}, \tag{3}$$

and L is a characteristic scale of variation of $f(\rho)$ and $r^2(\rho)$:

$$\frac{1}{L(\rho)} = \max\left\{\left|\frac{r'}{r}\right|,\ \left|\frac{f'}{f}\right|,\ \left|\frac{r'}{r}\sqrt{|\xi|}\right|,\ \left|\frac{f'}{f}\sqrt{|\xi|}\right|,\ \left|\frac{r''}{r}\right|^{1/2},\ \left|\frac{f''}{f}\right|^{1/2},\ \ldots\right\}. \tag{4}$$

The result of the calculation is (A. Popov , Phys. Rev. D **94**, 124033 (2016))

$$\begin{aligned}
&4\pi^2\left\langle\varphi^2\right\rangle_{ren} = \\
&\frac{4\mu^2}{r^2}J\left(a/\mu\right) - \frac{1}{4r^2}\left(2\xi - \frac{1}{4}\right) + \frac{1}{4r^2}\left[m^2r^2 + 2\left(\xi - \frac{1}{6}\right)\right]\ \ln\left|\frac{m^2r^2 + 2\xi - 1/4}{m_{_{DS}}^2 r^2}\right| \\
&-\frac{\mu^2}{r^2}I_1(\mu) + \frac{r^{2''}}{24r^2} - \frac{r^{2'^2}}{48r^4} + \frac{f'r^{2'}}{48fr^2} + \left[\frac{f''}{12f} + \frac{r^{2''}}{6r^2} - \frac{f'^2}{24f^2} - \frac{r^{2'^2}}{24r^4} + \frac{f'r^{2'}}{12fr^2}\right. \\
&\left.+\xi\left(-\frac{f''}{2f} - \frac{r^{2''}}{r^2} + \frac{f'^2}{4f^2} + \frac{r^{2'^2}}{4r^4} - \frac{f'r^{2'}}{2fr^2}\right)\right]\left(\frac{1}{2}\ln\left|\frac{m^2r^2 + 2\xi - 1/4}{m_{_{DS}}^2 r^2}\right| - \mu\frac{dI_0(\mu)}{d\mu}\right),
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
a &= \frac{2\pi T r}{\sqrt{f}},\ \mu^2 = m^2r^2 + 2\xi - \frac{1}{4}. \\
J\left(a/\mu\right) &= \int_1^\infty \frac{\sqrt{\eta^2 - 1}}{e^{2\pi\mu\eta/a} - 1}d\eta, \quad I_m(\mu) = \int_0^\infty \frac{\eta^{2m-1}\ln|1 - \eta^2|}{1 + e^{2\pi\mu\eta}}d\eta.
\end{aligned} \tag{6}$$

The constant $m_{_{DS}}$ is equal to the mass m of the field for a massive scalar field. For a massless field $m_{_{DS}}$ is an arbitrary parameter due to the infrared cutoff in $\langle\varphi^2\rangle_{_{DS}}$. A particular choice of the value of $m_{_{DS}}$ corresponds to a finite renormalization of the coefficients of terms in the gravitational Lagrangian and must be fixed by experiment or observation.

We have shown that in a long throat the effect of vacuum polarization of a quantized scalar field is local one for any finite mass m of the quantized field, including $m = 0$.

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