ON AN ESTIMATE OF THE TORSIONAL RIGIDITY OF A CONVEX DOMAIN THAT IMPROVES THE POLIA-SZEGO INEQUALITY

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Let G be a simply connected domain in the plane. One of the important physical characteristics of a domain in mathematical physics is the functional

$$\mathbf{P}(G) := 2\int_G u(x,G)\,\mathrm{d}A,$$

called torsional rigidity in elasticity theory. Here $u(x,G)$ is the stress function that satisfies the equation $\triangle u = -2$ in G and the boundary condition $u = 0$, while the differential area element is denoted by $\mathrm{d}A$ (see [1], [2]).

Denote by $\rho(x,G)$ the distance function from a point x to the boundary of a domain G, and $\rho(G)$ is the radius of the maximal circle contained in G.

G. Polya and G. Szege [1] showed that for any convex domain

$$\mathbf{P}(G) \geq \frac{1}{2}\mathbf{A}(G)\rho(G)^2, \tag{1}$$

where $\mathbf{A}(G)$ is the area of G. The equality in (1) is achieved for a disk.

Denote by $G(\mu)$ the level set of the distance function $\rho(x,G)$, $0 \leq \mu \leq \rho(G)$, and $l(\mu)$ is the length of the boundary curve of $G(\mu)$. Let $ł(\rho(G)) := \lim_{\mu\to\rho(G)} l(\mu)$.

Theorem 1. *Let G be a convex domain in the plane of the bounded area and $l(\rho(G)) \neq 0$. Then*

$$\mathbf{P}(G) > \frac{1}{2}\mathbf{A}(G)\rho(G)^2 + \frac{5}{12}l(\rho(G))\rho(G)^3. \tag{2}$$

References

1. Po'lya and G. Szego, *Isoperimetric Inequalities in Mathematical Physics* // Number 27 in Annals of Mathematical Studies. Princeton University Press, Princeton, N.J., 1951.

2. Arutyunyan N.Kh. *Torsion of elastic bodias. Moscow, Fizmatgiz, 1963, 688 p. (in Russian).*

BOUNDED COMPOSITION OPERATORS IN BV-SPACES ON CARNOT GROUPS

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Let $\varphi\colon \Omega \to \Omega'$ be a homeomorphism between two domains in a Carnot group G. We consider a situation when φ induces bounded composition operator between the BV spaces:

$$BV(\Omega') \ni u \mapsto \varphi^*(u) = u\circ\varphi \in BV(\Omega),$$