222. Measuring viscosity of liquid by the Stokes' method

A spherical body (metal ball or a droplet of liquid) falling in a liquid medium experiences the force of gravity F_g, the Archimedes (buoyant) force F_A, and resistance of medium (friction) F_r:

$F_g = mg = V\rho_b g$, $F_A = \rho V g$, and

$$F_r = 6\pi\eta r v. \qquad (1)$$

Here V is the ball's volume, ρ_b is its density, ρ is the density of the medium, and g is the free fall acceleration. Equation (1) is called the Stokes' formula. It was derived by an English physicist G. Stokes assuming that (1) the Reynolds number Re << 1 (the flow is laminar), (2) liquid fills the whole space, and (3) liquid wets the ball (that is, the closest layer of the liquid moves together with the ball).
The speed of the ball changes as

$$v(t) = \frac{gvV(\rho_b - \rho)}{6\pi\eta r}\left(1 - \exp\left(\frac{6\pi\eta r}{m}t\right)\right). \qquad (2)$$

Evidently, it tends exponentially to a certain limit value:

$$v_\infty = \frac{gV(\rho_b - \rho)}{6\pi\eta r}. \qquad (3)$$

The time characterizing the increase in the velocity of the ball is called relaxation time:

$$\tau = \frac{m}{6\pi\eta r}. \qquad (4)$$

After several τ periods have passed, the ball's speed can be assumed constant and equal to the limit value.
After substituting the expression of the spheres' volume $V=(4/3)\pi r_b^3$, we get:

$$\eta = \frac{2gr_b^2(\rho_b - \rho)}{9v_\infty}. \qquad (5)$$

Thus, the coefficient of the inner friction can be found experimentally if we know the values of r_b, ρ_b and ρ and measure v_∞.
The Stokes' equation assumes that the ball propagates through an infinite space filled with liquid. In the practical measurements, the ratio of the ball's radius r_b and the radius R of the tube with the liquid should be allowed for as in the expression below:

$$\eta = \frac{2gr_b^2(\rho_b - \rho)}{9v_\infty(1 + 2.4\frac{r_b}{R})}. \qquad (6)$$

If we use droplets of liquid in another liquid, the viscosity of the liquid of which the droplet is made (ρ_d) should also be allowed for. To do this, we introduce additional factor in formula (6):

$$\beta = \frac{1 + \eta_d/\eta}{2/3 + \eta_d/\eta},$$

so that finally

$$\eta = \beta\frac{2gr_b^2(\rho_b - \rho)}{9v_\infty(1 + 2.4\frac{r_b}{R})}.$$

If the ratio η_d/η is small (for example, if we consider a droplet of water propagating through oil), we can assume that $\beta = 3/2$.

Aim of the work:

- Getting acquainted with the theoretical basis if the Stokes' method;
- Measuring the viscosity of liquid.

Instruments:

Cylindrical glass vessel with level marks, filled with the investigated transparent oil; burette; distilled water; cap for the vessel in the form of a cap; stopwatch; ruler.

Algorithm of measurements

1. Measure the inner radius R of the cylinder.
2. Put the cap on.
3. Fill the burette with distilled water.
4. Accurately open the valve and find the position at which the droplets fall into the cap slowly, one by one.
5. Using the gauge on the burette, count how many droplets are contained in one milliliter of water. From this value, find the volume and radius (r_b) of an individual droplet.
6. Switch on the illumination behind the vessel.
7. Remove the cap and let one droplet fall into the oil. Return the cap to its place. Measure the time needed for the ball to pass between the two marks. Check if the speed is constant by comparing the times of passage of two intervals drawn of the tube.

8. Calculate the speed of falling (between the two marks at the biggest distance). Repeat this measurement 6–10 times and find the average value of the speed v_∞.
9. Calculate the viscosity of oil using Eq. (7). Density of the oil is written on the apparatus. Estimate the inaccuracy.
10. Calculate the relaxation time using Eq. (4).
11. Calculate the resistance force of the medium using the Stokes' formula (1).
12. Calculate the Reynolds number as $\mathrm{Re} = 2\rho r_b v/\eta$.

Questions

1. Inner friction in liquids.
2. Coefficient of dynamic viscosity (physical meaning).
3. Stokes' method of measuring the viscosity.
4. Which shape has the water droplet when it goes through oil?

Estimating the inaccuracy

RMSD for the time in the set of *n* measurement is $S_t \sqrt{\frac{\sum (t_i - \bar{t})^2}{n(n-1)}}$.

Inaccuracy of measuring the time is $\Delta t = c \cdot S_t$, where the Student's coefficient c is 2.4 for 6 or 7 measurements, 2.3 for 8 or 9, 2.2 for 10 measurements (assuming the confidence interval is 95%).

Relative error in the final value of the viscosity is $\frac{\Delta \eta}{\eta} = \frac{\partial \eta}{\partial t} \Delta t = \frac{\Delta t}{\bar{t}}$.