# 141. Measurement of the acceleration of gravity with the aid of a mathematical pendulum

Introduction

Acceleration of gravity **g** is the acceleration with respect to the Earth at which a released body begins to fall down. This acceleration is defined by the sum of the force of gravity (attraction to the Earth) and the centrifugal force of inertia.

Mathematical pendulum is an imaginary pendulum with all its mass placed at a single point, with the distance $l$ from this point to the centre of suspension being constant while it oscillates. Simple calculations show that at small angles of deviation from the vertical, the period of oscillations of the pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}}\ . \qquad (1)$$

Now the idea of one possible approach to determine the free fall acceleration is clear: the length and period of a mathematical pendulum should be measured.

However, a question arises whether properties of a real pendulum are properly described by the model of a mathematical pendulum?

Note that Eq. (1) show that the period of oscillations of the mathematical pendulum is proportional to $\sqrt{l}$. If this correlation is true for a given real pendulum, it may be assumed to be a mathematical pendulum, and the acceleration of gravity will by defined from the formula

$$g = \frac{4\pi^2 l}{T^2}\ . \qquad (2)$$

**Necessary notions:** definitions of a vector and vector's components; vector coordinates; projection of a vector on a given direction; vector of an infinitesimal rotation, angular velocity, and angular acceleration; coordinate frame; inertial and non-inertial coordinate frame; mass; moment of inertia; force and moment of force; centre of masses; inertial force; free fall acceleration; gravity force.

**You should know:** rotational dynamics equation and its limits of applicability; expressions for the forces of inertia; equations of motion of a mass point with respect to a rotating frame bound to the Earth; the reasons for the dependency of the free fall acceleration on the position on the Earth's surface.

**Necessary skills:** measurement of distances with a ruler; measurement of time with a stop-watch; estimating inaccuracies of direct and indirect measurements.

**Aim of the work:** measurement of the free fall acceleration.

**Problems being solved:**

- Acquaintance with the method of measuring of the acceleration of gravity with the aid of a mathematical pendulum.
- Estimation of feasibility of describing the given real pendulum by the model of a mathematical pendulum.
- Measurement of the free fall acceleration.

Experimental setup

**Materials:** massive ball on a hardly stretchable cord; ruler; stop-watch.

Algorithm of measurements

1. Shorten the cord so that its length between the ball and the steel ring is 10 – 15 cm. Attach the free end of the cord to the hook.
2. Measure the length of the pendulum $l$, which is the distance between the centre of suspension and the centre of the ball.
3. Deflect the ball so that the angle between the cord and the vertical does not exceed 10°, and release the ball.
4. Measure the time of 10 full oscillations, $t_{10}$, and find the period $T = t_{10}/10$.
5. Add 5 – 10 cm to the length of the pendulum (use a slip loop on the cord).
6. Repeat stages 2 – 4.
7. Repeat stages 5 – 6 until the length of the pendulum exceeds 100 cm.

Data interpretation and presentation

Fill in the table with results of your measurements and calculations.

| N | $l$, cm | $t_{10}$, s | $T$, s | $T^2$, s$^2$ |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
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Plot the graph showing the dependence of the squared oscillation period on the pendulum's length. Draw a straight line so that is deviate from the experimental points as little as possible. Find the acceleration of gravity from the slope of this line ($T^2 = k{\cdot}l$) using Eq. (2).

**Advice:** every pair of $T_j^2$ and $l_j$ gives you an individual estimate of $g$ – $g_j$. Perform a standard statistical analysis of this series: calculate mean value $\bar{g}$, dispersion $S_{ng}$, and error of determination of the free fall acceleration, $\Delta g$.