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Statistical Modelling of the Wind Load on a Structure in Kazan, Russian Federation

L. L. Salekhova[1*], I. I. Valeev[1], T. A. Zinnurov[2***], and Kamon Budsaba[3****]**

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[1]*Kazan (Volga Region) Federal University, Kazan, Tatarstan, 420008 Russia*

[2]*Kazan State University of Architecture and Engineering, Kazan, Tatarstan, 420043 Russia*

[3]*Department of Mathematics and Statistics, Thammasat University, Rangsit Center, Pathum Thani 12121, Thailand*

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Abstract—In this article we apply the normal, truncated normal, and Weibull distributions for modelling the annual extreme wind load obtained from the Kazan-"Basic" meteorological station for years 1905–2004. The statistical modelling of extreme wind load is important in the design of structures such as city buildings. More generally, it provides information about weather for modification and monitoring climate change. In particular, meteorologists can fit various statistical frequency distributions to historical wind data in order to estimate the magnitude of maximum wind.

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1. INTRODUCTION

Scientists from NASA declared that since the mid-1990s, there was an increase in the intensity of hurricanes. This increase demanded the reconsideration of the classification scale of hurricanes (F1–F4), adding categories F5, F6, and F7. Scientists also noted occurrences of hurricanes in territories where they had not occurred in centuries [1].

On 8 July, 2007, in Kazan, Russian Federation, a strong gusty wind was recorded. According to the Ministry of Civil Defense and Extreme Situations of Tatarstan Republic, the wind velocity $28-30$ m/s $= 101-108$ kph. Wind of this velocity is classified as a storm, almost hurricane. This is the first time such a wind has been recorded in Kazan. The direction of the air mass's movements was recorded from a satellite. From 7 a.m. until 7 p.m., the wind had a westerly direction. At 7 p.m., it unexpectedly changed to south-westerly direction; a hurricane had formed.

Severe damage occurred in Kazan and the nearby town of Laishevo. Some house roofs, lightweight constructions, and billboards were demolished; trees fell. Destruction was extensive. In wooded areas, in circular areas of several tens of meters in diameter, had downed trees suggesting tornadoes developed in the center of the hurricane. There is a hypothesis that the hurricane increased its power after passing through the Kuibyshev water basin.

In this article, the statistical modeling of the wind load is fitted by three distributions. The first model suggests the use of the normal distribution of the annual maximums; the second method is based on the truncated normal distribution; and the third method is based on the two-parameter Weibull distribution.

*E-mail: salekhova2009@gmail.com
**E-mail: valeev94@mail.ru
***E-mail: leongar@mail.ru
****E-mail: 2555kamon@gmail.com

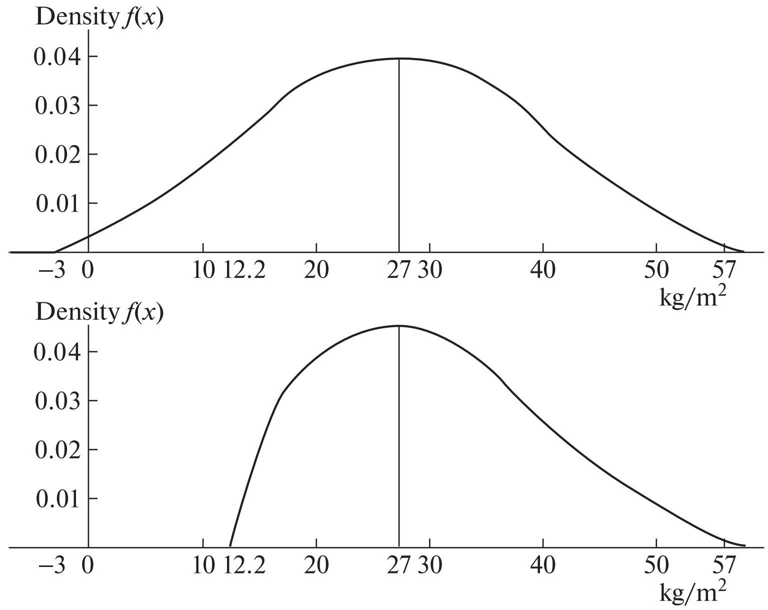


Fig. 1. Density functions of the normal (top part of the picture) and truncated normal (bottom part) distributions for the modeling of annual maximums of the wind pressure.

The criteria for model selection are the log-likelihood value, p-values of the chi-square and Kolmogorov–Smirnov goodness of fit tests, the Akaike information criterion (AIC), and the Bayesian information criterion (BIC).

Statistical modeling of wind loading is based on the data of meteorological observations on movements of air weights of a meteorological station Kazan-"Basic" for 100 years (1905–2004).

The content of the article is as follows. In Section 2, we describe the normal, truncated normal, and Wibull distributions that we use for the wind load modelling. Some details of statistical modelling with these distributions and corresponding accuracy measures are discussed in Section 3. The results and the summary of model performance are presented in Section 4. Additional descriptive statistics of the wind load are presented in Section 5.

In this article, the R Statistical Software [10] is applied to estimate the parameters of the distributions and perform the goodness of fit tests.

2. WIND LOAD MODELLING

We consider three models for the wind load in Kazan.

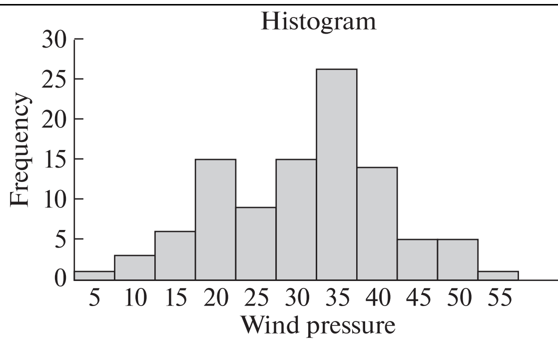
2.1. Modelling by the Normal Distribution

Random variable X has a *normal distribution* with parameters μ (mean) and σ^2 (variance), if its density function is $\phi\left(\frac{x-\mu}{\sigma}\right)$, where $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ is the density function of the standard normal distribution. In the following we denote $\Phi(\cdot)$ is the distribution function of the standard normal distribution.

2.2. Modelling by the Truncated Normal Distribution

Let $a < b$ be two real numbers. The *truncated normal distribution* is the probability distribution derived from that of a normally distributed random variable by bounding the random variable from below and above the interval values a and b.

Table 1

Bins for the wind pressure (kg/m^2)	Frequency
[0,5)	1
[5,10)	3
[10,15)	6
[15,20)	15
[20,25)	10
[25,30)	15
[30,35)	25
[35,40)	14
[40,45)	5
[45-50)	5
[50-55)	1
	

Suppose that random variable X has a normal distribution with mean μ and variance σ^2. The random variable X conditional on $a < X < b$ has a truncated normal distribution with parameters μ, σ^2, a, and b. Its density function is given by

$$f(x;\mu,\sigma,a,b) = \begin{cases} \frac{1}{\sigma}\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)}, & \text{if } a \le x \le b, \\ 0 \quad \text{otherwise.} \end{cases}$$

2.3. Modelling by the Weibull Distribution

The distribution function of the Weibull distribution with parameters $\alpha > 0$ and $\eta > 0$ takes the form

$$F(x) = \exp\left[-\left(\frac{x}{\eta}\right)^{\alpha}\right].$$

There is no closed form for the maximum likelihood estimators of the parameters α and η for the Weibull distribution, but they can be found numerically in any statistical software, for example in R.

3. STATISTICAL MODELLING

In report [2] (see also [3–9]), it is mentioned that fitting the wind values by normal (not truncated) distribution should be: mean $\mu = \overline{w}_0 = 27$ kg/m^2 and coefficient of variation $k_\nu = 0.37$. From this we can derive the standard deviation $\sigma = 9.99$ kg/m^2.

Table 2. Results of statistical modelling

	Chi-Square p-value	KS p-value	log-likelihood	AIC	BIC
Normal	0.162	0.454	−370.625	745.249	750.460
Weibull	0.091	0.410	−370.389	744.778	749.988
Truncated Normal	0.065	0.352	−339.346	682.692	687.903

Table 3. The monthly maximum wind velocity and gusts (m/s) for the year 2004

Months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Yearly
Maximum velocity	20	24	20	28	17	17	15	16	18	20	20	20	24
Wind gusts	20	26	24	20	20	25	20	19	24	28	28	24	28

Table 4. Distribution (%) of the monthly wind velocities (m/s) for the year 2004

Wind velocity	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Yearly
0–1	27.1	30.5	31.4	27.4	30.4	34.5	39.7	37.7	34.5	24.1	22.1	26.4	30.5
2–3	21.7	23.7	25.8	25.6	27.0	29.3	28.0	28.9	26.0	25.0	27.1	23.0	26.0
4–5	24.2	22.5	20.4	26.0	23.8	23.1	21.2	22.5	24.0	24.9	24.2	25.0	23.5
6–7	14.2	12.5	12.6	13.5	12.8	9.5	8.0	8.4	10.7	15.5	14.9	14.8	12.3
8–9	10.8	8.6	7.9	6.2	5.6	3.1	2.9	2.4	4.0	8.2	10.0	8.5	6.5
10–11	1.2	1.0	0.8	0.5	0.4	0.2	0.2	0.1	0.3	1.3	1.0	1.3	0.7
12–13	0.5	1.1	0.6	0.4	–	0.2	–	–	0.5	0.8	0.5	0.6	0.4
14–15	0.2	0.1	0.3	0.3	–	0.1	–	–	–	0.1	0.1	0.4	0.1
16–17	0.1	–	0.2	0.1	–	–	–	–	–	0.1	0.1	–	0.0
18–21	0.0	0.0	0.2	0.0	–	–	–	–	–	0.0	0.0	–	0.0
22–25	–	0.0	–	–	–	–	–	–	–	–	–	–	0.0

We use the maximum likelihood estimators for modelling by normal distribution. According the to data, the sample mean $\bar{X} = 28.668$ and sample standard deviation $s = 9.848$, which is very close to the numbers mentioned above.

For modelling by the truncated normal distribution, according to the data of meteorological observations for the city of Kazan over last 100 years, the minimal value of the annual extreme of the wind pressure was $a = 12.2$ kg/m^2, and the maximum value was $b = 76.6$ kg/m^2. These numbers provide exceptionally poor goodness of fit. Because of that, we varied the values of a and b to find the best values of the chi square statistics (chi-square method of estimation). It appears that the best fit is when $a = 12.5$ and $b = 72$. We use the same values for μ and σ as for the normal distribution case, namely mean $\bar{X} = 28.668$ and sample standard deviation $s = 9.848$.

We use the maximum likelihood estimators for modelling by Weibull distribution.

3.1. Criteria for the Accuracy of the Fit

The criteria for model selection are the highest value of the log-likelihood function and the highest of p-values based on the Chi-square and Kolmogorov–Smirnov goodness of fit tests, the lowest value of the Akaike information criterion (AIC), and the lowest value of the Bayesian information criterion (BIC).

Table 5. Distribution (%) of wind directions for the year 2004

Direction	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Yearly
North	8	8	9	9	13	14	15	14	12	11	6	5	10
North-East	5	7	6	8	11	10	10	9	5	5	4	3	7
East	5	8	8	9	9	10	11	10	8	4	5	5	8
South-East	15	16	16	13	9	11	10	9	11	8	11	15	12
East	27	24	27	23	14	15	12	13	16	19	26	31	21
South-West	13	11	11	13	10	9	8	9	12	16	19	16	12
West	16	17	13	14	18	15	16	18	19	20	19	16	17
North-West	10	10	6	8	13	15	14	15	14	16	9	8	12
Calm	11	12	15	11	13	14	19	17	14	11	9	9	13

4. RESULTS

We consider the following 11 bins for the Chi-square goodness of fit. The bins boundaries and corresponding frequencies (observed values) for the data are shown by Table 1.

Table 2 shows the final results of statistical modelling.

Statistical fitting of wind pressure for the territory of Kazan shows, that the minimum deviations from results of meteorological observations from the point of view of goodness of fit tests (Chi-square and Kolmogorov—Smirnov) takes place at the use of the normal distribution with the following parameters: mean value of the maximum annual pressure of wind $\mu = 28.668$ and sample standard deviation $\sigma = 9.848$.

From the point of view of the log-likelihood value and information criteria AIC and BIC, the best fit is the truncated normal distribution, while the difference between the normal and truncated normal distributions is quite small. Rule of thumb says that if a model is more than 2×AIC units lower than another, then it is considered significantly better than that model. As we can see, this is not the case for the normal and truncated normal distributions.

In any case, the Weibull distribution does not provide a good fit.

5. ADDITIONAL DESCRIPTIVE STATISTICS ABOUT WIND IN KAZAN

According to meteorological observations in the territory of Kazan, weak winds with a velocity 3.1—4.5 m/s prevail. In rare instances, occurrence of strong winds with speed above 14 m/s were observed. The average number of days with strong winds were 10/month, the greatest number of days was 22/month. The average of days with wind velocities in excess of 8 m/s was 89 ‘days/year, and the velocity of 15 m/s only 9 days/year.

Tables 3 and 4 show the monthly numbers of maximum velocity and wind gusts for the year 2004.

In winter, spring and autumn months, southerly winds with a repeatability of 49—56% blow. In the summer months, northwesterly winds with a repeatability of 44—47% prevail (see Table 5).

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