

# Analysis of orbital theories for the construction of the numerical theory of the lunar physical librations

A.A.Zagidullin<sup>1</sup>, N.K.Petrova<sup>1,2</sup>, V.S.Usanin<sup>1</sup>, A.O. Andreev<sup>1</sup>, Y. A.Nefedyev<sup>1</sup>

<sup>1</sup>Kazan Federal University, Institute of Physics, Kazan, 420008 Russia

<sup>2</sup>Kazan State Power Engineering University, Kazan, 420034 Russia

E-mail: arhtur.zagidullin@ya.ru

**Abstract:** When using the numerical approach to construct the theory of physical libration of the Moon, it is necessary to solve the problem of comparing the numerical and analytical theories of lunar orbital motion. During these studies, the impact on the parameters of physical libration from the Moon's center of mass motion is determined. The paper contains the results of determining the effects caused by distinction in the position of lunar center of mass whose location is obtained according to two various lunar orbital theories. The analytical theory by Gutzwiller and Schmidt [1] constructed within the main problem of the Moon's motion and DE432 numerical theory taking into account a large number of factors, that extend beyond the main issue and that would be complicated or even impossible to consider when obtaining the analytical solution, are compared. Some reductions had been carried out in order to bring both theories to a unified reference system. Then the positions of the Moon's center of mass obtained in those theories were compared at an interval of 800 years. As a result, it was established that the amplitude of residual differences at the interval did not exceed 80 arc seconds in longitude and 10 arc seconds in latitude. The main reason for the obtained differences is neglecting planetary perturbations in the analytical theory. Other effects to distinguish the numerical theory from the analytical one are: motion of plane of the ecliptic, the Earth's and Moon's flattening, tidal effects etc. Those effects also cause the difference between the theories but slightly.

## 1. Introduction

Currently, the problem of studying the dynamics and physics of the Moon is relevant in connection with the plans on its exploration in the near future. On the one hand, study of physical libration of the Moon (PLM) is necessary to analyze and interpret the data on long-term lunar laser ranging and for the planned experimental observation of the Moon's rotation using the equipment placed on its surface. On the other hand, simulation of libration for various parameters of the Moon's interior provides a sense of the structure and physical characteristics of the lunar body [3]. At Kazan Federal University there is some long-term experience in constructing the theory of PLM. However, at the present time, when investigating the Moon's rotation, the numerical approach providing a large number of various effects requires consideration. This method provides higher accuracy at comparing theory and observations. That is why at certain stage of constructing the theory of PLM, there is a need to transit from analytical description of the lunar center of mass motion (theory by Gutzwiller and Schmidt) to the numerical theory (dynamic ephemeris DE432 (JPL NASA)).

At the transition from theory [1] to theory [2] of PLM, the correct method of revealing the impact on physical libration from the divergence in position of the center of mass for these 2 types of theories was developed.

## 2. Problem on investigation of orbital theories statement

Theory of PLM is being constructed within the main problem of PLM whose essence lies in the fact that physical body of the Moon is considered rigid, orbital motion does not depend on the rotational one, and the point Sun and Earth are taken as perturbing sources. At the first stage of constructing the numerical theory of PLM, the analytical theory by Gutzwiller and Schmidt [1] was used to obtain the solution for similar analytical model of PLM [4]. This solution was carried out and satisfactory



agreement with the analytical one was achieved [5]. The next stage was related to transit to numerical analytical ephemeris DE432 [2].

It should be noted that the analytical theory by Gutzwiller and Schmidt is constructed within the main problem, solved with Hill-Brown method, and adapted by W. Eckert for computer calculations. The main problem in the Moon's motion considers 3 point bodies: the Moon, Earth, and Sun. In this theory, the ecliptic coordinate system is chosen as a reference system. Because due to precession from planets the ecliptic is exposed to secular motion, at significant time intervals the ecliptic coordinate system is not inertial. Authors of PLM theory [1] estimated maximum error of its each term to be 2 cm in radial direction and 20 cm in other directions. This is the internal accuracy of the theory. Comparison with observations of PLM theory [1] was not carried out.

Numerical dynamic ephemerides of DE series are constructed within post-Newtonian formalism. These models consider accelerations caused by tidal deformation of the Earth as well as shapes of the Earth and Moon. The equations in them are given for all planets of the Solar system, this is why indirect impacts from other planets and impacts from the main asteroid belt and Kuiper belt are taken into account in each ephemeris. Quasi-inertial ICRF system is chosen as a reference system. According to the analysis conducted by the authors of this series of ephemerides, the center of mass position is determined to within 1 meter.

At the transition to numerical theory of orbital motion, divergence from the results [5] in longitude at the very first stage of integration exceeded 200 arc seconds and at interval of a year reached 20000 arc seconds. There was also a marked tendency of residual differences with angular coefficient of 4.6 seconds. The reason for those enormous divergences could be both incorrect transition to dynamical ephemeris and significant impact of the divergences in these two types of theories, since the dynamical ephemeris included a large number of effects extending beyond the main problem.

This is why in the present paper search for the correct transition from ecliptic coordinate system of theory [1] to the one of theory [2] became a central problem.

### **3. Method and orbital theories of physical libration of the Moon results analysis**

The numerical ephemeris constructed in inertial coordinate system reduction to the rotating ecliptic coordinate system, in which theory by Gutzwiller and Schmidt is constructed, was carried out on the basis of complex precession matrix. After all the reduction adjustments had been introduced, the following results were obtained: at time interval of about 800 years the amplitude of residual differences in longitude was between -40 and +80 arc seconds and in latitude did not exceed 13 arc seconds. As a result, one may conclude that such an enormous difference in parameters in the considered theories of PLM is explained by neglecting planetary perturbations in analytical theory of PLM.

In the present work, algorithm of calculating the Earth's orbit elements in relation to the Moon's center of mass using dynamic ephemeris DE432 [5] is developed as well.

The algorithm of comparing orbital theories may be divided into 2 stages: the main problem effects consideration and the effects extending beyond the main problem consideration. Model of "the main problem" in lunar motion (see Fig. 1) is described by simplifying restrictions as follows:

- The lunar body is under the influence of gravitational pull of 2 point masses: the Earth and Sun;
- Heterogeneity of the lunar body is not considered; the lunar body is taken as rigid;
- The Moon's orbital motion does not affect the rotational one.

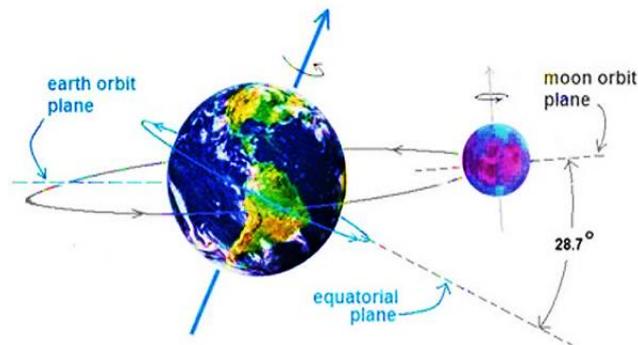


Fig. 1 The Earth – Moon dynamic system

Within the main problem reductions are carried out as follows:

- Consideration of non-linear (secular) terms of Delaunay arguments expansion, which was not performed in the analytical theory of PLM. Over the period of 800 years their impact become noticeable causing divergence in compared longitudes of 832 seconds. This is explained by slow but existing secular acceleration of the Moon;
- Transition from equatorial coordinate system to the ecliptic one used in the analytical theory of PLM;
- Transition from inertial coordinate system to the rotating one used in the numerical theory of PLM;
- Consideration of total precession in the analytical theory:
  - Transition to non-rotating coordinate system;
  - Consideration of total precession from planets;
  - Transition to rotating coordinate system.

After that, reductions to consider factors extending beyond the main problem were performed:

- Impact on the Moon's motion from the Earth's flattening;
- Resonance impact on the Moon from Venus.

Results of comparing residual differences at 2 types of reduction implementation are presented in Fig. 2 – 4.

In Fig. 2 results of comparing solutions for the Moon's latitude (upper line) and longitude (lower line) before and after conducting reductions for consideration of secular acceleration but without taking into account precession from planets (left and right diagrams correspondingly) are given. Obviously, there is no improvements for latitude, while for longitude there is a decrease in differential amplitude and even at the end of integration interval the difference increases up to 180 seconds compared with 300 seconds in non-reduced case.

Consideration of joint impact from secular acceleration and precession from planets (Fig. 3) leads to significant decrease in amplitudes of residual differences both in latitude ( $\pm 10$  arc seconds) and longitude (between  $-30$  to  $+70$  arc seconds). Similar and even slightly better effect is achieved when the Earth's flattening is taken into account:  $\pm 6$  arc seconds in latitude and the same (between  $-30$  to  $+70$  arc seconds) in longitude.

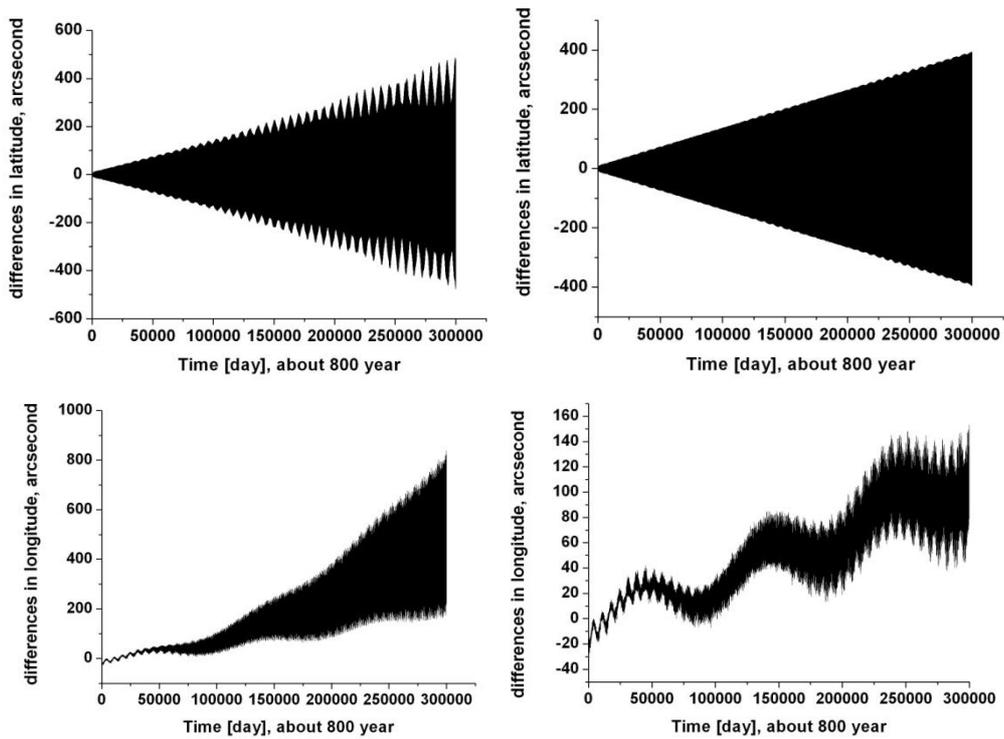


Fig. 2 Left line presents residual differences when comparing numerical and analytical theories of MPL in both latitude and longitude before reductions. Right line presents the same differences after considering all reductions.

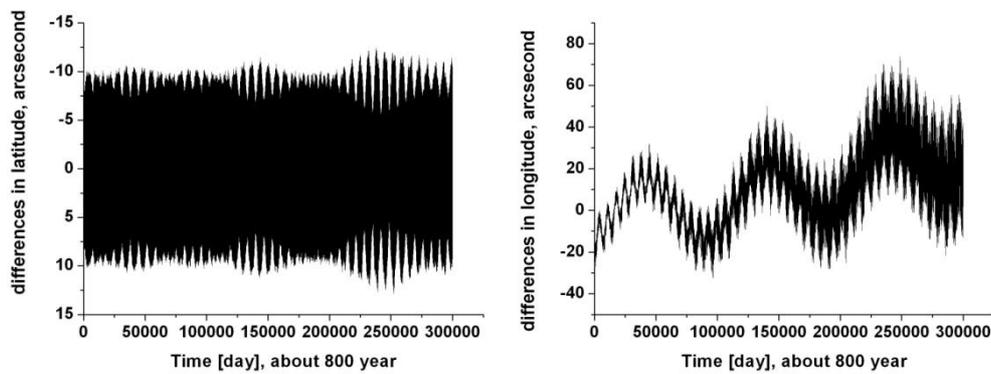


Fig. 3 Consideration of secular acceleration and precession from planets. To the left there is difference in latitude, to the right there is difference in longitude.

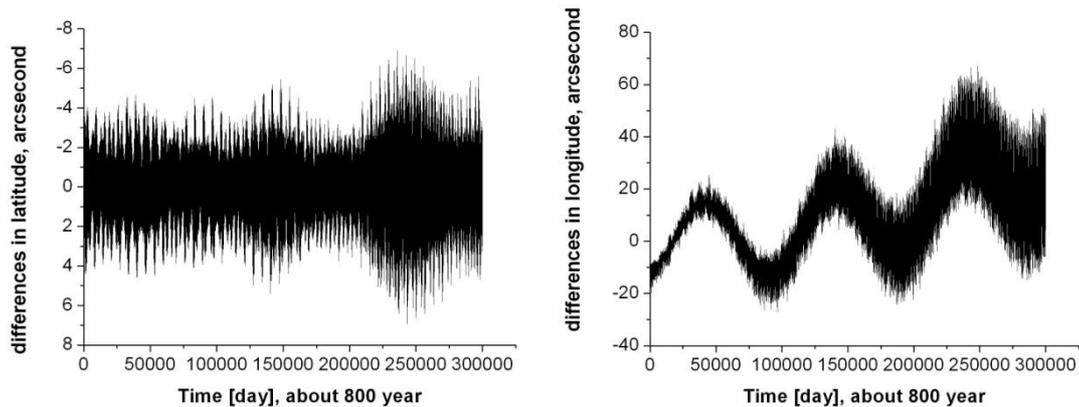


Fig. 4 Consideration of Earth's flattening. To the left there is difference in latitude, to the right there is difference in longitude.

#### 4. Summary and conclusions

This paper provides a short description of analytical theory by Gutzwiller and Schmidt for lunar orbital motion used for MPL analytical theory construction and numerical ephemeris DE432 used for MPL numerical theory construction. Construction method, considered perturbation factors, and accuracy of MPL orbital theories are analyzed. The emphasis is placed on developing an algorithm of bringing both considered theories to the single coordinate system. As a result, the system of MPL orbital theories has been brought to the mean ecliptic J2000, in which the analytical theory is constructed, and to the coordinate system of DE432 theory. During reductions, the effects of precession from planets making the planes of ecliptic to diverge for about 400 arc seconds at integration interval of 800 years are taken into account. The Moon's center of mass position at the same time interval has been determined for both theories as well; their divergences have been estimated. Based on comparative analysis, it has been concluded that at time intervals of about 800 years non-linear terms of higher order of Delaunay classic arguments expansion must not be neglected. Otherwise, there arises a false secular acceleration in the motion of the lunar center of mass: about 830 seconds for 800 years. Taking into account all the considered facts has led to decrease in amplitudes of residual differences down to 80 arc seconds in longitude and 16 arc seconds in latitude. All the results of the MPL theories comparison are presented in the figures. Based on the diagrams analysis, the main reason for the divergences of various theories is neglecting planetary perturbations in the analytical approach. Other effects, such as the Earth's flattening and resonance lunar orbit's perturbations from Venus, do have influence but significantly lower one compared with planetary perturbations.

#### Acknowledgments

The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University. This work was supported by the Russian Foundation for Basic Research, project nos. 16-02-00496 a, 18-02-01098 a, 18-32-00895 mol\_a.

#### References

- [1] Gutzwiller M.C, and Schmidt D S 1986 *Astronomical Papers prepared for the use of the American Ephemeris and Nautical Almanac* (U.S. Government Printing Office, Washington, D.C.) Vol. XXIII Part 1 pp 1–272
- [2] Folkner W M, Williams J G et al. 2014 *IPN Progress Report* pp 1-18.
- [3] Petrova N, Zagidullin A, Nefedyev Y, Kosulin V, Andreev 2017 *Advances in Space Research* Vol. 60 Issue 10 pp 2303-2313.
- [4] Petrova N 1996 *Earth, Moon and Planets* Vol. 73 No 1 pp 71-99

- [5] Zagidullin A, Petrova N, Nefed'ev Yu 2016 *Table of content ISBN 978-5-00015-013-9 Abstracts Moscow IKI N. 7MS3-PS-33*