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CLIMATE PROCESSES

Time Scales of the Global Carbon Cycle Response to External Forcing

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Abstract—The characteristic time scales of the response of the globally averaged climate model with the carbon cycle to external forcing with analysis of the spectrum of the linearized evolution operator of the corresponding dynamic system are evaluated. The model exhibits response time scales of about 4–6 years (related to the carbon dynamics in vegetation) and in the range of 20–100 years (related to the carbon dynamics in nonhumified soil reservoirs). When taking into account the effect of humification, the model reveals the time scale of the response, which is on the order of several millennia. For a closed carbon cycle, the time scale is 10^2 years, which characterizes the joint variations in the atmospheric and oceanic reservoirs. The proposed approach is highly universal and can be used for a wide range of tasks.

Keywords: conceptual climate model, global carbon cycle, time scales, spectral Decomposition

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INTRODUCTION

The analysis of the characteristic time scales of the response of the Earth's climate system to external forcings is among the important tasks in modern science dealing with climate. The analysis is especially significant in the case of nonstationary external forcings, including radiative forcing and natural and anthropogenic emissions of radioactive substances into the atmosphere, including greenhouse gases. It is necessary to estimate particularly the duration of climate changes after the cessation, stabilization, or abrupt alteration of external forcings ("legacy effect") [1]. Due to the existence of different time scales in the climate system, hysteresis-like effects [2–4] or peculiarities in climate cause-and-effect relationships [5–7] manifest themselves.

In geochemical problems, the ratio between the substance stock C in the reservoir of the system and the outgoing flux F from this reservoir is often used to estimate the time scale of the response [8–12]:

$$\tau_{res} = C/F. \quad (1)$$

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The time scale calculated this way characterizes the residence time of a substance within the reservoir. In [10, 11], calculations based on Earth's climate system models for terrestrial ecosystems as a whole (i.e., for the combined reservoir of vegetation, litter fall, and soil) indicated that τ_{res} is on the order of several years in regions dominated by forest ecosystems and on the order of decades to centuries in regions with grassland (including tundra) vegetation. It was noted in [10] that changes in τ_{res} for the terrestrial biomass reservoir due to climate change are a primary source of uncertainty in estimates of future carbon stock changes in terrestrial vegetation. It was found in [12] that the residence time of carbon decreased significantly both in terrestrial vegetation and in soil from the middle of the 19th century to the end of the 20th century during the accumulation of CO_2 in the atmosphere and associated climate warming.

We note that the residence time of a substance in the system reservoirs, determined by the stock and the intensity of the outgoing or incoming flow, accurately characterizes the time scales of the response to external forcing only in a state of dynamic equilibrium of the system. The purpose of this work is to assess the characteristic time scales of different versions of globally averaged climate system models with the carbon cycle based on the analysis of the spectrum of the evolutionary operator of the corresponding dynamic system.

MODEL AND ANALYSIS METHOD

The temporal structure of the trajectory of changes in the characteristics of the Earth's system is determined by a set of eigenvalues τ and the time scale τ_G of changes in the intensity of external forcing $\mathbf{G}(t)$: $\tau_\mathbf{G} = |\mathbf{G}(t)| \cdot |d\mathbf{G}/dt|^{-1}$. For example, a dynamic model with an n-dimensional state vector $\mathbf{Y}$ and an autonomous evolution operator $\mathbf{A}(\mathbf{Y})$ and a nonautonomous term in the right-hand side $\mathbf{G}(t)$

$$d\mathbf{Y}/dt = \mathbf{A}(\mathbf{Y}) + \mathbf{G}(t) \tag{2}$$

after linearization with respect to certain state $\mathbf{Y}(0)$ (replacing $\mathbf{A}(\mathbf{Y})$ with the product $\mathbf{JY}$, where $\mathbf{J}$ is the Jacobian of $\mathbf{A}$) is reduced to

$$d\mathbf{Y}/dt = \mathbf{JY} + \mathbf{G}(t). \tag{3}$$

Its formal solution in quadratures has the form

$$\mathbf{Y}(t) = \exp[-\mathbf{J}\cdot(t-t_0)]\cdot\mathbf{Y}(0) + \int_0^t \exp[-\mathbf{J}\cdot(t-\theta)]\cdot\mathbf{G}(\theta)\,d\theta. \tag{4}$$

In this case, the time scales of the response are entirely determined by the spectrum of the Jacobian $\mathbf{J}$, a set of its eigenvalues $\{\lambda_1, \lambda_2, ..., \lambda_n\}$, satisfying the condition

$$\mathbf{Jy}_j = \lambda_j \mathbf{y}_j, \tag{5}$$

where $\mathbf{y}_j$ ($1 \le j \le n$) are the eigenvectors. If

$$\tau_j = 1/\operatorname{Re}\lambda_j, \tag{6}$$

$$p_j = 2\pi/|\operatorname{Im}\lambda_j|, \tag{7}$$

then in (3) the interaction between different time scales takes the form of the displacement of the equilibrium position of the system that occurs with time scale $\tau_\mathbf{G}$, relaxation of the trajectories to this time-varying equilibrium position that occurs with time scale τ_j (or, with a negative sign of τ_j, increasing deviation from it with the same time scale), and oscillations with period p_j (in the case of $\operatorname{Im}\lambda_j \neq 0$). In turn, the eigenvectors characterize the distribution of the response to external forcing by the variables of the system state.

In this work, we analyze the globally averaged climate model with a carbon cycle of the form

$$a_0 dq_a/dt = E - F_L - F_O, \tag{8}$$

$$dC_v/dt = F_p - F_{rv} - F_l, \tag{9}$$

$$dC_s/dt = F_l - F_{rs} - F_{hum}, \tag{10}$$

$$dC_h/dt = F_{hum} - F_{rh}, \tag{11}$$

$$F_L = F_p - F_{ra} - F_{rs} - F_{rh}, \tag{12}$$

$$F_O = B_O\Delta q_a - \Gamma_O\Delta T, \tag{13}$$

$$c\,d\Delta T/dt = I_S(T) - I_T(T, q_a). \tag{14}$$

Here, the variables of the system are also carbon stocks in terrestrial vegetation C_v, in the nonhumified soil reservoir C_s, and in humus (a very slow soil reservoir) C_h, the concentration of carbon dioxide in the atmosphere q_a (in the well-mixed gas approximation), and the global annual average near-surface atmospheric temperature T. Symbols ΔX denotes deviations of variables X from their initial values. External (including anthropogenic) emissions of CO_2 into the atmosphere act as forcings of the system. The nonautonomous forcing of this system is external (e.g., anthropogenic or geological) emissions of CO_2 into the atmosphere with intensity E. Carbon exchange between reservoirs occurs through flows due to photosynthesis of terrestrial vegetation with intensity F_p, autotrophic respiration of terrestrial vegetation with intensity F_{rv}, litter fall/falling off with intensity F_l, decomposition of organic matter in nonhumified and humified soil reservoirs with intensities F_{rs} and F_{rh}, respectively, and humification with intensity F_{hum}. In (14), $I_S(T)$ and $I_T(T, q_a)$ represent the transfer of solar and thermal radiation in the atmosphere, respectively. The I_S(T) dependence is related to the albedo's dependence on snow and ice cover [13]. The dependence of $I_T(T, q_a)$ on the temperature is governed by the Stefan–Boltzmann law for thermal radiation, and this dependence on q_a is related to the greenhouse effect of CO_2. The parameter $a_0 = 2.12$ Pg C/ppm^{-1} is used to convert the atmospheric carbon dioxide concentration into the corresponding mass, and c is the effective heat capacity of the Earth's climate system (ECS) per unit area.

To calculate the intensity of CO_2 absorption from the atmosphere by the ocean F_O, we use dynamic equations with variable rather than constant coefficients B_O and Γ_O as in (13) [5, 6], but this would increase the dimension of the system. The aim of this study is to analyze the time scales of the response of the terrestrial carbon cycle, and for the sake of simplicity, we use relation (13).

For the intensities of flows in the model, we employ the following relations [5, 6, 8, 14]:

$$F_p = A_p g(q_a) f_p(\Delta T), \tag{15}$$

$$F_{rv} = A_{rv} C_v f_{rv}(\Delta T), \tag{16}$$

$$F_l = A_l C_v, \tag{17}$$

$$F_{rs} = A_{rs} C_s f_{rs}(\Delta T), \tag{18}$$

$$F_{rh} = A_{rh} C_h f_{rh}(\Delta T), \tag{19}$$

$$F_{hum} = A_{hum} C_s. \tag{20}$$

Here, A_i (i is one of the symbols "p", "rv", "l", "rs", "rh", "hum") are coefficients; functions $f_i(\Delta T)$ have the form

$$f_i(\Delta T) = Q_{10,i}^{\Delta T/T_0}, \tag{21}$$

with coefficients $Q_{10,\,i}$, indicating how much the corresponding flow changes with a temperature change by T_0 = 10 K. Relations (21) are equivalent to the Arrhenius relationship (represented in the conventional form for biogeochemical problems). Function $g(q_a)$ characterizes the influence of the CO_2 content in the atmosphere on the intensity of photosynthesis, known as the fertilization effect [15]. The absence of carbon stock in vegetation in expression (15) for photosynthesis intensity corresponds to relatively dense terrestrial vegetation. Wildfire contributions to the carbon cycle dynamics are not considered in this model.

The system (8)–(14), along with the energy conservation equation [13], includes the carbon mass conservation equations in the ECS. It is similar to the system used in [5, 6, 14] but extended to include soil organic matter humification in accordance with [16]. The representation of flows between reservoirs in terrestrial carbon cycle models, where the flow intensity is proportional to the carbon stock in the donor reservoir, ensures that all λ_j ($1 \le j \le n$) have negative real parts [17]. Thus, the system is stable, and the real parts of its eigenvalues correspond to relaxation times to the equilibrium position.

The linearization of the model (8)–(21) was performed with respect to the stationary state denoted as "*". The values of carbon stocks in individual reservoirs for the equilibrium state and corresponding flows are similar to those used in [15]. To be specific, we assumed that $g(q_{a,*}) = 1$. In particular, the equation for temperature (14) after linearization takes the form

$$cd\Delta T/dt = R(q_a) - \Lambda\Delta T. \quad (22)$$

Here, $R(q_a)$ is the radiative forcing of carbon dioxide and Λ is the climate sensitivity parameter characterizing the contribution of climate feedback to changes in the energy balance of the ECS. In particular, the parameter Λ takes into account changes in the content of water vapor, the primary greenhouse gas, in the atmosphere due to climate variations. In this case, the radiative forcing takes the form [1]

$$R(q_a) = R_0 \ln((q_a + q_{a,*})/q_{a,*})] \quad (23)$$

with $R_0 = 5.4$ W m^{-2}. After linearization, it becomes

$$R(q_a) = R_0 q_a/q_{a,*}. \quad (24)$$

The model considered in this work does not take into account the hydrological cycle and its influence on the carbon cycle explicitly. However, it is implicitly considered due to the choice of model coefficients with respect to the close relationship between temperature and precipitation changes on decadal and longer time scales in global averaging of both variables [18, 19].

RESULTS

In this work, we consider four variants of the model described in section 2.

(1) *Model with two reservoirs (vegetation and nonhumified soil) under constant climate and atmospheric* CO_2 *content.*

The dynamic core of this model consists of Eqs. (9) and (10) with $F_{hum} = 0$. This model can be regarded as the simplest model of terrestrial ecosystems under preindustrial climate conditions. For equilibrium values of the carbon stocks in vegetation and soil $C_{v,*}$ = 450–650 Pg C and $C_{s,*}$ = 1500–2400 Pg C, respectively, the intensities of photosynthesis are in the range of 93–125 g C/yr with approximately equal values of autotrophic respiration and litterfall intensity. This leads to values of coefficients A_p = 93–125 Pg C/yr, $A_{rv} = A_l = A_p/C_{v,*}$ = 0.07–0.14 yr^{-1}, $A_{rs} = A_{rv}C_{v,*}/C_{s,*}$ = 0.05–0.06 yr^{-1}.

The eigenvalues for the obtained system of equations are determined analytically and are $\lambda_1 = -(A_{rv} + A_l)$, $\lambda_2 = A_{rs}$. They correspond to time scales τ_1 = 3.5–7.0 yr and τ_2 = 15–20 yr. For the first time scale, the eigenvector is characterized by two nonzero components with $\Delta C_s/\Delta C_v = A_l/[A_{rs} - (A_{rv} + A_l)]$. For the mentioned intervals of values of coefficients A_{rs}, A_{rv}, and A_l, the ratio of $\Delta C_s/\Delta C_v$ varies from –0.61 to –0.88. Thus, changes in C_s and C_v on the time scale τ_1 are of similar magnitude but have opposite signs. For the second time scale, the variations are solely associated with the soil carbon reservoir.

(2) *Model with three reservoirs (vegetation, nonhumified, and humified soils) under constant climate and atmospheric* CO_2 *content conditions.*

The dynamic core of the model consists of Eqs. (9)–(11). This model can also be regarded as a model of terrestrial ecosystems under preindustrial climate conditions with respect to soil humus formation. For this model, λ_1 remains unchanged compared to variant 1 of the model, $\lambda_2 = A_{rs} - A_{hum}$, $\lambda_3 = A_{rh}$. As a result, τ_1 also remains unchanged compared to variant 1 of the model [16]. $A_{hum} = (0.5–3) \times 10^{-2}$ yr^{-1}, and $A_{rh} = (2–5) \times 10^{-4}$ yr^{-1}. In this case, λ_2 increases slightly compared to variant 1 of the model and corresponds to the time scale of 11–18 years. For the third eigenvalue, the time scale is 2000–5000 years.

For the second and third time scales, similarly to what was obtained for variant 1 of the model, the components of the eigenvector are dominated by changes in carbon stocks in nonhumified and humified soil reservoirs, with $(\Delta C_h/\Delta C_s)_{2,3} = A_{hum}/[A_{rh} - (A_{rs} + A_{hum})]$ = 0.09–0.33.

(3) *Closed carbon cycle* (8–13), (15–21) *with* $\Delta T \equiv 0$ (i is one of the symbols "p", "rv", "l", "rs", "rh", "hum").

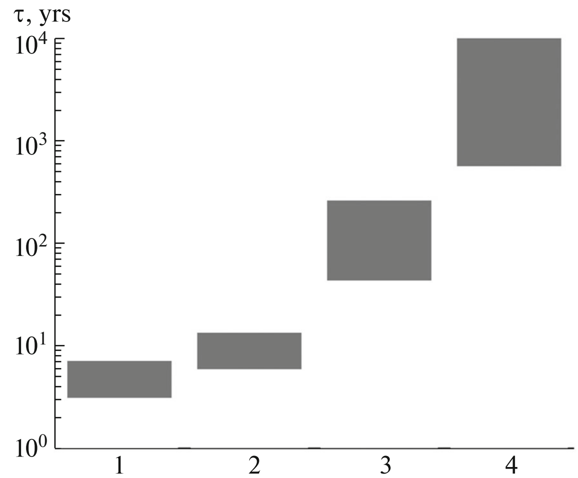

Fig. 1. Ranges of changes in time scales of the response to external forcing for variant 3 of the model (depending on the values of its parameters). The *x*-axis plots the eigenvalue number of the linearized evolution operator for the model considered.

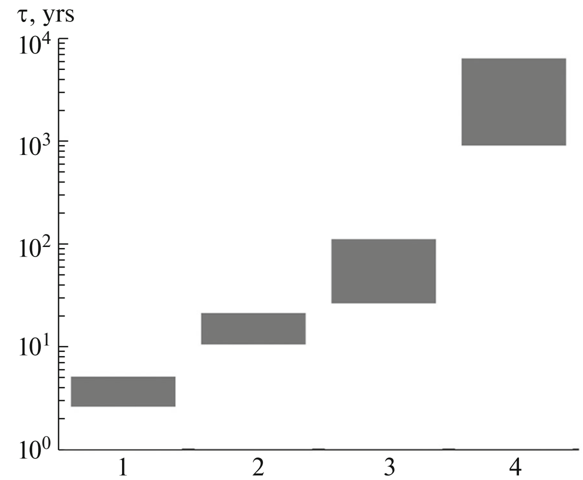

Fig. 2. Similar to Fig. 1 but for variant 4 of the model.

According to [1], this variant of the model corresponds to biogeochemical interactions in the Earth system (i.e., without considering climate changes). In this case,

$$g(q_a) = g_1(q_a)/g_1(q_{a,*}), \quad (25)$$

where

$$g_1(q_a) = q_a/(q_a + q_{a,1/2}), \quad (26)$$

with $q_{a,1/2} = 150$ ppmv [14]. The eigenvalues and eigenvectors of this model were determined numerically using the eigenfunction of the R computational system. (https://cran.r-project.org/web/packages/RcppEigen/index.html). For this system, one of the eigenvalues almost coincides with the largest eigenvalue in magnitude (i.e., corresponding to the shortest time scale) for variant 2. It turned out that λ_1, τ_1, and $\mathbf{y}_1$ almost do not change compared to the model variants considered previously (Fig. 1). Accordingly, the orders of magnitude of λ_2 and $\mathbf{y}_2$ do not change either. The variations on this time scale are mainly related to vegetation and the relatively fast soil carbon reservoir. Accounting for carbon exchange between the ocean and the atmosphere leads to the appearance of the time scale τ_4 = 50–250 yr, which decreases with increasing coefficient B_O [13]. Additionally, as in variant 2 of the model, the time scale τ_3 of the order of several thousand years associated with the response of carbon content in humus is identified.

(4) Closed carbon cycle interacting with climate (8)–(14), (15)–(22), and (24).

This variant can be regarded as a globally averaged carbon cycle model interacting with other components of the Earth's climate system. In this case, we used in (21) the values $Q_{10,p} = 1.5$, $Q_{10,l} = Q_{10,hum} = 1$, $Q_{10,rv} = 2.15$, $Q_{10,rs} = Q_{10,rh} = 2.4$ [5, 6, 14, 16]. For this variant of the model, the eigenvalues, corresponding time scales, and eigenvectors remain almost unchanged (Fig. 2). A time scale on the order of 10^6 yr appears, where variations in the carbon stock in humus play the dominant role. It is associated with the interaction of the slow dynamics of carbon stocks in humus with other carbon cycle reservoirs.

DISCUSSION AND CONCLUSIONS

The estimates of the time scales of the response from a globally averaged climate model with a carbon cycle to external forcings were obtained based on the analysis conducted. The use of the spectrum analysis of the linearized evolution operator of the corresponding dynamic system made it possible to generalize the results obtained for the states of the Earth's climate system far from the equilibrium positions of the system (but for which the linearized system adequately reproduces the dynamics of the initial system at least at a qualitative level).

It was established that, for all variants of the system, the time scales of response of about 4–6 years (associated with vegetation carbon dynamics) and in the span of 20–100 years (associated with carbon dynamics in nonhumified soil reservoirs) consistently manifest themselves. The stable presence of such time scales suggests, in particular, the possible use of active layer models that do not take into account soil carbon humification to analyze changes in the terrestrial carbon cycle on time scales from several years to centuries. The identified time scales are consistent with those obtained in [10, 11]. In particular, the time scale τ_1 of several years is close to that obtained in [10, 11] for the regions with forest vegetation, where vegetation plays a major role in carbon storage and turnover processes. The time scale τ_2 of several decades is consistent with the findings in [10, 11] for the regions with grass (including tundra) vegetation, where soil plays a major role in carbon storage and turnover processes.

When accounting for the humification effect in the model (variant 2), a response time scale on the order of millennia was identified. For the closed carbon cycle, a time scale on the order of 10^2 yr emerged, which characterized covariations in atmospheric and oceanic reservoirs in the model. Formally, this common reservoir can be divided into atmospheric and oceanic components (for example, by determining the change in the carbon stock in the ocean according to $\Delta C_O = \int_0^t F_O dt$), but this will not lead to the appearance of a new time scale of response in the model under consideration.

We note that the response of the linearized model to small changes in its parameters or to changes in external forcings occurs smoothly. For nonlinear systems, the solution may develop not only eigenfrequencies but also other time scales, or qualitative changes may occur in the system behavior; so-called critical points may arise [20]. However, from our experience, the nonlinearity of the system (8)–(21), (23) is not strong enough to exhibit such resonance phenomena and qualitative changes in solutions when external forcings change.

The approach we used in this work is highly universal and can be applied to a wide range of problems.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

REFERENCES

1. *Climate Change 2021: the Physical Science Basis. Working Group I Contribution to the 6th Assessment Report of the Intergovernmental Panel on Climate Change,* Ed. by V. Masson-Delmotte, (Cambridge Univ. Press., 2021).
2. A. V. Eliseev, P. F. Demchenko, M. M. Arzhanov, et al., Clim. Dyn. **42** (5–6), 1203–1215 (2014).
3. A. V. Eliseev, P. F. Demchenko, M. M. Arzhanov, and I. I. Mokhov, Dokl. Earth Sci. **444** (2), 725–729 (2012).
4. S.-K. Kim, J. Shin, S.-I. An, et al., Nat. Clim. Change **12** (9), 834–840 (2022).
5. K. E. Muryshev, A. V. Eliseev, I. I. Mokhov, and A. V. Timazhev, Dokl. Earth Sci. **463** (2), 863–868 (2015).
6. K. E. Muryshev, A. V. Eliseev, I. I. Mokhov, and A. V. Timazhev, Glob. Planet. Change **148**, 29–41 (2017).
7. I. I. Mokhov, Herald Russ. Acad. Sci. **92** (1), 1–12 (2022).
8. O. Kwon and J. L. Schnoor, Glob. Biogeochem. Cycles **8** (3), 295–305 (1994).
9. A. Friend, W. Lucht, T. Rademacher, et al., Proc. Nat. Acad. Sci. U.S.A. **111** (9), 3280–3285.
10. N. Carvalhais, M. Forkel, M. Khomik, et al., Nature **514** (7521), 213–217 (2014).
11. C. D. Koven, J. Q. Chambers, K. Georgiou, et al., Biogeosciences **12** (17), 5211–5228 (2015).
12. D. Wu, S. Piao, D. Zhu, et al., Global Change Biol. **26** (9), 5052–5062 (2020).
13. M. I. Budyko, Meteorol. Gidrol., No. 11, 3–12 (1968).
14. A. V. Eliseev and I. I. Mokhov, Theor. Appl. Climatol. **89** (1-2), 9–24 (2007).
15. A. V. Eliseev, Fundam. Prikl. Klimatol. **4**, 9–31 (2017).
16. S. Zaehle, S. Sitch, B. Smith, and F. Hatterman, Global Biogeochem. Cycles **19** (3), GB3020 (2005).
17. A. C. Lasaga, Geochim. Cosmochim. Acta **44** (6), 815–828 (1980).
18. I. I. Mokhov, *Diagnostics for the Climate System Structure* (Gidrometeoizdat, St. Petersburg, 1993) [in Russian].
19. I. M. Held and B. J. Soden, J. Clim. **19** (21), 5686–5699 (2006).
20. A. V. Eliseev, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **66** (2), 87–103 (2023).

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