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Propagation of gravitational waves in Chern–Simons axion $F(R)$ gravity

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ABSTRACT

In this paper we shall study the evolution of cosmological gravitational waves in the context of Chern–Simons axion $F(R)$ gravity. In the case of Chern–Simons axion $F(R)$ gravity there exist spin-0, spin-2 and spin-1 modes. As we demonstrate, from all the gravitational waves modes of the Chern–Simons axion $F(R)$ gravity, only the two tensor modes are affected, while the spin-0 and spin-1 modes are not affected at all. With regard to the two tensor modes, we show that these modes propagate in a non-equivalent way, so the resulting tensor modes are chiral. Notably, with regard to the propagation of the spin-2 graviton modes, the structure of the dispersion relations becomes more complicated in comparison with the Einstein gravity with the Chern–Simons axion, but the resulting qualitative features of the propagating modes are not changed. With regard to the spin-0 and spin-1 modes, the Chern–Simons axion $F(R)$ gravity contains two spin-0 modes and no vector spin-1 mode at all. We also find that for the very high energy mode, both the group velocity and the phase velocity are proportional to the inverse of the square root of the wave number, and therefore the velocities become smaller for larger wave numbers or even vanish in the limit that the wave number goes to infinity.

1. Introduction

The presence of a dark component of matter in the Universe was assumed early after the first galactic rotation curves appeared. Since then, theoretical physics studies were focused on proposing massive particles weakly interacting with luminous matter, the so-called WIMPs (Weakly Interacting Massive Particles), and there exist examples coming from various theoretical contexts, see for example [1–6]. To be honest, for the moment only indications exist that support the particle nature of dark matter, such as the observational data from the bullet cluster. However, nearly two decades of searches did not result in finding any WIMP. A crucial point to stress is that all the dark matter searches focused in mass ranges from a few MeV up to hundreds of GeV's. However, only recently the experimentalists focused their interest in searching WIMPs having masses a few eV or even sub-eV.

String theory is up to date the most prominent theory that may describe in a consistent way the quantum theory of gravity. One of the interesting predictions of string theory is the existence of low-mass axion [7–10] particles, other than the QCD axions. The axions are particularly appealing as dark matter candidates, since the mass range that these may have, is not investigated yet, and only the last 5 years experimentalists turned their focus on the axions. There is a plethora of experimental [11–19] and theoretical proposals related to the axions [20–68]. Axions can induce particularly interesting effects in the phenomenology of gravitational theories, since Chern–Simons terms of the form $U(\phi)\tilde{R}R$ are allowed in the theory [69–82]. The Chern–Simons terms produce non-equivalent polarizations in the tensor modes of the underlying gravitational theory [83,84], and in the literature there exist timely studies on chiral gravitational waves [85–92].

In this paper we shall study a general Chern–Simons Axion $F(R)$ gravity [93–99], by mainly focusing on the possibility of generating non-equivalent polarizations in the tensor modes of the gravitational waves. We shall be interested in the primordial

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5. Summary

In summary, we have investigated the gravitational wave in the context of Chern–Simons axion $F(R)$ gravity. For the spin-0 scalar mode and the spin-1 vector mode, we demonstrated that for these modes, the situation is not changed from the standard $F(R)$ gravity coupled with quintessence type scalar field, and in addition, the Chern–Simons axion term does not affect the propagation of these modes. This result was also known for the case of Chern–Simons axion Einstein gravity, as it was shown in Ref. [83]. Actually, the Chern–Simons term does not affect the scalar perturbations at all, and it affects solely the tensor perturbations. As a result, we have two propagating scalar modes and no propagating vector mode. With regard to the propagation of the spin-2 graviton mode, the structure of the dispersion relations become more complicated in comparison with the Einstein gravity with the Chern–Simons axion, but the qualitative features of the propagating modes are not changed, and actually non-equivalent polarization modes occur in both Einstein Chern–Simons and $F(R)$ gravity Chern–Simons theory. Our study is focused mainly on primordial gravitational modes, so in a future work we shall address several related issues, such as the conservation of the amplitude of gravitational waves at large scales, and the effect of the function $U(\phi)$ and of the $F(R)$ gravity itself on the polarization asymmetry of the primordial gravitational waves.

Although the dispersion relation (30) tells that the qualitative structure of the tensor modes is not extensively changed in comparison to that corresponding to the Chern–Simons axion Einstein gravity, there appear rather strange behaviors in the very high energy mode where $k \gg H$ and $k \gg \frac{F'}{\dot{U}}$. In the mode the frequency ω is always complex and proportional to the square root of the wave number k. Therefore an amplified and a decaying gravitational wave mode always occur, and also both the group velocity and the phase velocity are proportional to $\frac{1}{\sqrt{k}}$. Then both the group velocity and the propagating velocity become smaller for larger k, and even vanish in the limit of $k \to \infty$.

Finally let us note that in the present work we have found two propagating scalar modes, with the one being the scalar mode which appears commonly in the context of higher derivative gravity [102], and with the other being the pseudo-scalar mode corresponding to the axion scalar. Usually the scalar mode does not mix with the pseudo-scalar mode, but if the parity symmetry is broken by the non-trivial value of the Chern–Simons term, a mixing can occur in general.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. Detailed form of perturbed Einstein tensor components

In this Appendix we present the detailed form of the tensor expressions needed in the text, but have quite extended form. The full expression of the perturbed Einstein tensor is,

$$
\begin{aligned}
\delta G_{\mu\nu} =& \frac{1}{2} F\left(R^{(0)}\right) h_{\mu\nu} - \frac{1}{2} \Big(\nabla^{(0)}_{\mu} \nabla^{(0)\rho} h_{\nu\rho} + \nabla^{(0)}_{\nu} \nabla^{(0)\rho} h_{\mu\rho} \\
& - \Box^{(0)} h_{\mu\nu} - \nabla^{(0)}_{\mu} \nabla^{(0)}_{\nu} \left(g^{(0)\rho\lambda} h_{\rho\lambda} \right) \\
& - 2 R^{(0)\lambda\ \ \rho}_{\ \ \ \ \nu\ \ \mu} h_{\lambda\rho} + R^{(0)\rho}_{\ \ \ \ \mu} h_{\rho\nu} + R^{(0)\rho}_{\ \ \ \ \nu} h_{\rho\mu} \Big) F'\left(R^{(0)}\right) \\
& + \frac{1}{2} g^{(0)}_{\mu\nu} F'\left(R^{(0)}\right) \big(-h_{\rho\sigma} R^{(0)\rho\sigma} + \nabla^{(0)\rho} \nabla^{(0)\sigma} h_{\rho\sigma} \\
& - \Box^{(0)} \left(g^{(0)\rho\sigma} h_{\rho\sigma} \right) \big) \\
& + \left(-R^{(0)}_{\mu\nu} + \nabla^{(0)}_{\mu} \nabla^{(0)}_{\nu} - g^{(0)}_{\mu\nu} \Box^{(0)} \right) \big(F''\left(R^{(0)}\right) \big(-h_{\rho\sigma} R^{(0)\rho\sigma} \\
& + \nabla^{(0)\rho} \nabla^{(0)\sigma} h_{\rho\sigma} - \Box^{(0)} \left(g^{(0)\rho\sigma} h_{\rho\sigma} \right) \big) \big) \\
& + \frac{1}{2} g^{(0)\kappa\lambda} \left(\nabla^{(0)}_{\mu} h_{\nu\lambda} + \nabla^{(0)}_{\nu} h_{\mu\lambda} - \nabla^{0}_{\lambda} h_{\mu\nu} \right) \partial_{\kappa} F'\left(R^{(0)}\right) \\
& + g^{(0)}_{\mu\nu} g^{(0)\rho\tau} g^{(0)\sigma\eta} h_{\rho\sigma} \nabla^{(0)}_{\tau} \nabla^{(0)} \eta F'\left(R^{(0)}\right) \\
& - \frac{1}{2} g^{(0)}_{\mu\nu} g^{(0)\rho\sigma} g^{(0)\kappa\lambda} \big(\nabla^{(0)}_{\rho} h_{\sigma\lambda} + \nabla^{(0)}_{\sigma} h_{\rho\lambda} \\
& - \nabla^{0}_{\lambda} h_{\rho\sigma} \big) \partial_{\kappa} F'\left(R^{(0)}\right) \\
& + \frac{1}{2} \left(- \frac{\omega\left(\phi^{(0)}\right)}{2} g^{(0)\rho\sigma} \partial_{\rho} \phi^{(0)} \partial_{\sigma} \phi^{(0)} - V\left(\phi^{(0)}\right) \right) h_{\mu\nu} \\
& + \frac{\omega\left(\phi^{(0)}\right)}{4} g^{(0)}_{\mu\nu} h_{\rho\sigma} \partial^{\rho} \phi^{(0)} \partial^{\sigma} \phi^{(0)} \\
& + 2 \left(h_{\mu\xi} g^{(0)}_{\nu\sigma} + h_{\mu\sigma} g^{(0)}_{\nu\xi} + g^{(0)}_{\mu\xi} h_{\nu\sigma} + g^{(0)}_{\mu\sigma} h_{\nu\xi} \right) \\
& \times \tilde{\epsilon}^{(0)\zeta\eta\rho\xi} \nabla^{(0)}_{\tau} \nabla^{(0)}_{\rho} \left(U\left(\phi^{(0)}\right) R^{(0)\tau\sigma}_{\ \ \ \ \ \ \ \zeta\eta} \right) \\
& + 2 \left(g^{(0)}_{\mu\xi} g^{(0)}_{\nu\sigma} + g^{(0)}_{\mu\sigma} g^{(0)}_{\nu\xi} \right) \Big\{ - \frac{1}{2} g^{(0)\alpha\beta} h_{\alpha\beta} \tilde{\epsilon}^{(0)\zeta\eta\rho\xi} \nabla^{(0)}_{\tau} \nabla^{(0)}_{\rho} \\
& \times \left(U\left(\phi^{(0)}\right) R^{(0)\tau\sigma}_{\ \ \ \ \ \ \ \zeta\eta} \right) \\
& + \tilde{\epsilon}^{(0)\zeta\eta\rho\xi} h_{\alpha\beta} \nabla^{(0)\alpha} \nabla^{(0)}_{\rho} \left(U\left(\phi^{(0)}\right) R^{(0)\sigma\beta}_{\ \ \ \ \ \ \ \zeta\eta} \right) \\
& - \frac{1}{2} \tilde{\epsilon}^{(0)\zeta\eta\rho\xi} g^{(0)\tau\alpha} \nabla^{(0)}_{\tau} \nabla^{(0)}_{\rho} \big(2U\left(\phi^{(0)}\right) g^{(0)\sigma\beta} \\
& \times \left(\nabla^{(0)}_{\zeta} \left(\nabla^{(0)}_{\eta} h_{\alpha\beta} + \nabla^{(0)}_{\alpha} h_{\eta\beta} - \nabla^{(0)}_{\beta} h_{\eta\alpha} \right) \right) \big) \\
& - \frac{1}{2} \tilde{\epsilon}^{(0)\zeta\eta\rho\xi} g^{(0)\tau\alpha} \nabla^{(0)}_{\tau} \big(U\left(\phi^{(0)}\right) \big(g^{(0)\sigma\beta} \big(\nabla^{(0)}_{\rho} h_{\beta\gamma} \\
& + \nabla^{(0)}_{\gamma} h_{\rho\beta} - \nabla^{(0)}_{\beta} h_{\rho\gamma} \big) R^{(0)\gamma}_{\ \ \ \ \ \alpha\zeta\eta} \\
& - g^{(0)\gamma\beta} \left(\nabla^{(0)}_{\rho} h_{\beta\alpha} + \nabla^{(0)}_{\alpha} h_{\rho\beta} - \nabla^{(0)}_{\beta} h_{\rho\alpha} \right) R^{(0)\sigma}_{\ \ \ \ \ \gamma\zeta\eta} \\
& - g^{(0)\gamma\beta} \left(\nabla^{(0)}_{\rho} h_{\beta\zeta} + \nabla^{(0)}_{\zeta} h_{\rho\beta} - \nabla^{(0)}_{\beta} h_{\rho\zeta} \right) R^{(0)\sigma}_{\ \ \ \ \ \alpha\gamma\eta} \\
& - g^{(0)\gamma\beta} \left(\nabla^{(0)}_{\rho} h_{\beta\eta} + \nabla^{(0)}_{\eta} h_{\rho\beta} - \nabla^{(0)}_{\beta} hg_{\rho\eta} \right) R^{(0)\sigma}_{\ \ \ \ \ \alpha\zeta\gamma} \big) \big) \\
& - \frac{1}{2} \tilde{\epsilon}^{(0)\zeta\eta\rho\xi} g^{(0)\tau\alpha} \big(-g^{(0)\beta\gamma} \big(\nabla^{(0)}_{\tau} h_{\gamma\rho} + \nabla^{(0)}_{\rho} h_{\tau\gamma} \\
& - \nabla^{(0)}_{\gamma} h_{\tau\rho} \big) \nabla^{(0)}_{\beta} \left(U\left(\phi^{(0)}\right) R^{(0)\sigma}_{\ \ \ \ \ \alpha\zeta\eta} \right) \\
& + g^{(0)\sigma\gamma} \left(\nabla^{(0)}_{\tau} h_{\gamma\beta} + \nabla^{(0)}_{\beta} h_{\tau\gamma} - \nabla^{(0)}_{\gamma} h_{\tau\beta} \right) \\
& \times \nabla^{(0)}_{\rho} \left(U\left(\phi^{(0)}\right) R^{(0)\beta}_{\ \ \ \ \ \alpha\zeta\eta} \right) \\
& - g^{(0)\beta\gamma} \left(\nabla^{(0)}_{\tau} h_{\gamma\alpha} + \nabla^{(0)}_{\alpha} h_{\tau\gamma} - \nabla^{(0)}_{\gamma} h_{\tau\alpha} \right) \\
& \times \nabla^{(0)}_{\rho} \left(U\left(\phi^{(0)}\right) R^{(0)\sigma}_{\ \ \ \ \ \beta\zeta\eta} \right)
\end{aligned}
$$

[87] J.R. Pritchard, M. Kamionkowski, Ann. Physics 318 (2005) 2, http://dx.doi.org/10.1016/j.aop.2005.03.005, astro-ph/0412581.
[88] D.H. Lyth, C. Quimbay, Y. Rodriguez, J. High Energy Phys. 0503 (2005) 016, http://dx.doi.org/10.1088/1126-6708/2005/03/016, hep-th/0501153.
[89] S.H.S. Alexander, M.E. Peskin, M.M. Sheikh-Jabbari, eConf C 0605151 (2006) 0022, hep-ph/0701139.
[90] S.H.S. Alexander, M.E. Peskin, M.M. Sheikh-Jabbari, Phys. Rev. Lett. 96 (2006) 081301, http://dx.doi.org/10.1103/PhysRevLett.96.081301, hep-th/0403069.
[91] Y. Cai, Y.T. Wang, Y.S. Piao, J. High Energy Phys. 1703 (2017) 024, http://dx.doi.org/10.1007/JHEP03(2017)024, arXiv:1608.06508 [astro-ph.CO].
[92] F. Moretti, F. Bombacigno, G. Montani, arXiv:1906.01899 [gr-qc].
[93] S. Nojiri, S.D. Odintsov, V.K. Oikonomou, Phys. Rep. 692 (2017) 1, http://dx.doi.org/10.1016/j.physrep.2017.06.001, arXiv:1705.11098 [gr-qc].
[94] S. Nojiri, S.D. Odintsov, Phys. Rep. 505 (2011) 59, http://dx.doi.org/10.1016/j.physrep.2011.04.001, arXiv:1011.0544 [gr-qc].
[95] S. Nojiri, S.D. Odintsov, eConf C 0602061 (2006) 06; Int. J. Geom. Methods Mod. Phys. 4 (2007) 115, http://dx.doi.org/10.1142/S0219887807001928, hep-th/0601213.
[96] S. Capozziello, M. De Laurentis, Phys. Rep. 509 (2011) 167, http://dx.doi.org/10.1016/j.physrep.2011.09.003, arXiv:1108.6266 [gr-qc].
[97] V. Faraoni, S. Capozziello, Fundam. Theor. Phys. 170 (2010) http://dx.doi.org/10.1007/978-94-007-0165-6.
[98] A. de la Cruz-Dombriz, D. Saez-Gomez, Entropy 14 (2012) 1717, http://dx.doi.org/10.3390/e14091717, arXiv:1207.2663 [gr-qc].
[99] G.J. Olmo, Internat. J. Modern Phys. D 20 (2011) 413, http://dx.doi.org/10.1142/S0218271811018925, arXiv:1101.3864 [gr-qc].
[100] N. Seto, A. Taruya, Phys. Rev. D 77 (2008) 103001, http://dx.doi.org/10.1103/PhysRevD.77.103001, arXiv:0801.4185 [astro-ph].
[101] J. Bielefeld, R.R. Caldwell, Phys. Rev. D 91 (12) (2015) 123501, http://dx.doi.org/10.1103/PhysRevD.91.123501, arXiv:1412.6104 [astro-ph.CO].
[102] C. Bogdanos, S. Capozziello, M. De Laurentis, S. Nesseris, Astropart. Phys. 34 (2010) 236, http://dx.doi.org/10.1016/j.astropartphys.2010.08.001, arXiv:0911.3094 [gr-qc].