223. Measuring viscosity of liquid by the Stokes' method

A ball falling in a liquid medium experiences the force of gravity F_g, the Archimedes (buoyant) force F_A, and resistance of medium (friction) F_r:

$F_g = mg = V\rho_b g$, $F_A = \rho V g$, and

$$F_r = 6\pi\eta r v . \qquad (1)$$

Here V is the ball's volume, ρ_b is its density, ρ is the density of the medium, and g is the free fall acceleration. Equation (1) is called the Stokes' formula. It was derived by an English physicist G. Stokes assuming that (1) the Reynolds number Re << 1 (the flow is laminar), (2) liquid fills the whole space, and (3) liquid wets the ball (that is, the closest layer of the liquid moves together with the ball).
The speed of the ball changes as

$$v(t) = \frac{g v V(\rho_b - \rho)}{6\pi\eta r}\left(1 - \exp\left(\frac{6\pi\eta r}{m}t\right)\right). \qquad (2)$$

Evidently, it tends exponentially to a certain limit value:

$$v_\infty = \frac{gV(\rho_b - \rho)}{6\pi\eta r}. \qquad (3)$$

The time characterizing the increase in the velocity of the ball is called relaxation time:

$$\tau = \frac{m}{6\pi\eta r}. \qquad (4)$$

After several τ periods have passed, the ball's speed can be assumed constant and equal to the limit value.
After substituting the expression of the spheres' volume $V=(4/3)\pi r_b^3$, we get:

$$\eta = \frac{2gr_b^2(\rho_b - \rho)}{9v_\infty}. \qquad (5)$$

Thus, the coefficient of the inner friction can be found experimentally if we know the values of r_b, ρ_b and ρ and measure v_∞.
The Stokes' equation assumes that the ball propagates through an infinite space filled with liquid. In the practical measurements, the ratio of the ball's radius r_b and the radius R of the tube with the liquid should be allowed for as in the expression below:

$$\eta = \frac{2gr_b^2(\rho_b - \rho)}{9v_\infty(1 + 2.4\frac{r_b}{R})}. \qquad (6)$$

Aim of the work:

- Getting acquainted with the theoretical basis if the Stokes' method;
- Measuring the viscosity of liquid.

Instruments:

Cylindrical glass vessel with level marks, filled with the investigated transparent oil or glycerol; set of identical steel balls; stopwatch; ruler.

Algorithm of measurements

1. Measure the inner radius *R* of the cylinder and the distance between the level marks.
2. Drop the ball into the vessel (use the funnel-shaped cap, if provided). Measure the time needed for the ball to pass between the two marks. Check if the speed is constant by comparing the times of passage of two intervals drawn of the tube.
3. Calculate the speed of falling (between the two marks at the biggest distance). Repeat this measurement 6–10 times and find the average value of the speed v_∞.
4. Calculate the viscosity using Eq. (6). Density of the materials and the size of the ball are written on the apparatus. Estimate the inaccuracy.
5. Calculate the relaxation time using Eq. (4).
6. Calculate the resistance force of the medium using the Stokes' formula (1).
7. Calculate the Reynolds number as Re = $2\rho r_b v/\eta$.

Questions

1. Inner friction in liquids.
2. Coefficient of dynamic viscosity (physical meaning).
3. Stokes' method of measuring the viscosity.

Estimating the inaccuracy

RMSD for the time in the set of *n* measurement is $S_t \sqrt{\dfrac{\sum (t_i - \bar{t})^2}{n(n-1)}}$.

Inaccuracy of measuring the time is $\Delta t = c \cdot S_t$, where the Student's coefficient *c* is 2.4 for 6 or 7 measurements, 2.3 for 8 or 9, 2.2 for 10 measurements (assuming the confidence interval is 95%).

Relative error in the final value of the viscosity is $\dfrac{\Delta \eta}{\eta} = \dfrac{\partial \eta}{\partial t} \Delta t = \dfrac{\Delta t}{\bar{t}}$.