9) При каком условии, связывающем коэффициенты a, b, c, интеграл $\int \frac{ax^2+bx+c}{x^3(x-1)^2} dx$ является рациональной функцией?

10) При каких целых значениях n интеграл $\int \sqrt{1+x^n}\,dx$ выражается элементарными функциями?

§ 4.3. РАСЧЕТНЫЕ ЗАДАНИЯ

Задача 1. Найти неопределенные интегралы.

1. $\int (4-3x)e^{-3x}\,dx$.

2. $\int \operatorname{arctg}\sqrt{4x-1}\,dx$.

3. $\int (3x+4)e^{3x}\,dx$.

4. $\int (4x-2)\cos 2x\,dx$.

5. $\int (4-16x)\sin 4x\,dx$.

6. $\int (5x-2)e^{3x}\,dx$.

7. $\int (1-6x)e^{2x}\,dx$.

8. $\int \ln(x^2+4)\,dx$.

9. $\int \ln(4x^2+1)\,dx$.

10. $\int (2-4x)\sin 2x\,dx$.

11. $\int \operatorname{arctg}\sqrt{6x-1}\,dx$.

12. $\int e^{-2x}(4x-3)\,dx$.

13. $\int e^{-3x}(2-9x)\,dx$.

14. $\int \operatorname{arctg}\sqrt{2x-1}\,dx$.

15. $\int \operatorname{arctg}\sqrt{3x-1}\,dx$.

16. $\int \operatorname{arctg}\sqrt{5x-1}\,dx$.

17. $\int (5x+6)\cos 2x\,dx$.

18. $\int (3x-2)\cos 5x\,dx$.

19. $\int (x\sqrt{2}-3)\cos 2x\,dx$.

20. $\int (4x+7)\cos 3x\,dx$.

21. $\int (2x-5)\cos 4x\,dx$.

22. $\int (8-3x)\cos 5x\,dx$.

23. $\int (x+5)\sin 3x\,dx$.

24. $\int (2-3x)\sin 2x\,dx$.

25. $\int (4x+3)\sin 5x\,dx$.

26. $\int (7x-10)\sin 4x\,dx$.

27. $\int (\sqrt{2}-8x)\sin 3x\,dx$.

28. $\int \frac{x\,dx}{\cos^2 x}$.

29. $\int \frac{x\,dx}{\sin^2 x}$.

30. $\int x\sin^2 x\,dx$.

31. $\int \frac{x\cos x\,dx}{\sin^3 x}$.

Задача 2. Вычислить определенные интегралы.

1. $\int\limits_{-2}^{0} (x^2 + 5x + 6)\cos 2x\,dx$.

13. $\int\limits_{-1}^{0} (x^2 + 2x + 1)\sin 3x\,dx$.

2. $\int\limits_{-2}^{0} (x^2 - 4)\cos 3x\,dx$.

14. $\int\limits_{0}^{3} (x^2 - 3x)\sin 2x\,dx$.

3. $\int\limits_{-1}^{0} (x^2 + 4x + 3)\cos x\,dx$.

15. $\int\limits_{0}^{\pi} (x^2 - 3x + 2)\sin x\,dx$.

4. $\int\limits_{-2}^{0} (x + 2)^2 \cos 3x\,dx$.

16. $\int\limits_{0}^{\pi/2} (x^2 - 5x + 6)\sin 3x\,dx$.

5. $\int\limits_{-4}^{0} (x^2 + 7x + 12)\cos x\,dx$.

17. $\int\limits_{-3}^{0} (x^2 + 6x + 9)\sin 2x\,dx$.

6. $\int\limits_{0}^{\pi} (2x^2 + 4x + 7)\cos 2x\,dx$.

18. $\int\limits_{0}^{\pi/4} (x^2 + 17{,}5)\sin 2x\,dx$.

7. $\int\limits_{0}^{\pi} (9x^2 + 9x + 11)\cos 3x\,dx$.

19. $\int\limits_{0}^{\pi/2} (1 - 5x^2)\sin x\,dx$.

8. $\int\limits_{0}^{\pi} (8x^2 + 16x + 17)\cos 4x\,dx$.

20. $\int\limits_{\pi/4}^{3} (3x - x^2)\sin 2x\,dx$.

9. $\int\limits_{0}^{2\pi} (3x^2 + 5)\cos 2x\,dx$.

21. $\int\limits_{1}^{2} x\ln^2 x\,dx$.

10. $\int\limits_{0}^{2\pi} (2x^2 - 15)\cos 3x\,dx$.

22. $\int\limits_{1}^{e^2} \dfrac{\ln^2 x\,dx}{\sqrt{x}}$.

11. $\int\limits_{0}^{2\pi} (3 - 7x^2)\cos 2x\,dx$.

12. $\int\limits_{0}^{2\pi} (1 - 8x^2)\cos 4x\,dx$.

23. $\int\limits_{1}^{8} \dfrac{\ln^2 x\,dx}{\sqrt[3]{x^2}}$.

24. $\displaystyle\int_0^1 (x+1)\ln^2(x+1)\,dx$.

28. $\displaystyle\int_1^e \sqrt{x}\ln^2 x\,dx$.

25. $\displaystyle\int_2^3 (x-1)^3\ln^2(x-1)\,dx$.

29. $\displaystyle\int_{-1}^1 x^2 e^{-x/2}\,dx$.

26. $\displaystyle\int_{-1}^0 (x+2)^3\ln^2(x+2)\,dx$.

30. $\displaystyle\int_0^1 x^2 e^{3x}\,dx$.

27. $\displaystyle\int_0^2 (x+1)^2\ln^2(x+1)\,dx$.

31. $\displaystyle\int_{-2}^0 (x^2+2)e^{x/2}\,dx$.

Задача 3. Найти неопределенные интегралы.

1. $\displaystyle\int \frac{dx}{x\sqrt{x^2+1}}$.

2. $\displaystyle\int \frac{1+\ln x}{x}\,dx$.

3. $\displaystyle\int \frac{dx}{x\sqrt{x^2-1}}$.

4. $\displaystyle\int \frac{x^2+\ln x^2}{x}\,dx$.

5. $\displaystyle\int \frac{x\,dx}{\sqrt{x^4+x^2+1}}$.

6. $\displaystyle\int \frac{(\arccos x)^3-1}{\sqrt{1-x^2}}\,dx$.

7. $\displaystyle\int \mathrm{tg}\,x\ln\cos x\,dx$.

8. $\displaystyle\int \frac{\mathrm{tg}(x+1)}{\cos^2(x+1)}\,dx$.

9. $\displaystyle\int \frac{x^3}{(x^2+1)^2}\,dx$.

10. $\displaystyle\int \frac{1-\cos x}{(x-\sin x)^2}\,dx$.

11. $\displaystyle\int \frac{\sin x-\cos x}{(\cos x+\sin x)^5}\,dx$.

12. $\displaystyle\int \frac{x\cos x+\sin x}{(x\sin x)^2}\,dx$.

13. $\displaystyle\int \frac{x^3+x}{x^4+1}\,dx$.

14. $\displaystyle\int \frac{x\,dx}{\sqrt{x^4-x^2-1}}\,dx$.

15. $\displaystyle\int \frac{x\,dx}{\sqrt[2]{x-1}}$.

16. $\displaystyle\int \frac{1+\ln(x-1)}{x-1}\,dx$.

17. $\displaystyle\int \frac{(x^2+1)\,dx}{(x^3+3x+1)^5}$.

18. $\displaystyle\int \frac{4\arctan x-x}{1+x^2}\,dx$.

19. $\displaystyle\int \frac{x^3}{x^2+4}\,dx$.

20. $\displaystyle\int \frac{x+\cos x}{x^2+2\sin x}\,dx$.

21. $\displaystyle\int \frac{2\cos x+3\sin x}{(2\sin x-3\cos x)^3}\,dx$.

22. $\displaystyle\int \frac{8x-\mathrm{arctg}\,2x}{1+4x^2}\,dx$.

23. $\displaystyle\int \frac{1/(2\sqrt{x})+1}{(\sqrt{x}+x)^2}\,dx$.

24. $\int \dfrac{x}{x^4+1}\,dx.$

25. $\int \dfrac{x+1/x}{\sqrt{x^2+1}}\,dx.$

26. $\int \dfrac{x-1/x}{\sqrt{x^2+1}}\,dx.$

27. $\int \dfrac{\operatorname{arctg} x + x}{1+x^2}\,dx.$

28. $\int \dfrac{x-(\operatorname{arctg} x)^4}{1+x^2}\,dx.$

29. $\int \dfrac{x^3}{x^2+1}\,dx.$

30. $\int \dfrac{(\arcsin x)^2+1}{\sqrt{1-x^2}}\,dx.$

31. $\int \dfrac{1-\sqrt{x}}{\sqrt{x}(x+1)}\,dx.$

Задача 4. Вычислить определенные интегралы.

1. $\displaystyle\int_{e+1}^{e^2+1} \dfrac{1+\ln(x-1)}{x-1}\,dx.$

2. $\displaystyle\int_{0}^{1} \dfrac{(x^2+1)\,dx}{(x^3+3x+1)^2}.$

3. $\displaystyle\int_{0}^{1} \dfrac{4\operatorname{arctg} x - x}{1+x^2}\,dx.$

4. $\displaystyle\int_{0}^{2} \dfrac{x^3\,dx}{x^2+4}.$

5. $\displaystyle\int_{\pi}^{2\pi} \dfrac{x+\cos x}{x^2+2\sin x}\,dx.$

6. $\displaystyle\int_{0}^{\pi/4} \dfrac{2\cos x + 3\sin x}{(2\sin x - 3\cos x)^3}\,dx.$

7. $\displaystyle\int_{0}^{1/2} \dfrac{8x - \operatorname{arctg} 2x}{1+4x^2}\,dx.$

8. $\displaystyle\int_{1}^{4} \dfrac{1/(2\sqrt{x})+1}{(\sqrt{x}+x)^2}\,dx.$

9. $\displaystyle\int_{0}^{1} \dfrac{x\,dx}{x^4+1}.$

10. $\displaystyle\int_{\sqrt{3}}^{\sqrt{8}} \dfrac{x+1/x}{\sqrt{x^2+1}}\,dx.$

11. $\displaystyle\int_{\sqrt{3}}^{\sqrt{8}} \dfrac{x-1/x}{\sqrt{x^2+1}}\,dx.$

12. $\displaystyle\int_{0}^{\sqrt{3}} \dfrac{\operatorname{arctg} x + x}{1+x^2}\,dx.$

13. $\displaystyle\int_{0}^{\sqrt{3}} \dfrac{x-(\operatorname{arctg} x)^4}{1+x^2}\,dx.$

14. $\displaystyle\int_{0}^{1} \dfrac{x^3}{x^2+1}\,dx.$

15. $\displaystyle\int_{0}^{\sin 1} \dfrac{(\arcsin x)^2+1}{\sqrt{1-x^2}}\,dx.$

16. $\displaystyle\int_{1}^{3} \dfrac{1-\sqrt{x}}{\sqrt{x}(x+1)}\,dx.$

29. $\displaystyle\int_{\arccos(1/\sqrt{26})}^{\arccos(1/\sqrt{10})} \frac{12\,dx}{(6+5\,\mathrm{tg}\,x)\sin 2x}$.

30. $\displaystyle\int_{0}^{\pi/3} \frac{\mathrm{tg}^2\,x}{4+3\cos 2x}\,dx$.

31. $\displaystyle\int_{0}^{\arccos(1/\sqrt{6})} \frac{3\,\mathrm{tg}^2\,x - 1}{\mathrm{tg}^2\,x + 5}\,dx$.

Задача 10. Вычислить определенные интегралы.

1. $\displaystyle\int_{\pi/2}^{\pi} 2^8 \sin^8 x\,dx$.

9. $\displaystyle\int_{0}^{2\pi} \sin^2 x \cos^6 x\,dx$

2. $\displaystyle\int_{0}^{\pi} 2^4 \sin^6 x \cos^2 x\,dx$.

10. $\displaystyle\int_{0}^{2\pi} \cos^8 \frac{x}{4}\,dx$.

3. $\displaystyle\int_{0}^{2\pi} \sin^4 x \cos^4 x\,dx$.

11. $\displaystyle\int_{0}^{\pi} 2^4 \sin^8 \frac{x}{2}\,dx$.

4. $\displaystyle\int_{0}^{2\pi} \sin^2 \frac{x}{4} \cos^6 \frac{x}{4}\,dx$.

12. $\displaystyle\int_{-\pi}^{0} 2^8 \sin^6 x \cos^2 x\,dx$.

5. $\displaystyle\int_{0}^{\pi} 2^4 \cos^8 \frac{x}{2}\,dx$.

13. $\displaystyle\int_{\pi/2}^{\pi} 2^8 \sin^4 x \cos^4 x\,dx$.

6. $\displaystyle\int_{-\pi/2}^{0} 2^8 \sin^8 x\,dx$.

14. $\displaystyle\int_{0}^{\pi} 2^4 \sin^2 x \cos^6 x\,dx$.

7. $\displaystyle\int_{\pi/2}^{\pi} 2^8 \sin^6 x \cos^2 x\,dx$.

15. $\displaystyle\int_{0}^{2\pi} \cos^8 x\,dx$.

8. $\displaystyle\int_{0}^{\pi} 2^4 \sin^4 x \cos^4 x\,dx$.

16. $\displaystyle\int_{0}^{2\pi} \sin^8 \frac{x}{4}\,dx$.

17. $\displaystyle\int_0^\pi 2^4 \sin^6 \frac{x}{2} \cos^2 \frac{x}{2}\, dx.$

25. $\displaystyle\int_{\pi/2}^\pi 2^8 \cos^8 x\, dx.$

18. $\displaystyle\int_{-\pi/2}^0 2^8 \sin^4 x \cos^4 x\, dx.$

26. $\displaystyle\int_0^\pi 2^4 \sin^8 x\, dx.$

19. $\displaystyle\int_{\pi/2}^\pi 2^8 \sin^2 x \cos^6 x\, dx.$

27. $\displaystyle\int_0^{2\pi} \sin^6 x \cos^2 x\, dx.$

20. $\displaystyle\int_0^\pi 2^4 \cos^8 x\, dx.$

28. $\displaystyle\int_0^{2\pi} \sin^4 \frac{x}{4} \cos^4 \frac{x}{4}\, dx.$

21. $\displaystyle\int_0^{2\pi} \sin^8 x\, dx.$

29. $\displaystyle\int_0^\pi 2^4 \sin^2 \frac{x}{2} \cos^6 \frac{x}{2}\, dx.$

22. $\displaystyle\int_0^{2\pi} \sin^6 \frac{x}{4} \cos^2 \frac{x}{4}\, dx.$

23. $\displaystyle\int_0^\pi 2^4 \sin^4 \frac{x}{2} \cos^4 \frac{x}{2}\, dx.$

30. $\displaystyle\int_{-\pi/2}^0 2^8 \cos^8 x\, dx.$

24. $\displaystyle\int_{-\pi/2}^0 2^8 \sin^2 x \cos^6 x\, dx.$

31. $\displaystyle\int_0^{2\pi} \sin^4 3x \cos^4 3x\, dx.$

Задача 11. Вычислить определенные интегралы.

1. $\displaystyle\int_0^1 \frac{4\sqrt{1-x} - \sqrt{3x+1}}{\left(\sqrt{3x+1} + 4\sqrt{1-x}\right)(3x+1)^2}\, dx.$

2. $\displaystyle\int_1^{64} \frac{1 - \sqrt[6]{x} + 2\sqrt[3]{x}}{x + 2\sqrt{x^3} + \sqrt[3]{x^4}}\, dx.$

3. $\displaystyle\int_{-14/15}^{-7/8} \frac{6\sqrt{x+2}}{(x+2)^2 \sqrt{x+1}}\, dx.$

4. $\displaystyle\int_6^9 \sqrt{\frac{9-2x}{2x-21}}\, dx.$

17. $\displaystyle\int_0^{4\sqrt{3}} \frac{dx}{\sqrt{(64-x^2)^3}}$.

25. $\displaystyle\int_0^1 \sqrt{4-x^2}\,dx$.

18. $\displaystyle\int_{\sqrt{2}}^{2\sqrt{2}} \frac{\sqrt{x^2-2}}{x^4}\,dx$.

26. $\displaystyle\int_2^4 \frac{\sqrt{x^2-4}}{x^4}\,dx$.

19. $\displaystyle\int_0^{2\sqrt{2}} \frac{x^4\,dx}{(16-x^2)\sqrt{16-x^2}}$.

27. $\displaystyle\int_0^2 \frac{dx}{(4+x^2)\sqrt{4+x^2}}$.

20. $\displaystyle\int_{-3}^3 x^2\sqrt{9-x^2}\,dx$.

28. $\displaystyle\int_0^{\sqrt{2}} \frac{x^4\,dx}{(4-x^2)^{3/2}}$.

21. $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{\sqrt{(1+x^2)^3}}$.

29. $\displaystyle\int_0^{1/\sqrt{2}} \frac{dx}{(1-x^2)\sqrt{1-x^2}}$.

22. $\displaystyle\int_0^2 \frac{dx}{\sqrt{(16-x^2)^3}}$.

30. $\displaystyle\int_0^1 \frac{x^2\,dx}{\sqrt{4-x^2}}$.

23. $\displaystyle\int_0^2 \frac{x^4\,dx}{\sqrt{(8-x^2)^3}}$.

31. $\displaystyle\int_0^{3/2} \frac{x^2\,dx}{\sqrt{9-x^2}}$.

24. $\displaystyle\int_3^6 \frac{x^2-9}{x^4}\,dx$.

Задача 13. Найти неопределенные интегралы.

1. $\displaystyle\int \frac{\sqrt{1+\sqrt{x}}}{x\sqrt[4]{x^3}}\,dx$.

5. $\displaystyle\int \frac{\sqrt[3]{1+\sqrt[3]{x^2}}}{x\sqrt[9]{x^8}}\,dx$.

2. $\displaystyle\int \frac{\sqrt[3]{1+\sqrt{x}}}{x\sqrt[3]{x^2}}\,dx$.

6. $\displaystyle\int \frac{\sqrt[3]{(1+\sqrt[3]{x})^2}}{x\sqrt[9]{x^5}}\,dx$.

3. $\displaystyle\int \frac{\sqrt{1+\sqrt[3]{x}}}{x\sqrt{x}}\,dx$.

7. $\displaystyle\int \frac{\sqrt[3]{(1+\sqrt[3]{x^2})^2}}{x^2\sqrt[9]{x}}\,dx$.

4. $\displaystyle\int \frac{\sqrt[3]{1+\sqrt[3]{x}}}{x\sqrt[9]{x^4}}\,dx$.

8. $\displaystyle\int \frac{\sqrt[3]{(1+\sqrt{x})^2}}{x\sqrt[6]{x^5}}\,dx$.

9. $\displaystyle\int \frac{\sqrt{1 + \sqrt[3]{x^2}}}{x^2}\,dx.$

10. $\displaystyle\int \frac{\sqrt{1 + x}}{x^2\sqrt{x}}\,dx.$

11. $\displaystyle\int \frac{\sqrt[4]{(1 + \sqrt{x})^3}}{x\sqrt[8]{x^7}}\,dx.$

12. $\displaystyle\int \frac{\sqrt[4]{(1 + \sqrt[3]{x})^3}}{x\sqrt[12]{x^7}}\,dx.$

13. $\displaystyle\int \frac{\sqrt[4]{(1 + \sqrt[3]{x^2})^3}}{x^2\sqrt[6]{x}}\,dx.$

14. $\displaystyle\int \frac{\sqrt{1 + \sqrt[4]{x^3}}}{x^2\sqrt[8]{x}}\,dx.$

15. $\displaystyle\int \frac{\sqrt[3]{1 + \sqrt[4]{x^3}}}{x^2}\,dx.$

16. $\displaystyle\int \frac{\sqrt[3]{(1 + \sqrt[4]{x^3})^2}}{x^2\sqrt[4]{x}}\,dx.$

17. $\displaystyle\int \frac{\sqrt[5]{(1 + \sqrt{x})^4}\,dx}{x\sqrt[10]{x^9}}.$

18. $\displaystyle\int \frac{\sqrt[5]{(1 + \sqrt[3]{x})^4}\,dx}{x\sqrt[5]{x^3}}.$

19. $\displaystyle\int \frac{\sqrt[5]{(1 + \sqrt[3]{x^2})^4}}{x^2\sqrt[5]{x}}\,dx.$

20. $\displaystyle\int \frac{\sqrt[5]{(1 + \sqrt[4]{x^3})^4}}{x^2\sqrt[20]{x^7}}\,dx.$

21. $\displaystyle\int \frac{\sqrt[5]{(1 + \sqrt[5]{x^4})}}{x^2\sqrt[25]{x^{11}}}\,dx.$

22. $\displaystyle\int \frac{\sqrt{1 + \sqrt[5]{x^4}}}{x^2\sqrt[5]{x}}\,dx.$

23. $\displaystyle\int \frac{\sqrt[3]{1 + \sqrt[5]{x^4}}}{x^2\sqrt[15]{x}}\,dx.$

24. $\displaystyle\int \frac{\sqrt[3]{(1 + \sqrt[5]{x^4})^2}}{x^2\sqrt[3]{x}}\,dx.$

25. $\displaystyle\int \frac{\sqrt[4]{(1 + \sqrt[5]{x^4})^3}\,dx}{x^2\sqrt[5]{x^2}}.$

26. $\displaystyle\int \frac{\sqrt[3]{1 + \sqrt[4]{x}}}{x\sqrt[3]{x}}\,dx.$

27. $\displaystyle\int \frac{\sqrt[3]{(1 + \sqrt[4]{x})^2}}{x\sqrt[12]{x^5}}\,dx.$

28. $\displaystyle\int \frac{\sqrt[4]{1 + \sqrt[3]{x}}}{x\sqrt[12]{x^5}}\,dx.$

29. $\displaystyle\int \frac{\sqrt[4]{1 + \sqrt[3]{x^2}}}{x\sqrt[6]{x^5}}\,dx.$

30. $\displaystyle\int \frac{\sqrt[3]{1 + \sqrt[5]{x}}}{x\sqrt[16]{x^4}}\,dx.$

31. $\displaystyle\int \frac{\sqrt[5]{1 + \sqrt[3]{x}}}{x\sqrt[5]{x^2}}\,dx.$

Задача 14. Вычислить площади фигур, ограниченных графиками функций.

1. $y = (x - 2)^3$, $y = 4x - 8$.

2. $y = x\sqrt{9 - x^2}$, $y = 0$ $(0 \leqslant x \leqslant 3)$.

3. $y = 4 - x^2$, $y = x^2 - 2x$.

4. $y = \sin x \cos^2 x$, $y = 0$ $(0 \leqslant x \leqslant \pi/2)$.

5. $y = \sqrt{4 - x^2}$, $y = 0$, $x = 0$, $x = 1$.

9) Доказать, что ряд $\sum_{n=1}^{\infty} \frac{\sin n^2 x}{n^2}$ сходится равномерно в интервале $(-\infty, +\infty)$. Можно ли его почленно дифференцировать в этом интервале?

10) Доказать, что если ряд $\sum_{n=1}^{\infty} c_n e^{-nx}$ сходится в точке x_0, то он сходится абсолютно $\forall x > x_0$.

§ 6.3. РАСЧЕТНЫЕ ЗАДАНИЯ

Задача 1. Найти сумму ряда.

1. $\displaystyle\sum_{n=9}^{\infty} \frac{2}{n^2 - 14n + 48}.$

2. $\displaystyle\sum_{n=9}^{\infty} \frac{18}{n^2 - 13n + 40}.$

3. $\displaystyle\sum_{n=8}^{\infty} \frac{4}{n^2 - 12n + 35}.$

4. $\displaystyle\sum_{n=8}^{\infty} \frac{36}{n^2 - 11n + 28}.$

5. $\displaystyle\sum_{n=7}^{\infty} \frac{6}{n^2 - 10n + 24}.$

6. $\displaystyle\sum_{n=7}^{\infty} \frac{54}{n^2 - 9n + 18}.$

7. $\displaystyle\sum_{n=6}^{\infty} \frac{8}{n^2 - 8n + 15}.$

8. $\displaystyle\sum_{n=6}^{\infty} \frac{72}{n^2 - 7n + 10}.$

9. $\displaystyle\sum_{n=5}^{\infty} \frac{10}{n^2 - 6n + 8}.$

10. $\displaystyle\sum_{n=5}^{\infty} \frac{90}{n^2 - 5n + 4}.$

11. $\displaystyle\sum_{n=4}^{\infty} \frac{12}{n^2 - 4n + 3}.$

12. $\displaystyle\sum_{n=4}^{\infty} \frac{18}{n^2 - n - 2}.$

13. $\displaystyle\sum_{n=0}^{\infty} \frac{16}{n^2 + 4n + 3}.$

14. $\displaystyle\sum_{n=0}^{\infty} \frac{36}{n^2 + 7n + 10}.$

15. $\displaystyle\sum_{n=10}^{\infty} \frac{30}{n^2 - 14n + 48}.$

16. $\displaystyle\sum_{n=9}^{\infty} \frac{54}{n^2 - 11n + 28}.$

17. $\displaystyle\sum_{n=9}^{\infty} \frac{36}{n^2 - 12n + 35}.$

18. $\displaystyle\sum_{n=8}^{\infty} \frac{72}{n^2 - 9n + 18}.$

19. $\displaystyle\sum_{n=8}^{\infty} \frac{12}{n^2 - 10n + 24}.$

20. $\displaystyle\sum_{n=7}^{\infty} \frac{18}{n^2 - 7n + 10}.$

21. $\displaystyle\sum_{n=7}^{\infty} \frac{60}{n^2 - 8n + 15}.$

22. $\displaystyle\sum_{n=6}^{\infty} \frac{36}{n^2 - 5n + 4}.$

23. $\sum_{n=6}^{\infty} \dfrac{48}{n^2 - 6n + 8}.$

24. $\sum_{n=3}^{\infty} \dfrac{54}{n^2 + n - 2}.$

25. $\sum_{n=5}^{\infty} \dfrac{6}{n^2 - 4n + 3}.$

26. $\sum_{n=3}^{\infty} \dfrac{18}{n^2 - n - 2}.$

27. $\sum_{n=1}^{\infty} \dfrac{24}{n^2 + 4n + 3}.$

28. $\sum_{n=2}^{\infty} \dfrac{36}{n^2 + n - 2}.$

29. $\sum_{n=0}^{\infty} \dfrac{72}{n^2 + 6n + 8}.$

30. $\sum_{n=0}^{\infty} \dfrac{54}{n^2 + 5n + 4}.$

31. $\sum_{n=1}^{\infty} \dfrac{72}{n^2 + 5n + 4}.$

Задача 2. Исследовать на сходимость ряд.

1. $\sum_{n=1}^{\infty} \dfrac{\sin^2 n\sqrt{n}}{n\sqrt{n}}.$

2. $\sum_{n=1}^{\infty} \dfrac{\operatorname{arctg}^2 n}{n^3}.$

3. $\sum_{n=1}^{\infty} \dfrac{\operatorname{arctg} n^2}{n(n+1)(n+2)}.$

4. $\sum_{n=1}^{\infty} \dfrac{\ln n}{\sqrt[3]{n^7}}.$

5. $\sum_{n=1}^{\infty} \dfrac{3 - \sin n}{n - \ln n}.$

6. $\sum_{n=1}^{\infty} \dfrac{1 - \cos n}{n^3 + 2}.$

7. $\sum_{n=1}^{\infty} \dfrac{n(2 + \cos n\pi)}{2n^2 - 1}.$

8. $\sum_{n=2}^{\infty} \dfrac{3 + \sin n}{\sqrt[3]{n^3 - n}}.$

9. $\sum_{n=1}^{\infty} \dfrac{\sin^2 n}{n^2 + 1}.$

10. $\sum_{n=2}^{\infty} \dfrac{\ln \sqrt{n^2 + 3n}}{\sqrt{n^2 - n}}.$

11. $\sum_{n=1}^{\infty} \dfrac{1 + \cos n}{n^2 + 2}.$

12. $\sum_{n=1}^{\infty} \dfrac{n \cos^2 n}{n^3 + 5}.$

13. $\sum_{n=2}^{\infty} \dfrac{n \ln n}{n^2 - 3}.$

14. $\sum_{n=1}^{\infty} \dfrac{n^2 + 3}{n^3(2 + \cos n\pi)}.$

15. $\sum_{n=1}^{\infty} \dfrac{3 - \cos n}{\sqrt[4]{n^3}}.$

16. $\sum_{n=1}^{\infty} \dfrac{\ln n}{n^3 + n + 1}.$

17. $\sum_{n=1}^{\infty} \dfrac{\sin^2 n}{n^2}.$

18. $\sum_{n=1}^{\infty} \dfrac{\operatorname{arctg}^3 n}{n^4 + 3}.$

15. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+3}} \left(e^{\frac{1}{\sqrt{n}}} - 1 \right).$

16. $\sum_{n=1}^{\infty} \ln \frac{n^2 + 1}{n^2 - n + 2}.$

17. $\sum_{n=1}^{\infty} \sqrt[3]{n} \operatorname{arctg} \frac{1}{n^3}.$

18. $\sum_{n=1}^{\infty} \ln \frac{n^3 + 2}{n^3 + 1}.$

19. $\sum_{n=3}^{\infty} n^3 \operatorname{tg}^5 \frac{\pi}{n}.$

20. $\sum_{n=2}^{\infty} \frac{n+1}{(\sqrt[3]{n} - 1)(n\sqrt[4]{n^3} - 1)}.$

21. $\sum_{n=1}^{\infty} \left(1 - \cos \frac{\pi}{n} \right).$

22. $\sum_{n=1}^{\infty} \sin \frac{\sqrt[3]{n}}{\sqrt{n^5 + 2}}.$

23. $\sum_{n=2}^{\infty} \left(e^{\frac{\sqrt{n}}{n^3 - 1}} - 1 \right).$

24. $\sum_{n=1}^{\infty} \sin \frac{2n + 1}{n^2(n+1)^2}.$

25. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin \frac{2\pi}{2n + 1}.$

26. $\sum_{n=1}^{\infty} \frac{3 + 7n}{5^n + n}.$

27. $\sum_{n=1}^{\infty} n \sin \frac{1}{\sqrt[3]{n^4}}.$

28. $\sum_{n=1}^{\infty} n \left(e^{\frac{1}{n}} - 1 \right)^2.$

29. $\sum_{n=2}^{\infty} \operatorname{arctg} \frac{1}{(n-1)\sqrt[5]{n^2 + 1}}.$

30. $\sum_{n=1}^{\infty} \sin \frac{n}{n^2 \sqrt[3]{n} + 5}.$

31. $\sum_{n=1}^{\infty} \arcsin \frac{n}{(n^2 + 3)^{5/2}}.$

Задача 4. Исследовать на сходимость ряд.

1. $\sum_{n=2}^{\infty} \frac{n+1}{2^n(n-1)!}.$

2. $\sum_{n=1}^{\infty} \frac{4^n}{(n!)^2}.$

3. $\sum_{n=1}^{\infty} \frac{2^{n+1}(n^3+1)}{(n+1)!}.$

4. $\sum_{n=1}^{\infty} \frac{10^n n!}{(2n)!}.$

5. $\sum_{n=1}^{\infty} \frac{(2n+2)!}{2^n(3n+5)}.$

6. $\sum_{n=1}^{\infty} \frac{n+5}{n!} \sin \frac{2}{3^n}.$

7. $\sum_{n=1}^{\infty} \frac{1}{n!} \operatorname{arctg} \frac{5}{n}.$

8. $\sum_{n=1}^{\infty} \frac{n^n}{3^n n!}.$

9. $\sum_{n=1}^{\infty} \frac{n!}{(2n)!} \operatorname{tg} \frac{1}{5^n}.$

10. $\sum_{n=1}^{\infty} \frac{6^n(n^2 - 1)}{n!}.$

11. $\sum_{n=1}^{\infty} \frac{4^n n^2}{(n+2)!}.$

12. $\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}.$

13. $\sum_{n=1}^{\infty} \frac{7^{2n}}{(2n-1)!}.$

14. $\sum_{n=1}^{\infty} \frac{4^n n!}{(3n)!}.$

15. $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{3^n(n+1)!}$.

24. $\displaystyle\sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n+1)}{2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n-1)}$.

16. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^{n-1}}$.

25. $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \cdot \ldots \cdot (3n-2)}{7 \cdot 9 \cdot 11 \cdot \ldots \cdot (2n+5)}$.

17. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(3^n+1)(2n)!}$.

26. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{2^n+3}$.

18. $\displaystyle\sum_{n=1}^{\infty} n! \sin \frac{\pi}{2^n}$.

27. $\displaystyle\sum_{n=1}^{\infty} \frac{(3n+2)!}{10^n n^2}$.

19. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!}{n^n}$.

28. $\displaystyle\sum_{n=2}^{\infty} \frac{4^{n-1}\sqrt{n^2+5}}{(n-1)!}$.

20. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n \sqrt[3]{n^2}}{(n+1)!}$.

29. $\displaystyle\sum_{n=1}^{\infty} \frac{n! \sqrt[3]{n}}{3^n+2}$.

21. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$.

30. $\displaystyle\sum_{n=1}^{\infty} \frac{n!(2n+1)!}{(3n)!}$.

22. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n(n+1)!}{(2n)!}$.

31. $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \cdot \ldots \cdot (3n-2)}{2^{n+1} n!}$.

23. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{(n+2)! 4^n}$.

Задача 5. Исследовать на сходимость ряд.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n} \left(\frac{n}{n+1} \right)^{-n^2}$.

6. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+2}{3n+1} \right)^n n^3$.

2. $\displaystyle\sum_{n=1}^{\infty} n^4 \left(\frac{2n}{3n+5} \right)^n$.

7. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n-3}{5n+1} \right)^{n^2}$.

3. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n^2+1}{n^2+1} \right)^{n^2}$.

8. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{10n+5} \right)^{n^2}$.

4. $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{n} \right)^{n^2} \cdot \frac{1}{4^n}$.

9. $\displaystyle\sum_{n=1}^{\infty} n \arcsin^n \frac{\pi}{4n}$.

5. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n-2} \right)^{n^2}$.

10. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+2}{3n-1} \right)^{n^2}$.

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n-1}{n}\right)^n \frac{n}{5^n}.$ **22.** $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^5}{(2n+1)^n}.$

12. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+3}{n+1}\right)^{n^2}$ **23.** $\displaystyle\sum_{n=1}^{\infty} 2^{n-1} e^{-n}.$

13. $\displaystyle\sum_{n=1}^{\infty} n^2 \left(\frac{3n+2}{4n-1}\right)^n$ **24.** $\displaystyle\sum_{n=1}^{\infty} n \left(\frac{3n-1}{4n+2}\right)^{2n}$

14. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{n+1}{2n-3}\right)^{n^2}.$ **25.** $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n}{4n+3}\right)^{n^2}$

15. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^{2n+1}$ **26.** $\displaystyle\sum_{n=1}^{\infty} \frac{n^{n+2}}{(2n^2+1)^{n/2}}$

16. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n-1}{3n+1}\right)^{\frac{n}{2}}.$ **27.** $\displaystyle\sum_{n=1}^{\infty} \sqrt{n} \left(\frac{n}{3n-1}\right)^{2n}.$

17. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n+1}}{n^n}.$ **28.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n} \left(\frac{n+1}{n}\right)^{n^2}$

18. $\displaystyle\sum_{n=1}^{\infty} n^2 \sin^n \frac{\pi}{2n}.$ **29.** $\displaystyle\sum_{n=1}^{\infty} \frac{n 3^{n+2}}{5^n}.$

19. $\displaystyle\sum_{n=2}^{\infty} \frac{n^3}{(\ln n)^n}.$ **30.** $\displaystyle\sum_{n=2}^{\infty} \sqrt[3]{n} \left(\frac{n-2}{2n+1}\right)^{3n}.$

20. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{3n-1}\right)^{n^3}.$ **31.** $\displaystyle\sum_{n=1}^{\infty} n^4 \operatorname{arctg}^{2n} \frac{\pi}{4n}.$

21. $\displaystyle\sum_{n=1}^{\infty} n^3 \operatorname{arctg}^n \frac{\pi}{3n}.$

Задача 6. Исследовать на сходимость ряд.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln^2 (3n+1)}.$ **4.** $\displaystyle\sum_{n=3}^{\infty} \frac{1}{(3n-5)\ln^2 (4n-7)}.$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n \ln^2 (2n+1)}.$ **5.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(3n+4)\ln^2 (5n+2)}.$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+3)\ln^2 (2n+1)}.$ **6.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)\ln^2 (n\sqrt{5}+2)}.$

Задача 7. Исследовать на сходимость ряд.

1. $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\dfrac{2n+1}{n(n+1)}$.

15. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{2^{2n}(n+1)}$.

2. $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\left(\dfrac{n}{2n+1}\right)^{n}$.

16. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n}}{\sqrt[3]{3n}\cos(\pi/3n)}$.

3. $\sum\limits_{n=2}^{\infty}\dfrac{(-1)^{n+1}}{\ln(n+1)}$.

17. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{(3/2)^{n}(n+1)}$.

4. $\sum\limits_{n=3}^{\infty}\dfrac{(-1)^{n}}{n\ln n(\ln\ln n)}$.

18. $\sum\limits_{n=1}^{\infty}(-1)^{n}\dfrac{2n-1}{3n}$.

5. $\sum\limits_{n=1}^{\infty}(-1)^{n}\dfrac{2n^{2}}{n^{4}-n^{2}+1}$.

19. $\sum\limits_{n=1}^{\infty}(-1)^{n}\dfrac{(n+3)!}{2^{n}}$.

6. $\sum\limits_{n=3}^{\infty}\dfrac{(-1)^{n}}{(n+1)\ln n}$.

20. $\sum\limits_{n=1}^{\infty}(-1)^{n}\dfrac{n+1}{\sqrt{n^{3}}}$.

7. $\sum\limits_{n=3}^{\infty}\dfrac{(-1)^{n}}{n\ln(n+1)}$.

21. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n}}{\sqrt{5n-1}}\operatorname{tg}\dfrac{\pi}{4\sqrt{n}}$.

8. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{n\sqrt[4]{2n+3}}$.

22. $\sum\limits_{n=0}^{\infty}\dfrac{(-1)^{n}}{2^{2n+1}(2n+1)}$.

9. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n}}{\sqrt{3n+1}}\sin\dfrac{\pi}{2\sqrt{n}}$.

23. $\sum\limits_{n=1}^{\infty}(-1)^{n}\dfrac{\sin(n\sqrt{n})}{n\sqrt{n}}$.

10. $\sum\limits_{n=1}^{\infty}(-1)^{n}\left(\dfrac{3n-1}{n}\right)^{n}$.

24. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n}}{n+\cos(2/\sqrt{n+4})}$.

11. $\sum\limits_{n=1}^{\infty}\dfrac{\sin n}{n!}$.

25. $\sum\limits_{n=1}^{\infty}(-1)^{n}\sin\dfrac{\pi}{2^{n}}$.

12. $\sum\limits_{n=3}^{\infty}\dfrac{(-1)^{n}}{n\ln(2n)}$.

26. $\sum\limits_{n=1}^{\infty}(-1)^{n}\sin^{n}\dfrac{\pi}{2n}$.

13. $\sum\limits_{n=1}^{\infty}(-1)^{n}\operatorname{tg}\dfrac{1}{n}$.

27. $\sum\limits_{n=1}^{\infty}(-1)^{n}\dfrac{\sin 3^{n}}{3^{n}}$.

14. $\sum\limits_{n=1}^{\infty}\dfrac{\cos n}{n^{2}}$.

28. $\sum\limits_{n=1}^{\infty}(-1)^{n}\ln\left(1+\dfrac{1}{n^{2}}\right)$.

24. $\displaystyle\int\limits_{-\sqrt{2}}^{-1} dy \int\limits_{-\sqrt{2-y^2}}^{0} f\,dx + \int\limits_{-1}^{0} dy \int\limits_{y}^{0} f\,dx.$

25. $\displaystyle\int\limits_{0}^{1} dx \int\limits_{0}^{x^3} f\,dy + \int\limits_{1}^{2} dx \int\limits_{0}^{2-x} f\,dy.$

26. $\displaystyle\int\limits_{0}^{\sqrt{3}} dx \int\limits_{0}^{2-\sqrt{4-x^2}} f\,dy + \int\limits_{\sqrt{3}}^{2} dx \int\limits_{0}^{\sqrt{4-x^2}} f\,dy.$

27. $\displaystyle\int\limits_{0}^{1} dx \int\limits_{-\sqrt{x}}^{0} f\,dy + \int\limits_{1}^{2} dx \int\limits_{-\sqrt{2-x}}^{0} f\,dy.$

28. $\displaystyle\int\limits_{0}^{1} dx \int\limits_{0}^{x} f\,dy + \int\limits_{1}^{\sqrt{2}} dx \int\limits_{0}^{\sqrt{2-x^2}} f\,dy.$

29. $\displaystyle\int\limits_{0}^{1} dy \int\limits_{0}^{\sqrt{y}} f\,dx + \int\limits_{1}^{\sqrt{2}} dy \int\limits_{0}^{\sqrt{2-y^2}} f\,dx.$

30. $\displaystyle\int\limits_{0}^{1} dx \int\limits_{0}^{\sqrt{x}} f\,dy + \int\limits_{1}^{2} dx \int\limits_{0}^{\sqrt{2-x}} f\,dy.$

31. $\displaystyle\int\limits_{-2}^{-\sqrt{3}} dx \int\limits_{0}^{\sqrt{4-x^2}} f\,dy + \int\limits_{-\sqrt{3}}^{0} dx \int\limits_{0}^{2-\sqrt{4-x^2}} f\,dy.$

Задача 2. Вычислить.

1. $\displaystyle\iint\limits_{D} (12x^2y^2 + 16x^3y^3)\,dx\,dy;\ D: x = 1,\ y = x^2,\ y = -\sqrt{x}.$

2. $\displaystyle\iint\limits_{D} (9x^2y^2 + 48x^3y^3)\,dx\,dy;\ D: x = 1,\ y = \sqrt{x},\ y = -x^2.$

3. $\displaystyle\iint\limits_{D} (36x^2y^2 - 96x^3y^3)\,dx\,dy;\ D: x = 1,\ y = \sqrt[3]{x},\ y = -x^3.$

4. $\displaystyle\iint\limits_{D} (18x^2y^2 + 32x^3y^3)\,dx\,dy;\ D: x = 1,\ y = x^3,\ y = -\sqrt[3]{x}.$

21. $\iint\limits_{D} (44xy + 16x^3y^3)\,dx\,dy$, D: $x = 1$, $y = x^2$, $y = -\sqrt[3]{x}$
$(x \geqslant 0)$.

22. $\iint\limits_{D} (4xy + 176x^3y^3)\,dx\,dy$, D: $x = 1$, $y = \sqrt[3]{x}$, $y = -x^3$
$(x \geqslant 0)$.

23. $\iint\limits_{D} (xy - 4x^3y^3)\,dx\,dy$; D: $x = 1$, $y = x^3$, $y = -\sqrt{x}$.

24. $\iint\limits_{D} (4xy + 176x^3y^3)\,dx\,dy$; D: $x = 1$, $y = \sqrt{x}$, $y = -x^3$.

25. $\iint\limits_{D} \left(6x^2y^2 + \frac{25}{3}x^4y^4\right) dx\,dy$, D: $x = 1$, $y = x^2$, $y = -\sqrt{x}$.

26. $\iint\limits_{D} (9x^2y^2 + 25x^4y^4)\,dx\,dy$, D: $x = 1$, $y = \sqrt{x}$, $y = -x^2$.

27. $\iint\limits_{D} \left(3x^2y^2 + \frac{50}{3}x^4y^4\right) dx\,dy$, D: $x = 1$, $y = \sqrt[3]{x}$, $y = -x^3$.

28. $\iint\limits_{D} (9x^2y^2 + 25x^4y^4)\,dx\,dy$; D: $x = 1$, $y = x^3$, $y = -\sqrt[3]{x}$.

29. $\iint\limits_{D} (54x^2y^2 + 150x^4y^4)\,dx\,dy$; D: $x = 1$, $y = x^2$, $y = -\sqrt[3]{x}$
$(x \geqslant 0)$.

30. $\iint\limits_{D} (xy - 9x^5y^5)\,dx\,dy$; D: $x = 1$, $y = \sqrt[3]{x}$, $y = -x^2$ $(x \geqslant 0)$.

31. $\iint\limits_{D} 54x^2y^2 + 150x^4y^4)\,dx\,dy$; D: $x = 1$, $y = x^3$, $y = -\sqrt{x}$.

Задача 3. Вычислить.

1. $\iint\limits_{D} ye^{xy/2}\,dx\,dy$; D: $y = \ln 2$, $y = \ln 3$, $x = 2$, $x = 4$.

2. $\iint\limits_{D} y^2 \sin\frac{xy}{2}\,dx\,dy$; D: $x = 0$, $y = \sqrt{\pi}$, $y = \frac{x}{2}$.

3. $\iint\limits_{D} y\cos xy\,dx\,dy$; D: $y = \frac{\pi}{2}$, $y = \pi$, $x = 1$, $x = 2$.

4. $\iint\limits_{D} y^2 e^{-xy/4}\,dx\,dy$; D: $x = 0$, $y = 2$, $y = x$.

5. $\iint\limits_{D} y\sin xy\,dx\,dy$; D: $y = \frac{\pi}{2}$, $y = \pi$, $x = 1$, $x = 2$.

6. $\iint\limits_{D} y^2 \cos \dfrac{xy}{2}\, dx\, dy$; $D: x = 0$, $y = \sqrt{\dfrac{\pi}{2}}$, $y = \dfrac{x}{2}$.

7. $\iint\limits_{D} 4y e^{2xy}\, dx\, dy$; $D: y = \ln 3$, $y = \ln 4$, $x = \dfrac{1}{2}$, $x = 1$.

8. $\iint\limits_{D} 4y^2 \sin xy\, dx\, dy$; $D: x = 0$, $y = \sqrt{\dfrac{\pi}{2}}$, $y = x$.

9. $\iint\limits_{D} y \cos 2xy\, dx\, dy$; $D: y = \dfrac{\pi}{2}$, $y = \pi$, $x = \dfrac{1}{2}$, $x = 1$.

10. $\iint\limits_{D} y^2 e^{-xy/8}\, dx\, dy$; $D: x = 0$, $y = 2$, $y = \dfrac{x}{2}$.

11. $\iint\limits_{D} 12y \sin 2xy\, dx\, dy$; $D: y = \dfrac{\pi}{4}$, $y = \dfrac{\pi}{2}$, $x = 2$, $x = 3$.

12. $\iint\limits_{D} y^2 \cos xy\, dx\, dy$; $D: x = 0$, $y = \sqrt{\pi}$, $y = x$.

13. $\iint\limits_{D} y e^{xy/4}\, dx\, dy$; $D: y = \ln 2$, $y = \ln 3$, $x = 4$, $x = 8$.

14. $\iint\limits_{D} 4y^2 \sin 2xy\, dx\, dy$; $D: x = 0$, $y = \sqrt{2\pi}$, $y = 2x$.

15. $\iint\limits_{D} 2y \cos 2xy\, dx\, dy$; $D: y = \dfrac{\pi}{4}$, $y = \dfrac{\pi}{2}$, $x = 1$, $x = 2$.

16. $\iint\limits_{D} y^2 e^{-xy/2}\, dx\, dy$; $D: x = 0$, $y = \sqrt{2}$, $y = x$.

17. $\iint\limits_{D} y \sin xy\, dx\, dy$; $D: y = \pi$, $y = 2\pi$, $x = \dfrac{1}{2}$, $x = 1$.

18. $\iint\limits_{D} y^2 \cos 2xy\, dx\, dy$; $D: x = 0$, $y = \sqrt{\dfrac{\pi}{2}}$, $y = \dfrac{x}{2}$.

19. $\iint\limits_{D} 8y e^{4xy}\, dx\, dy$; $D: y = \ln 3$, $y = \ln 4$, $x = \dfrac{1}{4}$, $x = \dfrac{1}{2}$

20. $\iint\limits_{D} 3y^2 \sin \dfrac{xy}{2}\, dx\, dy$; $D: x = 0$, $y = \sqrt{\dfrac{4\pi}{3}}$, $y = \dfrac{2}{3}x$.

21. $\iint\limits_{D} y \cos xy\, dx\, dy$; $D: y = \pi$, $y = 3\pi$, $x = \dfrac{1}{2}$, $x = 1$.

22. $\iint\limits_{D} y^2 e^{-xy/2}\, dx\, dy$; $D: x = 0$, $y = 1$, $y = \dfrac{x}{2}$.