On Resonant Effects in the Semi-Infinite Waveguides with Barriers

Nikolai Pleshchinskii, Garnik Abgaryan and Bulat Vildanov

Abstract The problems of electromagnetic wave diffraction by thin conductive barriers in a semi-infinite parallel-plate waveguide are reduced to infinite sets of linear algebraic equations concerning the expansion coefficients of the field by its eigen waves. Values of resonant frequencies are obtained for which there is a sharp increase in the characteristics of the electromagnetic field in the area between the barrier and the metal wall.

1 Introduction

In the design of radiotechnical devices with optimal characteristics the situations when there is a resonant growth of certain parameters of the electromagnetic field are of particular interest. Barriers in waveguide structures are widely used in the production of converters, filters, splitters and other elements.

In this paper, we explore the resonant effects that occur when the diffraction of eigen electromagnetic wave, which attacks a thin conductive barriers in a semi-infinite parallel-plate waveguide with metal walls.

As it is known [1], any electromagnetic field in the parallel-plate waveguide can be presented as a sum of its eigen waves propagating or damping in different directions. The theory of equivalent chains was used in [2] as a simple model of the process of electromagnetic wave diffraction. In recent years, during the investigation of the resonant properties of waveguides with heterogeneities, the method of equivalent

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circuits [3], the method of moments [4] and more rigorous methods as well as method of the Riemann-Hilbert problem [5] and method of integral equations [6] are used. Some numerical results can be found in the works [7], [8].

In this paper, the method of integral-series identities is used to reduce the paired series functional equations of diffraction problems by the screens to regular infinite sets of linear algebraic equations (ISLAE) [9]. Early we investigated the resonant properties of the diaphragms in the semi-infinite waveguides [10], [11].

2 Lateral Barrier in a Waveguide

Let us consider the two-dimensional problem of TE-wave diffraction by a lateral barrier in a half-infinite parallel-plate waveguide $0 < x < a,\ z < d$. The barrier is located in the plane $z = 0$. The part $\mathcal{M} = (\alpha, \beta)$ of the cross-section $[0, a]$ of the waveguide corresponds to it (Fig. 1). Let's denote by $\mathcal{N}$ supplement of $\mathcal{M}$ up to $[0, a]$.

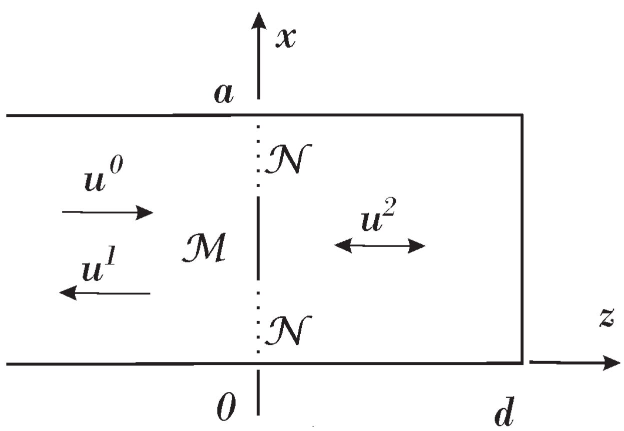

Fig. 1 Lateral barrier in a plane waveguide.

Let free currents and charges be absent, the medium be homogeneous and isotropic, electromagnetic field harmoniously depend on time ($\exp(-i\omega t)$). Denote

$$\varphi_n(x) = \sqrt{2/a}\sin\frac{\pi n x}{a}, \quad \gamma_n = \sqrt{\kappa^2 - (\pi n/a)^2},$$

where κ is wave number, $\operatorname{Re}\gamma > 0$ or $\operatorname{Im}\gamma_n > 0$ and $n = 1, 2, \ldots$

From the region $z < 0$ on the barrier runs its eigen wave

$$u^0(x, z) = \varphi_l(x)\, e^{i\gamma_l z}.$$

We will look for the wave reflected to the left in the form of

$$u^1(x,z) = \sum_{n=1}^{+\infty} a_n \varphi_n(x)\, e^{-i\gamma_n z},$$

and we will look for the wave passed to the right in the form of

$$u^2(x,z) = \sum_{n=1}^{+\infty} b_n \varphi_n(x) \big(e^{i\gamma_n z} - e^{2i\gamma_n d}\, e^{-i\gamma_n z}\big).$$

For the wave $u^2(x,z)$ a boundary condition is fulfilled on the metal wall $z = d$, and this wave is bounded for $n \to +\infty$.

Let's write down the boundary conditions on the $\mathcal{M}$:

$$\varphi_l(x) + \sum_{n=1}^{+\infty} a_n\, \varphi_n(x) = 0, \quad \sum_{n=1}^{+\infty} b_n \big(1 - e^{2i\gamma_n d}\big)\, \varphi_n(x) = 0$$

and the conjugation conditions on the $\mathcal{N}$:

$$\varphi_l(x) + \sum_{n=1}^{+\infty} a_n\, \varphi_n(x) = \sum_{n=1}^{+\infty} b_n \big(1 - e^{2i\gamma_n d}\big)\, \varphi_n(x),$$

$$\gamma_l\, \varphi_l(x) - \sum_{n=1}^{+\infty} a_n\, \gamma_n\, \varphi_n(x) = \sum_{n=1}^{+\infty} b_n\, \gamma_n \big(1 + e^{2i\gamma_n d}\big)\, \varphi_n(x).$$

It follows that $1 + a_l = b_l\big(1 - e^{2i\gamma_l d}\big)$ and $a_n = b_n\big(1 - e^{2i\gamma_n d}\big),\ n \neq l$. We exclude the unknowns a_n.

To regularize the pair series functional equations, we use an integral-series identity

$$\int_0^a \Big(\sum_{n=1}^{+\infty} b_n \big(1 - e^{2i\gamma_n d}\big)\, \varphi_n(t)\Big)\, K(t,x)\, dt = \sum_{n=1}^{+\infty} b_n \gamma_n\, \varphi_n(x),$$

here

$$K(t,x) = \sum_{m=1}^{+\infty} \frac{\gamma_m}{1 - e^{2i\gamma_m d}}\, \varphi_m(t)\, \varphi_m(x).$$

It is assumed that $\gamma_m d \neq \pi j$.

Finally, after projecting on the function $\varphi_k(x)$ we get ISLAE ($k = 1, 2, \dots$)

$$b_k \gamma_k - \sum_{n=1}^{+\infty} b_n \big(1 - e^{2i\gamma_n d}\big) \sum_{m=1}^{+\infty} \frac{\gamma_m}{1 - e^{2i\gamma_m d}}\, J_{nm}\, I_{mk} = \gamma_l\, J_{lk},$$

where

$$I_{nm} = \int_{\mathcal{M}} \varphi_n(t)\, \varphi_m(t)\, dt, \quad J_{nm} = \int_{\mathcal{N}} \varphi_n(t)\, \varphi_m(t)\, dt.$$

3 Computing Experiments, I

The computing experiments are based on the multiple solving the truncated ISLAE in the case when the frequency of the excitatory wave changes with a small step. We will look for approximate solution of ISLAE by truncation method. It is enough to take the parameter of truncated method $N = 30$. As a wave incoming on the barrier, we will consider the first mode of the waveguide.

Let's choose the following parameters: $a = 1.1$, $d = 1.3$; $\alpha = 0.1$; $\beta = 1.0$ in dimensionless quantities. As the computing experiment has shown, the modules of coefficients $b_1, b_2, \dots$ have sharp local maximums, with correspond to wave number ≈ 3.7410, ≈ 5.6135, $\approx 7.7910, \dots$. These values are close to the eigen wave numbers ≈ 3.7412, ≈ 5.6140, ≈ 7.7921 of a two-dimensional rectangular region of the size $a \times d$.

The dependence of coefficient b_1 module on wave number κ in the neighborhoods of resonant values is shown in Fig. 2–3.

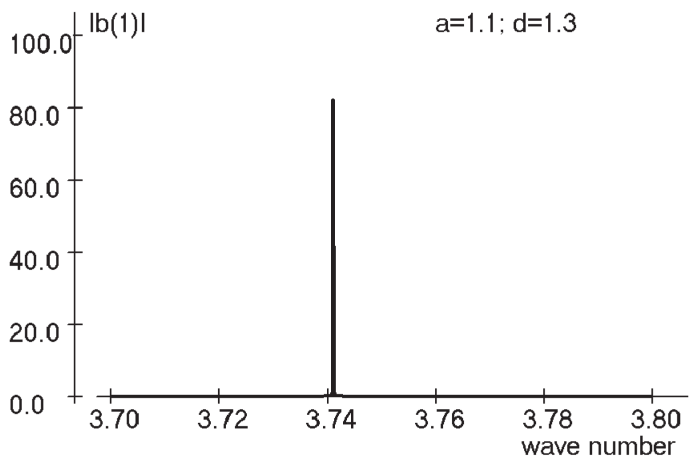

Fig. 2 Dependence of the module of b_1 on the wave number κ.

If the size of barrier decreases, then the resonant values of k decrease slightly also.

The dependence of the conditional number cond $A = ||A|| \cdot ||A^{-1}||$ on the parameter κ is also resonant.

If we balance the equations in SLAE (divide each equation by the largest in-module coefficient for the unknowns), then the conditioned number will be significantly reduced, but the solution of the SLAE will not change. But after balancing, it becomes possible to study the dependence on the parameter κ of the values of the determinant of the matrix of the SLAE coefficients. Now the modules of these values

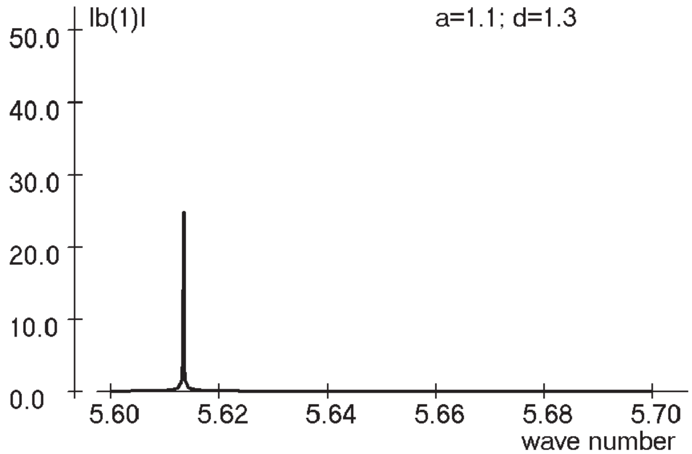

Fig. 3 Dependence of the module of b_1 on the wave number κ.

in the neighborhood of the resonant point do not exceed one. Before balancing, they had an order of 10^{45} or more.

As in the case of diaphragm in the semi-infinite waveguide [11], the resonant values of the parameter κ can be found: 1) when solving the SLAE of diffraction problem; 2) when calculating the conditioned number of the matrix of its coefficients; 3) when analyzing the values of the determinant of this matrix.

4 Longitudinal Barrier in a Waveguide

Now let the thin conductive barrier with a length of d be placed at a height of b from the lower wall of the waveguide (Fig. 4).

We will use the following notations:

$$\varphi_n^a(x) = \sqrt{2/a}\sin\frac{\pi n x}{a}, \quad \gamma_n^a = \sqrt{\kappa^2 - (\pi n/a)^2},$$

$$\varphi_n^b(x) = \sqrt{2/b}\sin\frac{\pi n x}{b}, \quad \gamma_n^b = \sqrt{\kappa^2 - (\pi n/b)^2},$$

$$\varphi_n^c(x) = \sqrt{2/(a-b)}\sin\frac{\pi n(x-b)}{a-b},$$

$$\gamma_n^c = \sqrt{\kappa^2 - (\pi n/(a-b))^2}, \quad n = 1, 2, \ldots$$

Let the eigen wave

$$u^0(x,z) = \varphi_l^a(x)e^{i\gamma_l^a z}.$$

run on the barrier. We will look for the wave reflected to the left in the form of

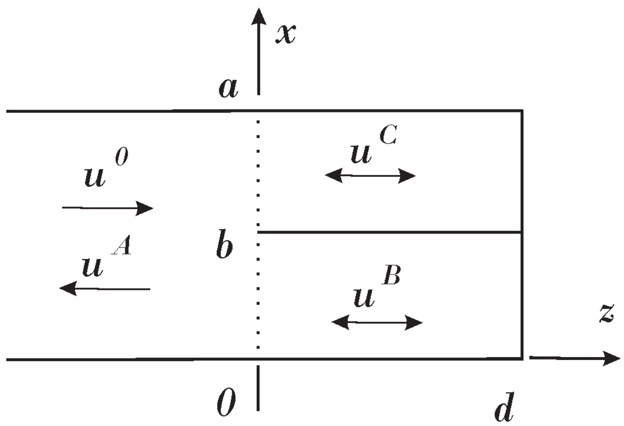

Fig. 4 Longitudinal barrier in a waveguide.

$$u^A(x,z) = \sum_{n=1}^{+\infty} a_n \varphi_n^a(x) e^{-i\gamma_n^a z}$$

and we will look for the wave in the regions B: $0 < x < b,\ 0 < z < d$ and C: $b < x < a,\ 0 < z < d$ in the form of

$$u^B(x,z) = \sum_{n=1}^{+\infty} b_n \varphi_n^b(x) \big(e^{i\gamma_n^b z} - e^{2i\gamma_n^b d} e^{-i\gamma_n^b z}\big),$$

$$u^C(x,z) = \sum_{n=1}^{+\infty} c_n \varphi_n^c(x) \big(e^{i\gamma_n^c z} - e^{2i\gamma_n^c d} e^{-i\gamma_n^c z}\big).$$

The equalities on the $(0, b)$

$$2\varphi_l^a(x) + \sum_{n=1}^{+\infty} d_n \varphi_n^a(x) = \sum_{n=1}^{+\infty} b_n \varphi_n^b(x) \big(1 - e^{2i\gamma_n^b d}\big),$$

$$-\sum_{n=1}^{+\infty} d_n \gamma_n^a \varphi_n^a(x) = \sum_{n=1}^{+\infty} b_n \gamma_n^b \varphi_n^b(x) \big(1 + e^{2i\gamma_n^b d}\big),$$

and on the (b, a)

$$2\varphi_l^a(x) + \sum_{n=1}^{+\infty} d_n \varphi_n^a(x) = \sum_{n=1}^{+\infty} b_n \varphi_n^c(x) \big(1 - e^{2i\gamma_n^c d}\big),$$

$$-\sum_{n=1}^{+\infty} d_n \gamma_n^a \varphi_n^a(x) = \sum_{n=1}^{+\infty} c_n \gamma_n^c \varphi_n^c(x) \big(1 + e^{2i\gamma_n^c d}\big)$$

should be fulfilled if $z = 0$. Here $d_l = a_l - 1$, $d_n = a_n,\ n \neq l$.

Let's exclude unknowns d_n using the integral-series identity

$$\sum_{n=1}^{+\infty} d_n \varphi_n^a(x) = \int_0^a \Big(\sum_{n=1}^{+\infty} d_n \gamma_n^a \varphi_n^a(t)\Big) K(t,x)\, dt, \quad x \in (0,a),$$

$$K(t,x) = \sum_{m=1}^{+\infty} \frac{1}{\gamma_m^a} \varphi_m^a(t) \varphi_m^a(x).$$

Replace x by t in the equalities of the second pair, multiply both parts by $K(t,x)$ and integrate from 0 to a. Then we get

$$-\sum_{n=1}^{+\infty} d_n \varphi_n^a(x) = \sum_{n=1}^{+\infty} b_n \gamma_n^b (1 + e^{2i\gamma_n^b d}) \sum_{m=1}^{+\infty} \frac{1}{\gamma_m^a} \varphi_m^a(x)\, I_{mn}^b$$

$$+ \sum_{n=1}^{+\infty} c_n \gamma_n^c (1 + e^{2i\gamma_n^c d}) \sum_{m=1}^{+\infty} \frac{1}{\gamma_m^a} \varphi_m^a(x)\, I_{mn}^c, \quad x \in (0,a).$$

Let's add the equations of the first pair and new equality. The equation

$$2\varphi_l^a(x) = \sum_{n=1}^{+\infty} b_n \varphi_n^b(x) \big(1 - e^{2i\gamma_n^b d}\big)$$

$$+ \sum_{n=1}^{+\infty} b_n \gamma_n^b \big(1 + e^{2i\gamma_n^b d}\big) \sum_{m=1}^{+\infty} \frac{1}{\gamma_m^a} \varphi_m^a(x)\, I_{mn}^b$$

$$+ \sum_{n=1}^{+\infty} c_n \gamma_n^c \big(1 + e^{2i\gamma_n^c d}\big) \sum_{m=1}^{+\infty} \frac{1}{\gamma_m^a} \varphi_m^a(x)\, I_{mn}^c, \quad x \in (0,b),$$

we multiply by $\varphi_k^b(x)$ and integrate from 0 to b. A similar equation on (b,a) is multiplied by $\varphi_k^c(x)$ and integrated from b to a. Then

$$2I_{lk}^b = b_k (1 - e^{2i\gamma_k^b d}) + \sum_{n=1}^{+\infty} b_n \gamma_n^b (1 + e^{2i\gamma_n^b d}) \sum_{m=1}^{+\infty} \frac{1}{\gamma_m^a}\, I_{mn}^b\, I_{mk}^b$$

$$+ \sum_{n=1}^{+\infty} c_n \gamma_n^c (1 + e^{2i\gamma_n^c d}) \sum_{m=1}^{+\infty} \frac{1}{\gamma_m^a}\, I_{mn}^c\, I_{mk}^b, \quad k = 1, 2, \ldots$$

$$2I_{lk}^c = c_k (1 - e^{2i\gamma_k^c d}) + \sum_{n=1}^{+\infty} b_n \gamma_n^b (1 + e^{2i\gamma_n^b d}) \sum_{m=1}^{+\infty} \frac{1}{\gamma_m^a}\, I_{mn}^b\, I_{mk}^c$$

$$+ \sum_{n=1}^{+\infty} c_n \gamma_n^c (1 + e^{2i\gamma_n^c d}) \sum_{m=1}^{+\infty} \frac{1}{\gamma_m^a}\, I_{mn}^c\, I_{mk}^c, \quad k = 1, 2, \ldots$$

where

$$I^b_{mn} = \int_0^b \varphi^a_m(t)\varphi^b_n(t)\,dt, \quad I^c_{mn} = \int_b^a \varphi^a_m(t)\varphi^c_n(t)\,dt.$$

So, the ISLAE to determine the coefficients b_n and c_n consists of two groups of equations. When truncated, we leave N unknown in each equation and N equations in each group.

5 Computing Experiments, II

A computational experiment has shown that at some values of electromagnetic oscillation frequencies there is a resonant increase in field expansion coefficients in regions B and C. The dependencies of the coefficient b_1 module on the frequency (more precisely, when the wave number k changes in the truncated ISLAU) are shown on Fig. 5-7.

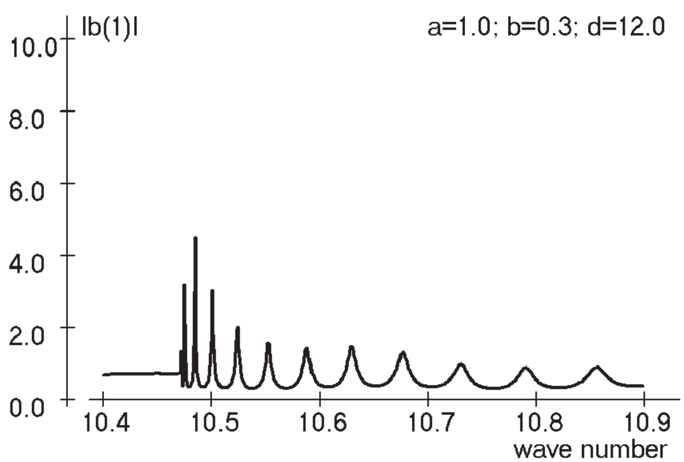

Fig. 5 Dependence of the module of b_1 on the wave number κ.

Resonant frequencies depend significantly on the value of the b. It's easy to see that the highest peak of the lines on the charts are observed when the frequencies are close to the eigen values $\pi\sqrt{1/b^2 + 1/d^2}$ of the frequencies of rectangular domain of the size $b \times d$. At low values of d resonances are not observed.

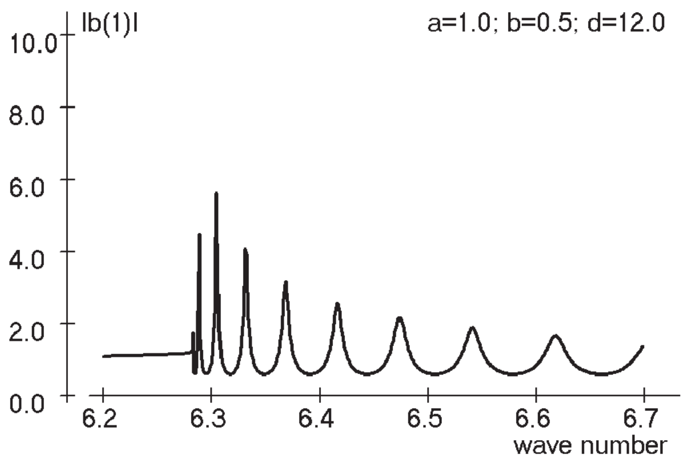

Fig. 6 Dependence of the module of b_1 on the wave number κ.

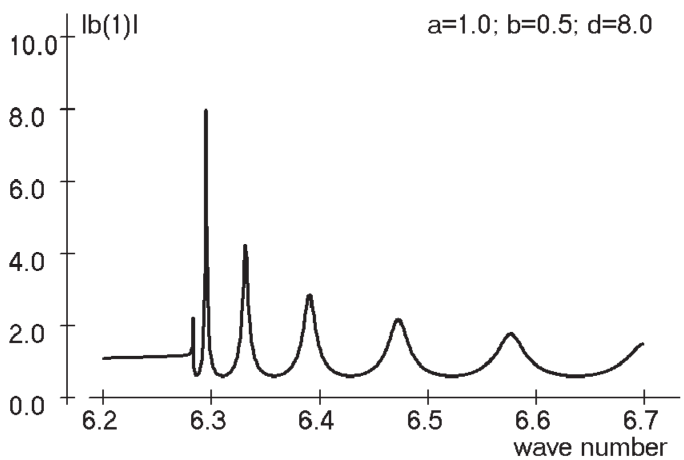

Fig. 7 Dependence of the module b_1 on the wave number κ.

6 Conclusion

In this paper the diffraction problems of the electromagnetic wave by the barrier in a semi-infinite waveguide are reduced to infinite sets of linear algebraic equations relative to coefficients of expansion by eigen waves of the waveguide. The computing experiment has shown that the dependence of the desired coefficients on the frequency of the excitatory wave is resonant.

Acknowledgements The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

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