

Test 3

1.
Two point charges, $q_1 = +25 \text{ nC}$ and $q_2 = -75 \text{ nC}$, are separated by a distance of 3 cm. Find the magnitude of electric force that q_1 exerts on q_2 .

$$\begin{aligned}k &= 9 \cdot 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \\ q_1 &= +25 \text{ nC} = +25 \cdot 10^{-9} \text{ C} \\ q_2 &= -75 \text{ nC} = -75 \cdot 10^{-9} \text{ C} \\ r &= 3 \text{ cm} = 0.03 \text{ m}\end{aligned}$$

$$F = k \cdot \frac{|q_1 \cdot q_2|}{r^2} = 9 \cdot 10^9 \cdot \frac{|25 \cdot 10^{-9} \cdot 75 \cdot 10^{-9}|}{0.03^2} \approx 0.019 \text{ (N)}$$

2.
A positive point charge $q = +3 \mu\text{C}$ is surrounded by a sphere with radius $r = 0.2 \text{ m}$. Find the electric flux Φ ($\text{N} \cdot \text{m}^2 \cdot \text{C}^{-1}$) through the sphere.

$$\begin{aligned}k &= 9 \cdot 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \\ q &= +3 \mu\text{C} = +3 \cdot 10^{-6} \text{ C} \\ \Phi &= \frac{q}{\epsilon_0} = 4 \cdot \pi \cdot k \cdot q \approx 4 \cdot 3.14 \cdot 9 \cdot 10^9 \cdot 3 \cdot 10^{-6} = 3.4 \cdot 10^5 \text{ (N} \cdot \text{m}^2 \cdot \text{C}^{-1}\text{)}\end{aligned}$$

3.
A parallel-plate capacitor (insulator is air) has a capacitance of $C = 1 \text{ F}$. If the plates are 1 mm apart, what is the area A (m^2) of the plates?

$$\begin{aligned}d &= 1 \text{ mm} = 1 \cdot 10^{-3} \text{ m} \\ A &= \frac{C \cdot d}{\epsilon_0} = 4 \cdot \pi \cdot k \cdot C \cdot d = 4 \cdot 3.14 \cdot 9 \cdot 10^9 \cdot 1 \cdot 1 \cdot 10^{-3} = 1 \cdot 10^8 \text{ (m}^2\text{)}\end{aligned}$$

4.
Find the equivalent capacitance, when the two capacitors $C_1 = 1 \text{ F}$ and $C_2 = 3 \text{ F}$ are connected in series.

$$\begin{aligned}\frac{1}{C} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1} + \frac{1}{3} = \frac{4}{3} \\ C &= \frac{3}{4} = 0.75 \text{ (F)}\end{aligned}$$

5.

Find the equivalent capacitance,

when the two capacitors $C_1 = 1 \text{ F}$ and $C_2 = 3 \text{ F}$ are connected in parallel.

$$C = C_1 + C_2 = 1 + 3 = 4 \text{ (F)}$$

6.

The potential difference is $V = 2 \text{ V}$.

A current is $I = 0.5 \text{ A}$.

Find the resistance $R \text{ (}\Omega\text{)}$.

$$R = \frac{V}{I} = \frac{2}{0.5} = 4 \text{ (}\Omega\text{)}$$

7.

A wire has a diameter of $d = 1 \text{ mm}$, the length $l = 314 \text{ m}$, and the resistivity $\rho = 2 \cdot 10^{-8} \text{ }\Omega \cdot \text{m}$.

Find the resistance $R \text{ (}\Omega\text{)}$.

$$d = 1 \text{ mm} = 10^{-3} \text{ m}$$
$$R = \rho \cdot \frac{l}{A} = \rho \cdot \frac{l}{\pi \cdot \frac{d^2}{4}} = 2 \cdot 10^{-8} \cdot \frac{314}{3.14 \cdot \frac{(10^{-3})^2}{4}} \approx 8 \text{ (}\Omega\text{)}$$

8.

Find the equivalent resistance,

when the two resistors $R_1 = 1 \text{ }\Omega$ and $R_2 = 3 \text{ }\Omega$ are connected in parallel.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{1} + \frac{1}{3} = \frac{4}{3}$$
$$R = \frac{3}{4} = 0.75 \text{ (}\Omega\text{)}$$

9.

Find the equivalent resistance,

when the two resistors $R_1 = 1 \text{ }\Omega$ and $R_2 = 3 \text{ }\Omega$ are connected in series.

$$R = R_1 + R_2 = 1 + 3 = 4 \text{ (}\Omega\text{)}$$

10.

Find the power of energy dissipation $P \text{ (W)}$ in the resistor $R = 10 \text{ }\Omega$, if current $I = 2 \text{ A}$.

$$P = I^2 \cdot R = 2^2 \cdot 10 = 40 \text{ (W)}$$

11.

The proton ($q = 1.6 \cdot 10^{-19}$ C) has velocity $v = 2 \cdot 10^5$ m/s.

The uniform magnetic field with magnitude $B = 5$ T has angle with velocity direction $\alpha = 30^\circ$.

Find Lorentz force F (N).

$$\sin(30^\circ) = 0.5$$

$$F = q \cdot v \cdot B \cdot \sin(\alpha) = 1.6 \cdot 10^{-19} \cdot 2 \cdot 10^5 \cdot 5 \cdot 0.5 = 8 \cdot 10^{-14} \text{ (N)}$$

12.

Two straight, parallel, superconducting cables 4.5 mm apart carry equal currents of $I = 15000$ A.

Length of cables is $l = 1$ m.

Find force F (N) of interaction between these cables.

$$\mu_0 = 4 \cdot \pi \cdot 10^{-7} \text{ (T} \cdot \text{m} \cdot \text{A}^{-1}\text{)}$$

$$r = 4.5 \text{ mm} = 4.5 \cdot 10^{-3} \text{ m}$$

$$F = \frac{\mu_0 \cdot I_1 \cdot I_2 \cdot l}{2 \cdot \pi \cdot r} = \frac{4 \cdot \pi \cdot 10^{-7} \cdot 15000 \cdot 15000 \cdot 1}{2 \cdot \pi \cdot 4.5 \cdot 10^{-3}} = 1 \cdot 10^4 \text{ (N)}$$

13.

The solenoid consists of a helical winding of wire on a cylinder, usually circular in cross section.

The solenoid has $n = 10^7$ turns of wire per meter of length and carries a current $I = 2$ A.

Find the magnitude of magnetic field B (T) at the center of the solenoid's length.

$$\mu_0 = 4 \cdot \pi \cdot 10^{-7} \text{ (T} \cdot \text{m} \cdot \text{A}^{-1}\text{)}$$

$$B = \mu_0 \cdot n \cdot I = 4 \cdot \pi \cdot 10^{-7} \cdot 10^7 \cdot 2 = 25.12 \text{ (T)}$$

14.

The speed of sound at 20°C is $v = 344$ m/s.

The frequency is $f = 172$ Hz.

Find the wavelength λ (m).

$$\lambda = v/f = 344/172 = 2 \text{ (m)}$$

15.

The frequency of wave is $f = 500$ Hz.

Find the period T (s).

$$T = 1/f = 1/500 = 0.002 \text{ (s)}$$