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# Analysis of dynamic ephemeris and physical libration of the Moon in order to create a lunar navigational system

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Currently, there are a number of projects on robotic and manned exploration of the Moon are being developed. With this purpose, the space agencies pay particular attention to providing coordinate and time support for the upcoming space missions. In particular, Roskosmos is planning to extend GLONASS navigational system to the lunar orbit. Construction of navigational selenocentric systems implies significant development and refinement of the physical libration of the Moon (PLM) numerical theory that defines position of the lunar axes of inertia in relation to the mean ecliptic. In the absence of reliable PLM theory coordinate and time support for the lunar space missions does not have the required accuracy. The present work is mainly concerned with the construction of PLM theory and analysis of its reliability. The authors' algorithm of building PLM theory and calculating lunar orbit parameters is described. As a result, a numerical PLM theory is developed using DE421 ephemeris. The accuracy of the PLM theory has been assessed by analyzing its residual differences on the basis of comparison with corresponding theories by Rambaux, Williams (Rambaux, N., *et al.*[1], Rambaux, N., *et al.*[2]).

**Nomenclature**

| | | |
|---|---|---|
| GLONASS | = | Global Navigation Satellite System |
| PLM | = | Physical Libration of the Moon |
| LLR | = | Lunar Laser Ranging |

## I. Introduction

Long-term lunar laser ranging (LLR) of the angle reflectors installed on lunar surface by the astronauts of "Apollo" mission showed the lunar rotation did not achieve its minimum of potential energy. It appeared that tidal influence from the Earth and Sun as well as hydrodynamic effects at the core-mantle boundary had been able to cause the sufficient dissipation of the lunar rotation energy that was determined from observations. The dissipation manifests as a periodic displacement of the rotation axis from the Cassini plane. The greatest harmonic of this oscillation has an amplitude of 0.27 arcseconds and a period of 27.3 days. After the laser data had been accumulated and its accuracy had increased, 4 harmonics in the Moon's libration series responsible for the dissipation were revealed Williams, J. G., *et al.*[3]. As a result of numerical simulation, it was established that only 2/3 of the amplitude of 0.27 arcsecond could be explained by monthly tides at the mantle and the remaining third could be caused by dissipation at the core-mantle boundary.

Analysis of modern LLR observations allowed making an important discovery. Free libration harmonics and derived from them were found in the lunar rotational motion. The very existence of them is surprising, since according to calculations, the free libration decay time in the presence of significant dissipation is rather short Williams, J. G., *et al.*[3]. The problems of finding the source of these excitations and explaining the process of free libration maintenance still remain unsolved.

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## II. Algorithm of developing a theory of PLM and calculating lunar orbit parameters

The first stage of developing a PLM theory consisted in the implementation of “main problem” model where the lunar orbit was described using the theory by Gutzwiller & Schmidt (Gutzwiller, M. C., *et al.*[4]). The next stage consisted in the use of a highly accurate numerical ephemeris of the Moon and planets followed by taking into account the direct and indirect perturbations from planets. DE421 (Folkner W. M., *et al.*[5]) dynamic ephemeris was used, since it allowed making comparison with semi-empirical PLM series **(**Rambaux, N., *et al*.[1], Rambaux, N., *et al*.[2]**)** obtained on the basis of the given ephemeris. The dynamic ephemeris was constructed in ICRF system, and to adapt it for our task it was necessary to transform DE421 parameters (values of librational angles and orbit parameters for the Moon and planets) into the system of average ecliptic on the corresponding date and J2000 epoch correctly.

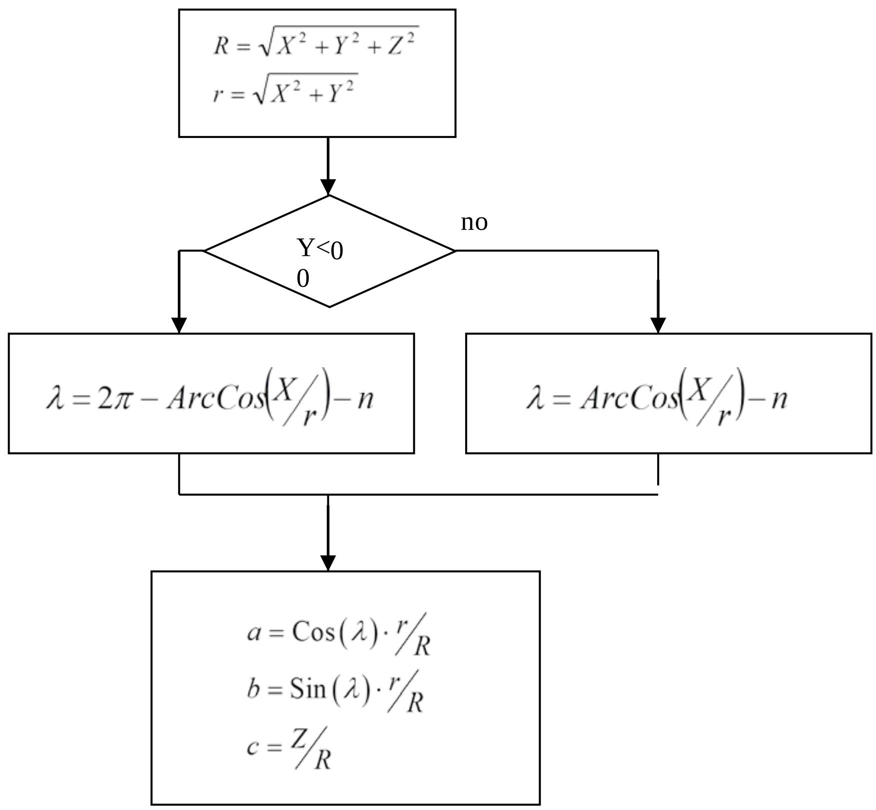


Fig. 1 Block diagram of calculating directing cosines of the ecliptic pole’s radius vector

Since DE421 ephemeris is constructed in ICRS, it is necessary to rotate ICRS system about OX axis by the average angle of ICRS equator’s inclination to the average ecliptic $\varepsilon_0$ in order to obtain the ephemeris in standard ecliptic system. Usually, position of an object on the celestial sphere is defined by spherical coordinates rather than Cartesian ones. In the present case, longitude, latitude, and parallax are used to determine the position of the lunar ephemeris center. When determining longitude, the coordinate system is considered rotating. In this case, one should subtract the mean rotational velocity of central body (which is $n = F + \Omega - p_A - 180^0$, where F is Delaunay argument, $\Omega$ is longitude of the lunar orbit’s ascending node at average ecliptic of J2000 epoch, $p_A$ is total precession in longitude) from longitude. Below is a block diagram demonstrating how longitude and directing cosines (a, b, c) of the ecliptic pole are calculated from Cartesian ecliptic coordinates. In Fig.1 X, Y, Z stand for Cartesian coordinates in the ecliptic coordinate system with a center in the Moon; $\lambda$ is Earth's longitude in rotating ecliptic system; a, b, c are directing cosines of the ecliptic pole’s radius vector.

In order to assess reliability of the described algorithm, Fig. 2 presents calculated residual differences in Cartesian coordinate system over the interval of 180 years from the comparison of coordinates determined with

DE421 and the ones taken from HORIZONS internet resource for the Earth's center of mass in relation to the lunar one. Here ICRS coordinate system is used.

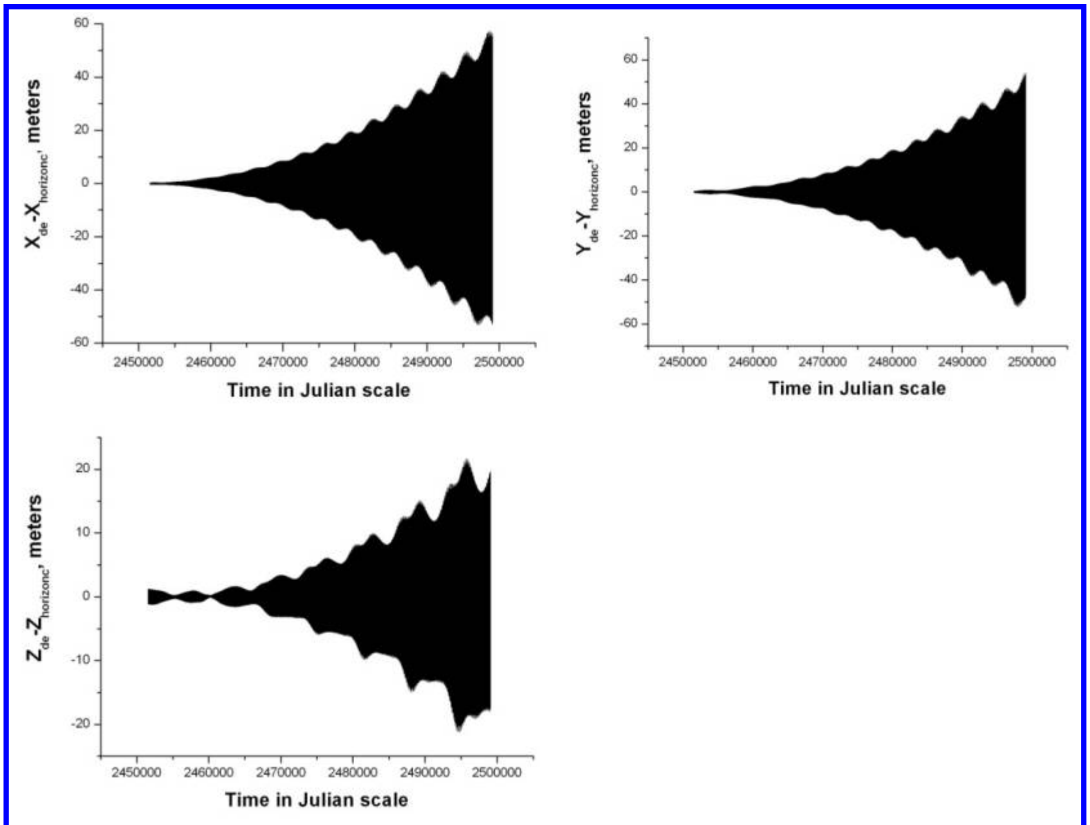


Fig.2 Residual differences between DE421 and HORIZONS in Cartesian coordinate system

### III. Analysis of physical libration parameters

When constructing the lunar physical libration theory, various orientation angles of a coordinate system may be used: Euler, Andoyer, and plane ones. We are developing the PLM theory using plane angles (Khabibullin, Sh. T., *et al.*[6], Zagidullin A., *et al.*[7], Petrova N, *et al.*[8]) (Fig. 3). In this connection, it is necessary to transit from Iσ, ρ, τ (libration in node, in inclination, in longitude) angles to the plane ones μ, ν, π (libration in longitude and latitude).

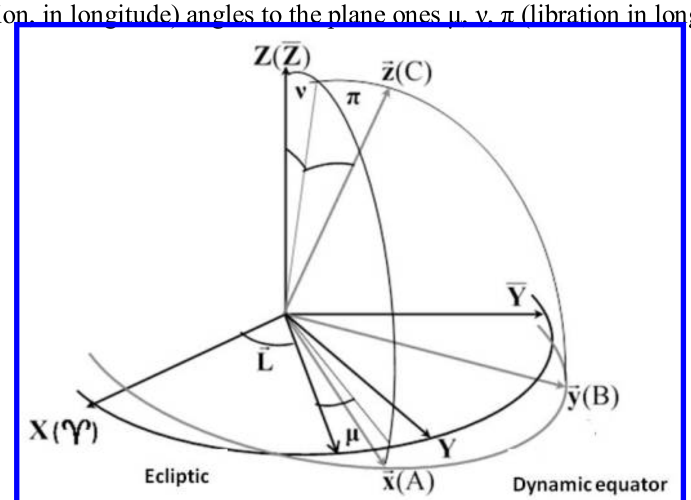


Fig.3 Transition from ecliptic coordinate system (X, Y, Z) to the dynamic one (x(A),y(A),z(C)) using μ, ν, π plane angles and intermediate rotating coordinate system ($\overline{X}, \overline{Y}, \overline{Z}$).

The transformation from ecliptic coordinate system to dynamic coordinate system (DCS) may be performed using a number of methods. A classic transit from ecliptic coordinate system to DSC is implemented using the Euler angles. Since the Euler angles are related to Iσ, ρ, τ libration angles, the transition from one coordinate system to the other can be performed using the rotation matrix as follows: $R_z(\psi) \cdot R_x(-\theta) \cdot R_z(\varphi)$, where ψ is angle counted from the intersection of lunar equator and plane of ecliptic to the first lunar meridian, θ is inclination of lunar equator to plane of ecliptic, φ is angle between the direction towards equinoctial point and intersection of lunar equator and plane of ecliptic. The same transition expressed through the plane angles is as follows: $R_x(-\pi) \cdot R_y(\nu) \cdot R_z(\mu)$. Since both expressions transform ecliptic coordinate system into the dynamic one, the transition matrix should be the same in both cases. Relation between the two sets of angles describing PLM could be expressed as follows:

$$\begin{aligned} &\sin(\nu) = \sin(\psi)\sin(\theta) \\ &\sin(\pi)\cos(\upsilon) = \cos(\psi)\sin(\theta) \\ &\sin(\mu)\cos(\upsilon) = \cos(\psi)\sin(\varphi) + \sin(\psi)\cos(\theta)\cos(\varphi) \\ &\cos(\mu)\cos(\upsilon) = \cos(\psi)\cos(\varphi) - \sin(\psi)\cos(\theta)\sin(\varphi) \end{aligned} \tag{1}$$

In order to examine the adequacy of the developed algorithm, a theory whose internal properties do not contradict DE421 theory is required. Unfortunately, the analytical theory (Petrova N., *et al.*[8]) used at the first stage of the analysis does not take into account a number of perturbation factors such as perturbations from planets, Earth's flattening effects, the 4$^{th}$ harmonic of selenopotential, and other ones considered in DE421. In this connection, it is only possible to assess an amplitude of the residual differences on the basis of following harmonics: ω, F+ω, F-ω. The total maximum contribution of these harmonics is about 20 arcseconds (Rambaux, N., *et al.*[2]). Fig.4 shows residual differences calculated for ν and π angles. This data was obtained in accordance with the theory by Petrova, and after the transformations, with taking into account DE421 ephemeris and using equations (1). Here it should be noted that libration angle in longitude μ is very sensitive to the effects falling outside the main problem. Thus, the obtained differences for latitudinal components of libration do not relate to the given libration.

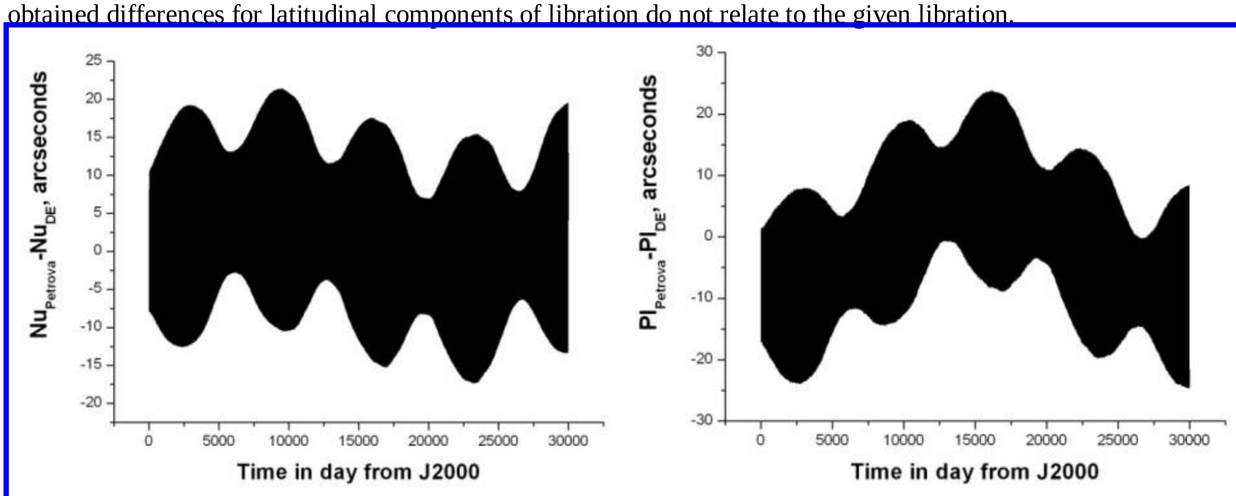


Fig. 4 Residual differences in ν and π angles between theory by Petrova and DE421

**IV. Analysis of results and Conclusion**

We may conclude that the application of a dynamic ephemeris allows obtaining directly an accurate numerical solution of N-body problem for orbit parameters of any planet of the Solar system. There also arises an opportunity to calculate the parameters in relation to both the Sun and other celestial bodies.

In the present paper, we have analyzed the lunar libration angles and directing cosines of the ecliptic pole's radius vector in the reference system related to the center of the Moon around which the Earth orbits.

When constructing the numerical theory of lunar physical libration, a comparison between this theory and observations is required. In this work, the PLM ephemeris constructed on the basis of lunar laser ranging data is compared with libration angles from theories (Zagidullin A., *et al.*[7], Petrova N, *et al.*[8]) constructed in other coordinate systems. Comparison with the results obtained in (Rambaux, N., *et al.*[1]) has shown a good agreement in libration angles, except libration in inclination. In latter case, one needs to carry out additional calculations. Relations (1) derived by the authors allow obtaining plane angles from the classic Euler angles of libration.

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