

## Modules close to the automorphism-invariant and coinvariant

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The aim of this paper is to introduce a general setting where some well-known results on essentially injective modules, automorphism-(co)invariant modules and small projective modules can be obtained by developing a general theory of modules which are (co)invariant under automorphisms of their covers and envelopes.

*Keywords:* Essentially injective module; small projective module; automorphism-invariant module; automorphism-coinvariant module.

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### 1. Introduction

Throughout this paper, all rings will be associative rings with identity and our modules will be unitary right modules unless otherwise is stated. We will use the notation  $N \leq M$ ,  $N \leq^e M$ ,  $N \ll M$  to stress that  $N$  is the submodule of  $M$ ,  $N$  is the essential submodule of  $M$  and  $N$  is the small submodule of  $M$ , respectively. We write  $J(R)$ ,  $J(M)$ ,  $\text{Soc}(R_R)$ ,  $\text{Soc}(M)$ ,  $Z(R_R)$  for the Jacobson radical of the ring

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