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On τ-Compactness of Products of τ-Measurable Operators

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Abstract Let $\mathcal{M}$ be a von Neumann algebra of operators on a Hilbert space $\mathcal{H}$, τ be a faithful normal semifinite trace on $\mathcal{M}$. We obtain some new inequalities for rearrangements of τ-measurable operators products. We also establish some sufficient τ-compactness conditions for products of selfadjoint τ-measurable operators. Next we obtain a τ-compactness criterion for product of a nonnegative τ-measurable operator with an arbitrary τ-measurable operator. We construct an example that shows importance of nonnegativity for one of the factors. The similar results are obtained also for elementary operators from $\mathcal{M}$. We apply our results to symmetric spaces on $(\mathcal{M}, \tau)$. The results are new even for the *-algebra $\mathcal{B}(\mathcal{H})$ of all linear bounded operators on $\mathcal{H}$ endowed with the canonical trace $\tau = \mathrm{tr}$.

Keywords Hilbert space · Linear operator · Von Neumann algebra · Normal semifinite trace · τ-measurable operator · τ-compact operator · Elementary operator · Integrable operator · Rearrangement

1 Introduction

Let $\mathcal{M}$ be a von Neumann algebra of operators on a Hilbert space $\mathcal{H}$, τ be a faithful normal semifinite trace on $\mathcal{M}$. Products of τ-measurable operators appear in various problems of the noncommutative integration theory (for example, in [19] in definiton of Köthe dual space; in Hölder inequality [28], in Golden-Tompson inequality [7] and the Peierls-Bogoliubov inequality [8], among others). Some sufficient conditions for integrability of τ-measurable operators products can be found in [14]. The present paper continues the

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