Simulation of Interaction of Vortex Structures

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***Abstract*—The results of numerical experiments, made on the basis of the modified CD method, on study of the dynamics of interaction of vortical structures for various configurations of their relative positioning, signs of vorticity and distances between borders of vortical regions are presented. The examples of modeling results for real vortex systems in atmosphere and plasma are given.**

Keywords—vortices, modeling, atmosphere, plasma, CD method

I. Introduction. Basic Equations

In this paper we study numerically the interaction of vortex structures of type of the finite area vortex regions (FAVR) [1] in continuous media such as the atmosphere, hydrosphere and plasma in two-dimensional (2D) approximation when Euler-type equations are used to describe motion. The Euler equation for an inviscid incompressible fluid,

$$\mathrm{d}\mathbf{u}/\mathrm{d}t = \mathbf{F} - (1/\rho)\,\mathrm{grad}\,p \tag{1}$$

in variables "vorticity – flow function" in the case of flat motion takes the form of set consisting of the equation of carry of a vortex and the Poisson equation for flow function [2]:

$$\partial_t \zeta + u\,\partial_x \zeta + v\,\partial_y \zeta = 0\,, \qquad \Delta\psi = -\zeta \tag{2}$$

where $\zeta = [\nabla, \mathbf{v}]$ is the vorticity, $\mathbf{v} = [\nabla, \psi]$ where ψ is the flow function (ψ is positive when streamlines are directed counter-clockwise); $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$. Note that the equation of carry of a vortex is nonlinear since u and v are functions of ζ. The last two terms in it describe convection, and this means that vortex is carried by a stream.

Let us give examples of real physical problems when such a description is valid. A simple model of a 2D magnetized plasma of Tailor-McNamara [3] is the charged filaments aligned in a uniform magnetic field **B**, which move with the central-directed velocity $\mathbf{E}\times\mathbf{B}/B^2$, has the form

$$\begin{aligned} e_i\,\mathrm{d}x_i/\mathrm{d}t &= (1/B)(\partial H/\partial y_i),\\ e_i\,\mathrm{d}y_i/\mathrm{d}t &= -\partial H/\partial y_i \end{aligned} \tag{3}$$

where e_i is charge on unit of length, $H = \sum -e_i e_j \ln(|\mathbf{r}_i - \mathbf{r}_j|)$ is the energy of Coulomb interaction. In a continuous limit, this 2D plasma satisfies the equations:

$$\begin{aligned} \partial\rho/\partial t + \mathbf{v}\cdot\nabla\rho = 0\,, &\quad \mathbf{v} = -(1/B)\,(\hat{\mathbf{z}}\times\nabla\psi)\,,\\ \nabla^2\psi = -\rho\,, &\quad \mathbf{v} = (v_x, v_y) \end{aligned} \tag{4}$$

where ρ is the charge density, $\nabla = (\partial/\partial x,\ \partial/\partial y)$, ψ is the potential of an electric field. One can see that these equations are identical to the equations of 2D motion of an inviscid incompressible fluid, when ρ is the z-component of vorticity ζ, ψ is the flow function, and for discrete vortices B = 1. The equations of motion of clouds of ideal ionospheric plasma have a similar form. Other 2D continuous models include the Debye shielding radius $k^2\psi$ in the Poisson equation (4) [4]:

$$\nabla^2\psi - k^2\psi = -\rho\,. \tag{5}$$

In these models the Hamiltonian H describes the shielded interaction between filaments.

The Hasegawa-Mima model [5] includes ionic-polarized current through the equation of motion of ions:

$$\mathrm{d}\mathbf{v}/\mathrm{d}t = (e/M)(-\nabla\varphi + \mathbf{v}\times\mathbf{B})\,. \tag{6}$$

In this case k^{-1} is not Debye length but it is the ion Larmour radius (electron temperature), and the shielding is an indirect effect of the ion-polarized current.

The model of hydrodynamic fluids Charney [6], describing the motion of the earth's atmosphere, also formally corresponds to the screened interaction. Atmospheric flows in the horizontal plane are described by the equation

$$\mathrm{d}\mathbf{v}/\mathrm{d}t = -g\nabla h + \mathbf{R}v\times\hat{\mathbf{z}} \tag{7}$$

where h is an atmospheric depth, **R** is the Coriolis force, and the Rossby radius $\sqrt{gH_0}/R$ plays the role of the shielding length.

There are many other examples of vortex motions in plasma and rotating fluids (see, e.g., [2], [7]–[9]) when the hydrodynamics description is also used and the equations can be reduced to the form (2) or (4).

So, the set of the equations describing a motion of fluid, gas or plasma, in generalized variables has form

$$\begin{aligned} \partial\rho/\partial t + \mathbf{v}\cdot\nabla\rho = 0, &\quad \mathbf{v} = -(1/B)(\hat{\mathbf{z}}\times\nabla\psi),\\ \nabla^2\psi - f &= -\rho. \end{aligned} \tag{8}$$

Depending of the medium under consideration, functions and variables here will have a different physical sence (Table 1), and the system will have form of the equations discussed above.

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TABLE I. SENSE OF VARIABLES IN DEPENDENCE ON MEDIUM

Function	Fluid, gas	Plasma
ρ	z-component of vorticity	line density of a charge
ψ	flow function	potential of an electric field
B	$B = 1$	module of a vector of a magnetic induction
f	$f = 0$	$f = 0$ – plasma with Coulomb interaction $f = k^2\psi$ – plasma with shielded Coulomb interaction

We solved numerically the set (8) for different physical systems using the modified method of contour dynamics (the CD method) developed and described in detail in [10]-[12].

II. THE RESULTS OF SIMULATION AND CONCLUSION

At the first stage, we studied the regimes of 2-vortex (pair) interaction depending on the value of a number of critical parameters [2], [9], [13], [14]. We have found that for vortices with the same polarities the result of evolution depends on the value of parameter δ which is the initial distance between boundaries of the interacting vortex regions, namely:

- For rather big δ (Fig. 1a) the vortices, on a level with rotation around of their own axes, rotate also around their common center, their interaction is weak and is reduced to small cyclic change of their form – we observe the "quasi-recurrence" phenomenon, noted for the first time in [1].
- At rather small δ (Fig. 1c) the vortices, on a level with the same rotation, interact forming a common vortex region which consists of the vortices of more small scale, thus the regime of active interaction with the "phase intermixing" takes place. The critical initial distance between two interacting vortices dividing two types of interaction is $\delta_{cr} = 3d/4$ where d is the diametr of the vortex.
- At $\delta \sim \delta_{cr}$ an intermediate case is realized (Fig. 1b).

For two vortices with the opposite polarity we have found that they do not interact practically, independently on the value of initial distance δ between their boundaries [15].

The problem for N-vortex systems with N = 3, 4 was considered in two variants: for vortices linearly disposed at initial time, and for vortices disposed in the corners of appropriate equilateral figure at initial time. In the numerical experiments we have found (see Fig. 2) that:

- For rather big and equal initial distance δ_i between vortices the evolution leads to formation of two vorticity regions as a result of more strong interaction of each of the "outer" vortices with closest "inner" vortex (Fig. 2a). Thus, the interaction of forming pairs is similar to that of two vortex case.
- For $\delta_i \leq d/2$ $(i = 1, \ldots, 4)$ we observed the formation of a complex vortex structure which consists of many vorticities of more small scales (Fig. 2b). Further evolution leads to formation of complex turbulent field. We can also see that the interaction between outer vortices is stronger.

As a result we have obtained the critical value of parameter δ for such complex systems: $\delta_{cr} = \delta_i = d/2$ $(i = 1, \ldots, 4)$.

Further, on the basis of the results on pair interaction we also simulated the interaction dynamics of the 3D plane-rotating vortex structures in the "2D approximation" within the framework of many-layer model of medium. Fig. 3 shows the result of simulation of the interaction of two 3D vortices with the exponential decreasing of their vorticity with z-coordinate. One can see that, in the beginning, the vortices' central regions start to interact and only then other areas are involved in interaction. Such behavior is explained by stronger interaction of central regions, which locate at the relatively short distance each other and their vorticities have relatively big values, so that the ratio ζ/δ is big in comparison with that for top and bottom of vortices.

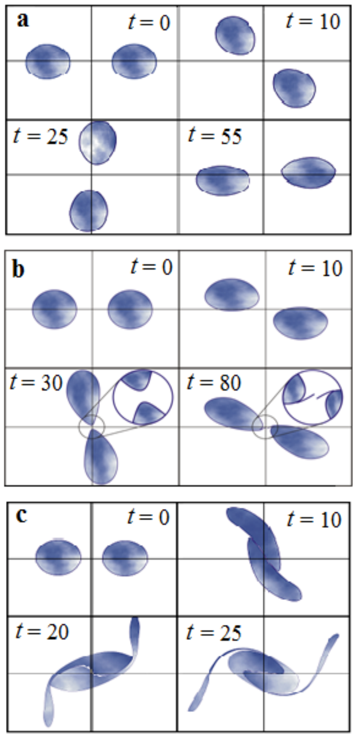

Fig. 1. Modes of pair interaction of vortices of anti-cyclonic type.

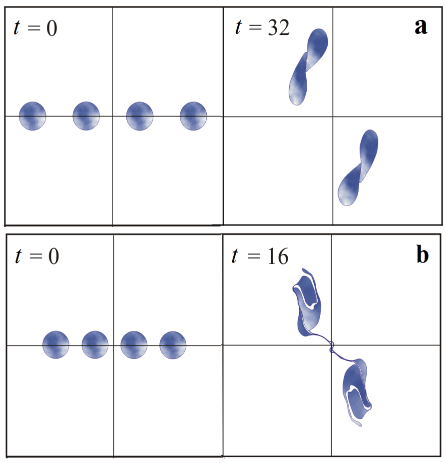

Fig. 2. 4-vortex interaction for the vortices linearly disposed at initial time.

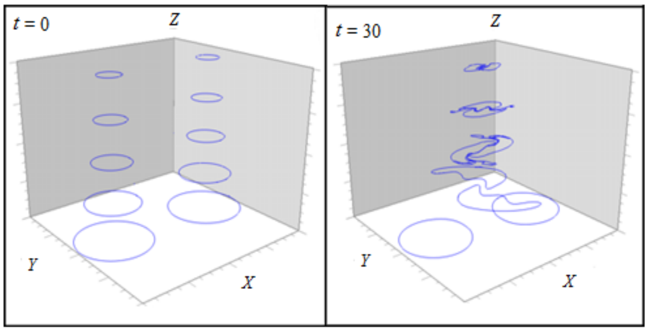

Fig. 3. Interaction of the 3D plane-rotating vortical structures (many-layer model of medium).

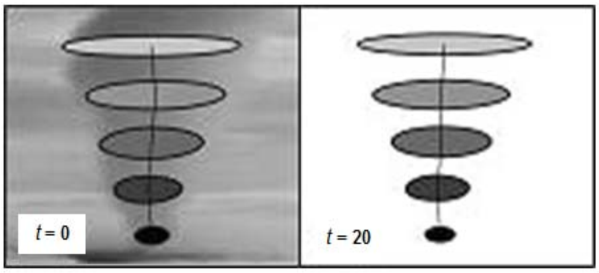

Fig. 4. Example of simulation of the tornado evolution using quasi-2D approach.

As an application to real media, we have simulated the tornado evolution with use of quasi-2D approach with many-layer approximation of the 3D vortical structure by the vortex system (Fig. 4). Our model vortex (system of FAVRs) has been associated with real tornado from video-record. In particular, in our numerical experiments [2], [9] we investigated an influence of the perturbation imposed on the tornado axis on its dynamics. We established as a result, that small cross-section indignation leads to inappreciable fluctuations of an axis and, as a whole, does not influence on structure and stability of a vortex. So, our simulation reflects the basic features of evolution of a tornado and, therefore, we can forecast a tornado evolution and can also simulate interaction of such type of 3D vortices.

Other application studied with use of the approach presented here is the formation of complex vortex structures and folds at cross-section perturbations of the charged filaments in a uniform magnetic field **B** (using 2D model of plasma of Taylor-McNamara [3]) with transition of the particles' streams in an unstable state (Fig. 5). We have found that

- The structures of vortical type are forming especially quickly and more intensively, than more amplitude of perturbations and quantity of the filaments participating in interaction, and also than is more close to each other filaments are located;
- The cross-section perturbations of velocity of a stream lead to its transition in an unstable state with formation of folds and complex vortical structures;
- This process leads to deformations of the magnetic field in the polar cusp area.

One more application investigated by us is the interaction between "hydrodynamic" vortex structures and dust particles in a dust plasma. The theoretical analysis and the experimental results show that in plasma with gradient of dust charge the vorticity of dust particles can exist [2],

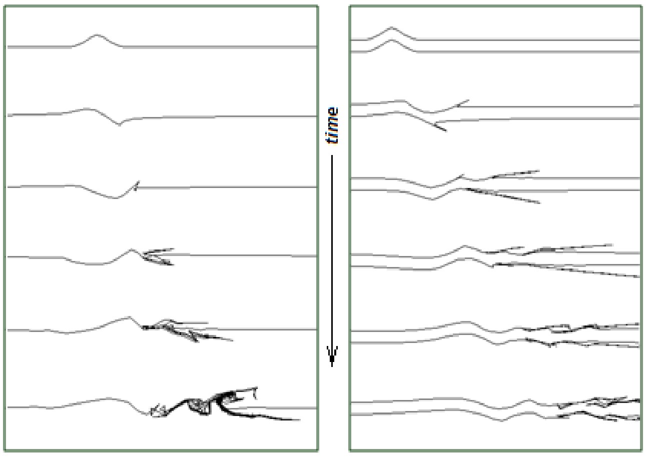

Fig. 5. Vortex structures formation at cross-section perturbation of charged filaments: a – one perturbed line; b – two lines with perturbations of the same polarity.

[9], [10]. This gives a possibility to study the interaction between the "hydrodynamic" vortex structures and dust particles considering the dust particles as vortices of very small scales.

In our numerical experiments we have found (see Fig. 6) that the character of interaction in this case depends on value of particles' vorticity:

- If particles' vorticity is very small that the interaction does not observed;
- When the vorticity of dust particles becomes like vorticity of the "hydrodynamic" vortex, the interaction becomes significant, and particles are involved by a vortex in large-scale rotation.

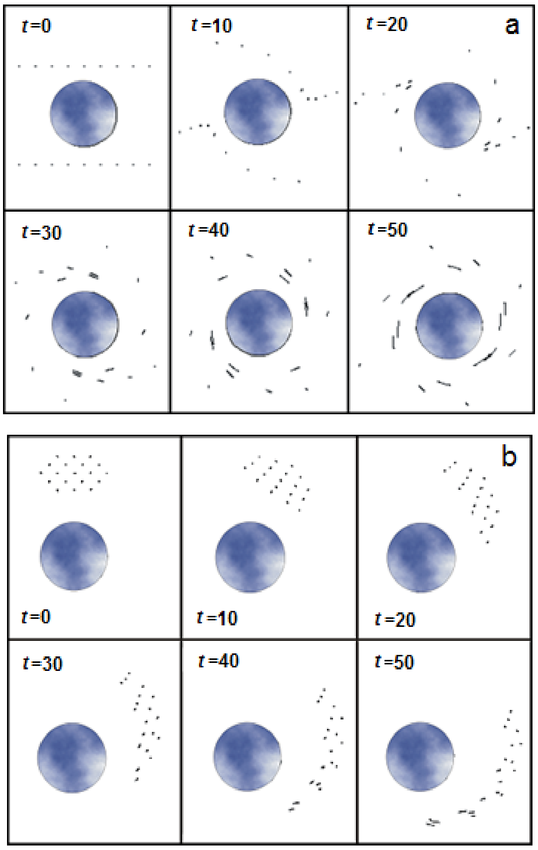

Fig. 6. Interaction of dust particles with vortex: a – linear dust layers; b – dust clouds.

III. Conclusion

In conclusion, we have presented here our results of numerical study of the dynamics and interaction of 2D and 3D vortex structures described by a system of differential equations of Euler type. From the Euler equation for inviscid incompressible fluid we have obtained the set of the equations in generalized variables "vorticity – flow function" describing a motion of fluid, gas or plasma, which in dependence of the medium under consideration, functions and variables are applicable to study the vortex motions in different physical media. We have considered their applications to description of vortex evolution and interaction in a plasma (in the framework of the Tailor-McNamara and Hasegawa-Mima models) and in the Earth's atmosphere (the Charney model).

For simulation we used the generalized contour dynamics (CD) method which has been developed by us earlier. We investigated numerically the regimes of 2-vortex (pair) interaction depending on the value of a number of critical parameters which define the regimes of interaction. As a result, we have found that in our generalized system two cases of pair interaction can be realized: the "quasi-recurrence" phenomenon (like noted for the first time in [1]) and the "phase intermixing". In some cases also an intermediate case is realized. For the N-vortex systems we have found that in some cases, in dependence on the values of critical parameters, the formation of a complex vortex structure which consists of many vorticities of more small scales can be observed. Further evolution of such systems leads to formation of complex turbulent field.

We have considered some applications of our general results in physics of real physical media. So, we have showed that the models considered play an important role not only in studying general vortex dynamics but also in modeling vortical processes in the atmosphere (including the ionosphere) and hydrosphere, as well as in studying the propagation of vortex structures in a magnetized plasma. The results obtained can be useful in investigations in fields such as plasma physics, hydrodynamics and physics of atmosphere, ionosphere and hydrosphere.

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