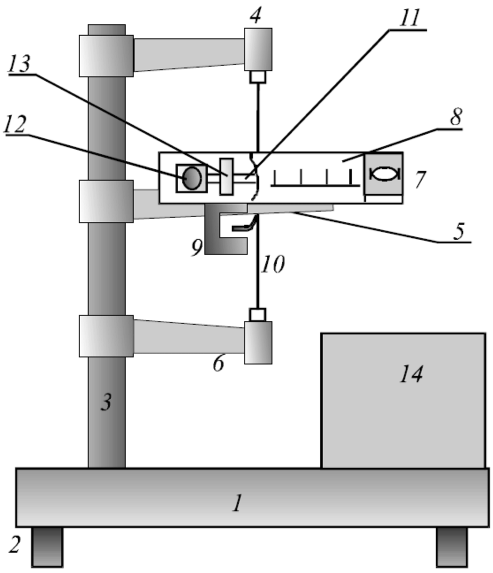
Measuring the speed of a bullet with the aid of a torsional pendulum

Aim of the work
Studying the laws of conservation in dynamics of rotational motion and their application in a practical task of finding the speed of a bullet.

Experimental setup
The work is performed on a rotational ballistic pendulum with an electronic timer and bullets. The general view of the pendulum is shown in Fig. 1. The basis 1 stands on regulating legs 2 used for levelling. The basis holds the column 3 with fixed arms 4, 5, 6. The middle arm 5 holds a shooting device 7, a transparent screen 8 with an angle gauge, and a photoelectric sensor 9. Arms 4 and 6 have clamps for fastening a steel wire 10. The pendulum hangs on this wire and consists of two rods 11, plasticine-covered plates 12, and two movable plummets 13. Photoelectric sensor is connected with a timer 14 which is used for measuring the oscillation period of the pendulum.



Deriving the working formula
After the bullet hits the pendulum, is begins to oscillate around the vertical axis. If the interaction of the bullet and the pendulum proceed quick enough (the time of interaction is much shorter than the period of oscillation) then the angular momentum should remain the same before and after the shot:

$$mvl = \left(J_1 + ml^2\right)\omega, \qquad (1)$$

where m is the bullet's mass, v is its speed, l is the distance from the rotation axis to the point at which the bullet hits, J_1 is the moment of inertia of the pendulum with respect to the rotation axis, and ω is the angular speed which the pendulum acquires after the shot.
If friction is neglected, the mechanic energy should conserve during the oscillations. The maximal value of the kinetic energy is then equal to the maximal value of the potential energy:

$$\left(J_1 + ml^2\right)\omega^2 / 2 = D\alpha_m^2 / 2\,. \qquad (2)$$

Here D is the torsion modulus (the proportionality coefficient in the correlation between the moment of strain forces and the angle of rotation) and α_m is the angle of the maximal deviation of the pendulum.
Equations (1) and (2) allow finding the following expression for the bullet's speed:

$$v = \frac{\alpha_m}{ml}\sqrt{D\left(J_1 + ml^2\right)}\,. \qquad (3)$$

Since in our experiment $ml^2 << J_1$, we can simplify this formula and write down:

$$v = \frac{\alpha_m}{ml}\sqrt{DJ_1}\,. \qquad (4)$$

To find J_1 and D, free oscillations of the pendulum can be used. The motion of equation is written using the derivative of the unknown function of time α(t):

$$J_1\alpha'' = -D\alpha\,. \qquad (5)$$

The solution of this differential equation is the harmonic function $\alpha = \alpha_m\cos(2\pi t/T + \varphi_0)$ with the period T depending on the properties of the system as

$$T = 2\pi\sqrt{\frac{J_1}{D}}\,. \qquad (6)$$

If the distance between plummets 13 is changed (see Fig. 1), the moment of inertia of the pendulum and, hence, the period of oscillations also change. We can write for two different positions:
periods are

$$T_1 = 2\pi\sqrt{\frac{J_1}{D}}\,,\; T_2 = 2\pi\sqrt{\frac{J_2}{D}}\,, \qquad (7)$$

moments of inertia are

$$J_1 = J_0 + 2MR_1^2\,,\; J_2 = J_0 + 2MR_2^2\,. \qquad (8)$$

Here J_0 is the moment of inertia of the pendulum itself without the plummets; R_1 and R_2 are the distances from the rotation axis to the plummets' centres, and M is the mass of the plummet. Values of I_1 and D can be calculated from these equations, but we do not need to know the numerical values. If corresponding analytical expressions are substituted to Eq. (3), the working formula for finding the bullet's speed can be written as

$$v = \frac{4\pi\alpha_m M}{ml}\frac{T_1}{T_1^2 - T_2^2}\left(R_1^2 - R_2^2\right). \qquad (9)$$

Algorithm of measurements

1. Switch on the setup by pressing subsequently the buttons “Сеть”/“Power” and “Сброс”/“Clear” of the front panel. Zeros should starts glowing on the digital display. Check if the lamp in the photoelectric sensor is also working/
2. Locate the plummets 13 on the rods at the biggest distance from each other; measure R_1.
3. Set the angle in the zero position, if necessary, by rotating the element holding the wire in the arms 4 and 6.
4. Cock the spring of the shooting device, put the bullet and make a shot.
5. Measure the maximal angle of deviation of the pendulum.
6. Omit two or three first oscillation and measure the duration of next 10–15 periods by starting the timer with the button “Сброс”/“Clear” and stopping it with the button “Стоп”/“Stop”. The number of oscillations counted is shown on the display “Периоды”/“Periods”. Calculate T_1.
7. Locate the plummets 13 at the smallest distance R_2; measure it.

8. Rotate the pendulum by hand at an angle similar to α_m and release it. Find T_2 like in the step 6.
9. Measure the bullet's mass m.
10. Calculate the bullet's speed from Eq. (9). The mass of the plummet is written on the apparatus.
11. Repeat the measurement for other positions (R_1^k, $k = 2, 3, \ldots$) of the plummets 13.

Questions

1. Mechanics of collision (impact), basic features of this phenomenon.
2. Absolutely elastic and absolutely inelastic collisions: common features and differences.
3. Examples of absolutely inelastic collisions.
4. Describe the experimental setup and the method of measuring the bullet's speed.
5. Is it possible to measure any speed of a bullet? Is it possible to deal with a bullet of any possible mass?