

#111 * Rectilinear motion

October 8, 2018

Purpose

- Determine the dependence of the distance, speed, and acceleration on time for the case of uniform rectilinear motion.
 - Determine the dependence of the distance, speed, and acceleration on time for the linear motion with uniform acceleration.
 - Analyse the obtained results, compare them with the theory.
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Motion is the way that matter exists. In a general sense, motion includes any change, all processes occurring in the Universe, from mechanical motion to the thinking process.

Mechanical motion is the simplest form of motion. It is change of the mutual disposition of bodies or their parts in space with time. The field of science studying general laws of mechanical motion and equilibrium of material bodies is called mechanics. Classical mechanics considers motion of absolutely rigid bodies (which are not deformed when moving) with velocities negligible compared to the speed of light. Motion of a mass point is an example of the simplest mechanical motion.

Mass point is a body whose size and shape are unimportant. If the sizes of a body are small compared to the way which it passes and to the distance to other bodies considered in a given task, then this body may be considered as a mass point.

Motion of material bodies takes place in space and time.

Space and time are forms of existence and changing the states of the matter; they are inherent attributes of the matter. To describe the motion, we need a frame of reference. The reference frame should include a reference body (a real or abstract one), a system of coordinates bound to that body, and a method of measuring time. For example, position of a mass point in any instance may be fixed in a Cartesian frame of reference (see Fig. 1).

As the mass point moves, its coordinates change with time:

$$\begin{aligned}x &= f(t), \\y &= g(t), \\z &= h(t).\end{aligned}\tag{1}$$

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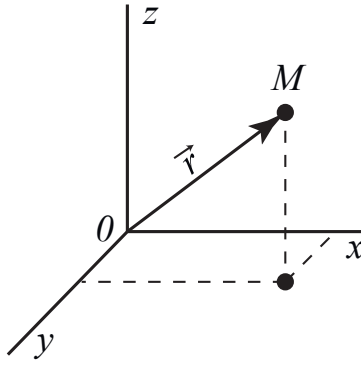


Figure 1: Cartesian frame of reference and a radius vector pointing to a mass point M .

Motion of the point is fully described, if equations (1) are known. Three equations can be rewritten in a brief vector form:

$$\vec{r} = \vec{r}(t), \quad (2)$$

where \vec{r} is the radius vector connecting the origin of the reference frame with the moving mass point.

Any mechanical motion has a relative character. A particular way of motion, defined by the trajectory, path, displacement, velocity and acceleration, depends on the choice of the reference frame. Let us clarify it by examples.

Imagine a ship travelling uniformly along a straight line on a river. A material body detaches from the top of its mast and falls down. In the reference frame bound to the ship (in other words, to an observer on the ship's deck) the trajectory of the body is a vertical line, but for an observer staying on the river bank the trajectory of the same body looks like a curved line (a parabola).

Imagine also a man sitting in a carriage in a moving train. In the reference frame bound to the carriage, the speed of the man is zero, whereas with respect to the railway the speed of the man equals the speed of the train.

Ways covered by a body may also be different in different reference frames, and so on.

1 Kinematics

Kinematics is a part of mechanics studying the laws of motion without considering the reasons which makes the bodies move.

1.1 Physical notions characterising the kinematics of a mass point

Trajectory is a line passed in space by a moving mass point (m.p.). Depending on the shape of the trajectory, one can speak about rectilinear or curvilinear motion.

Path (distance) is the length of a trajectory segment which was passed by the m.p. in a certain time range.

Displacement is the vector connecting the initial and current positions of the m.p.

In Fig. 2 A stands for the initial position (at $t = 0$) of the mass point; B is its position at time t ; the arc AaB is the trajectory, the length of the arc AaB is the distance (ΔS); and $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$ is the displacement.

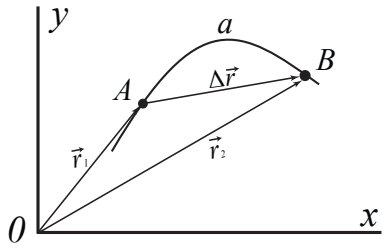


Figure 2: Trajectory, path and displacement.

Velocity is a vector value characterising the rate and direction of the motion of the m.p. It equals the first derivative of the displacement with respect to time:

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}. \quad (3)$$

It follows from Eq. (3) that in the case of curvilinear motion, the velocity vector is tangent to the trajectory in the corresponding point (Fig. 3).

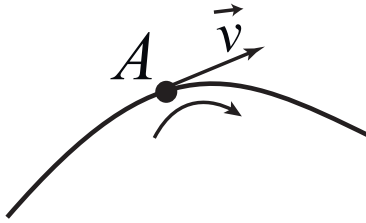


Figure 3: Instantaneous velocity.

In the limit, the length of the arc dS is equal to the chord length $d\vec{r}$, and hence

$$v = \frac{dS}{dt}, \quad (4)$$

that is, the speed value is the same as the first derivative of the distance with respect to time.

Acceleration is a vector value describing the rate at which the velocity changes (in a general case, both in the absolute value and direction). The acceleration vector is the first derivative of the velocity with respect to time:

$$\vec{a} = \frac{d\vec{v}}{dt}. \quad (5)$$

In the case of the rectilinear motion, the velocity may change its value but not the orientation. Then Eq. (5) may be rewritten as follows (without vectors):

$$a = \frac{dv}{dt}. \quad (6)$$

Evidently, acceleration of the rectilinear motion equals the first derivative of the absolute value of the velocity (the speed) with respect to time.

1.2 Uniform rectilinear motion

Motion in a fixed direction at a constant speed is called uniform and rectilinear. According to Eq. (4), $dS = vdt$ and $\int_0^S = v \int_0^t dt$. Hence

$$S = vt. \quad (7)$$

Equation (7) is the equation of uniform rectilinear motion, and it follows from it that

$$v = \frac{S}{t},$$

that is, the speed of such a motion equals the ratio of the way, passed by a body during some time period, to this period.

The unit in which the speed is measured is meter per second.

The dependence of the velocity $v(t)$ can be plotted as a diagram with time corresponding to the horizontal (x-coordinate) axis and the speed shown by the vertical axis. In the case of the uniform motion, $v = \text{const}$, and the plot looks like a horizontal line parallel to the x-coordinate axis (Fig. 4).

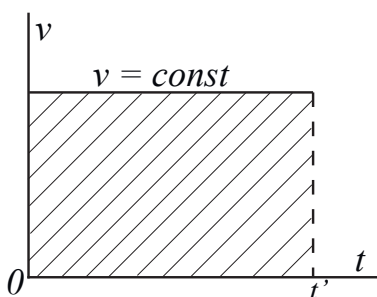


Figure 4: Speed is shown by the horizontal line, and way corresponds to the dashed area.

The area of the figure bounded by the velocity plot, horizontal time axis, and two vertical lines corresponding to the time moments of beginning and end of the motion, is equal to the distance passed by the body in the corresponding time range.

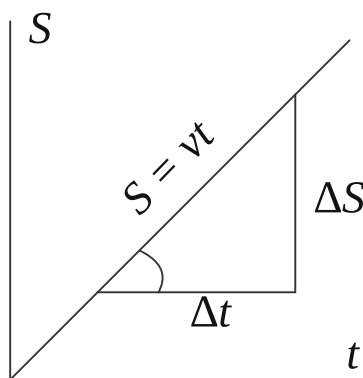


Figure 5: Change of distance with time in the case of the uniform motion. The slope of the plot is the speed: $\tan \alpha = \Delta S / \Delta t = v$.

The plot in Fig. 5 is a distance plot (the horizontal axis shows the time; the vertical axis, the path). According to Eq. (7) it is a straight line starting at the origin (at $t = 0, S = 0$).

1.3 Rectilinear motion with a constant acceleration

If the velocity of a body changes in time, the motion is called **variable motion**. If the speed (absolute value) changes by the same value in any equal time ranges, we call this uniformly variable motion. It can be decelerating or accelerating.

In any variable motion, the new concepts appear: mean and instantaneous velocity.

Mean speed of a variable motion in a given time period is defined as the speed of a uniform motion at which a m.p. would pass the same distance in the same period. Mean speed is the ratio of the path ΔS , covered by a certain time Δt , to this time range:

$$v_m = \frac{\Delta S}{\Delta t}. \quad (8)$$

Instantaneous speed is the speed in a given point on a trajectory, in a given time moment. It is equal to the limit value of the mean speed, to which the mean speed tends as the time period tends to zero:

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t} = \frac{dS}{dt}. \quad (9)$$

Instantaneous speed is the first derivative of the distance with respect to time.

Uniformly variable motion is characterised by some constant value of the acceleration $a = \text{const}$, and according to Eq. (6), $dv = a \cdot dt$, and

$$v = \int a \cdot dt = at + C. \quad (10)$$

The integration constant can be found from the boundary condition: $t = 0$, $v = v_0$, therefore, $C = v_0$. Finally,

$$\begin{aligned} v &= v_0 \pm at, \\ \pm a &= \frac{v - v_0}{t}. \end{aligned} \quad (11)$$

Acceleration of a uniformly variable rectilinear motion is equal to the change in the speed per time unit. Positive sign in Eq. (11) corresponds to the accelerated motion ($v > v_0$); negative, to the decelerated motion ($v < v_0$).

Unit of measurement for acceleration can be determined from Eq. (11): m/s^2 .

$$|a| = \frac{|v|}{t} = \left[\frac{m}{s^2} \right].$$

Now we can rewrite expression (4) as $dS = v \cdot dt$. Then, allowing for (11), we get:

$$S = \int_0^t v \cdot dt = \int_0^t (v_0 \pm at) dt = v_0 \int_0^t dt \pm a \int_0^t t dt;$$

and hence

$$S = v_0 t \pm \frac{at^2}{2}. \quad (12)$$

Let us consider now the diagrams of the speed and path of the uniformly variable motion. Analytical dependence of the speed on time (11) is the equation of a straight line which crosses the vertical axis at the value of v_0 (see Figs. 6, 7).

If the initial velocity is zero, then

$$v = at. \quad (13)$$

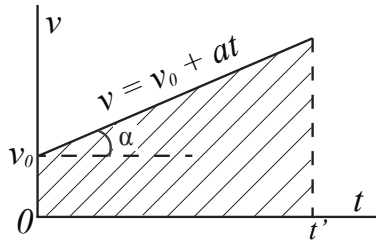


Figure 6: Velocity dependence of accelerated motion.

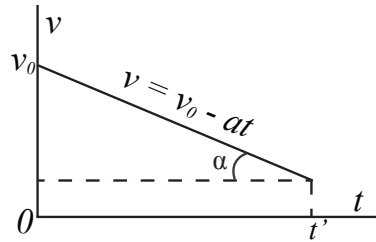


Figure 7: Velocity dependence of decelerated motion.

The dashed area in Fig. 6 shows the path covered by the body from time 0 to t' . The area of this trapezium is

$$\text{Trap.area} = \frac{1}{2} (v_0 + v) t' = \frac{1}{2} (v_0 + v_0 + at') t' = v_0 t' + \frac{at'^2}{2}.$$

The dependence (12) of the distance on time is the second order polynomial, and its plot is a curved line (a parabola). In the case of accelerated motion this parabola has its convex downwards; in the case of decelerated motions, upwards (Figs. 8, 9).

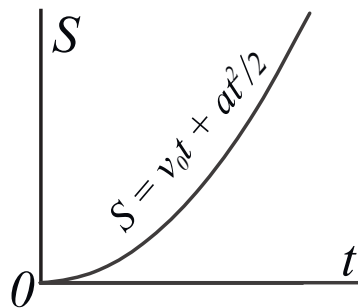


Figure 8: Path dependence of accelerated motion.

2 Dynamics

Dynamics is the part of mechanics studying motion of bodies and its reasons (interaction between the bodies). Dynamics is based on three laws of Newton.

Newton's first law states that every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force.

The second law: acceleration gained by a body as a result of some forces acting on it is proportional to the geometrical (vector) sum of these forces and inverse proportional to the body's mass.

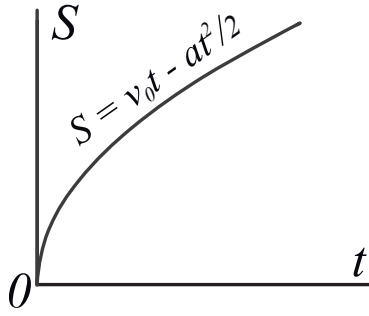


Figure 9: Path dependence of decelerated motion.

$$\vec{a} = \frac{\sum_{i=1}^n \vec{F}_i}{m} = \frac{\vec{F}_R}{m}. \quad (14)$$

Here \vec{a} is the acceleration of the body; m is its mass; $\sum_{i=1}^n \vec{F}_i$ is the vector sum of the forces acting on the body; \vec{F}_R is the symbol for the resultant force. Note that the vectors of the acceleration and the resultant force are collinear.

Force is a measure of influence of other bodies and fields on the considered body.

The third law states that for every action (force) in nature there is an equal and opposite reaction. In other words, if object A exerts a force on object B, then object B also exerts an equal force on object A. Notice that the forces are exerted on different objects. Mathematically this can be written as

$$\vec{F}_1 = -\vec{F}_2. \quad (15)$$

Experimental setup

- 1 Rails
- 2 CASSY
- 3 Trolley
- 4 Fixed pulley with passage light sensor
- 5 Thread
- 6 Fastening for weights (20 gram)
- 7 Weights (10 gram)

Assembly of the setup

Before starting the assembly, check if somebody has already done some steps.

1. Put the trolley on the rails. Check that it does not fall off the rails.
2. Take the thread, fasten one end of the thread to the trolley and the other to the fastening for weights. Pass the thread over the pulley.

Algorithm of measurements

1. Turn the laptop on.

2. Connect the CASSY unit to the laptop via USB.
3. Launch the file **Linear motion** (Desktop/Students/Labs). In the window which will appear press the button “Close.”
4. Move the trolley on the rail to the right side as far as the thread allows it. Zero the reading of the path s_1 . To do this, press the reading in the s_1 window with the right mouse button and then press $\triangleright 0 \triangleleft$ button in the pop-up menu.
5. Then release the trolley and start the measurements simultaneously. The measurements are started by either pressing **F9** on the keyboard or by the **Measurement...** button on the top of the programme’s window. When the trolley reaches the leftmost position, stop the measurements (again, either by the **F9** key or by the **Measurement...** button).
6. Two plots will appear in the window. They may be seen in the Path and Velocity tabs. They show the time dependences of the path and speed, respectively.
7. Repeat steps 4, 5 three times.
8. Analyse the obtained data on the speed (“Velocity” tab).
 - (a) Press the diagram area by the right mouse (or touchpad) button, choose **Approximation** \rightarrow **Line** in the appearing menu.
 - (b) Press the analysed graphic with the left mouse button and drag the cursor (while holding the button) till the end of the linear segment. Release the button. A straight line passing through the experimental points should appear.
 - (c) Press the keys **Alt + T**. then press “OK” in the arising window. Move the cursor to the graphic to be analysed and press the left mouse button. An equation should appear nearby.
 - (d) In this equation the coefficient A is the acceleration a with which the trolley had been moving.
9. Save the obtained plots
10. Write down the acceleration values and the mass of the weight used.
11. Add a new weight to the thread, and repeat steps 4–6.
12. Build a plot for the dependence of the acceleration on the weight mass $a(m)$ (mass on the horizontal axis, acceleration on the vertical axis).

Presenting the results

1. Write the title of the work, describe the tasks to solve.
2. Attach the graphics.
3. Make conclusions corresponding to the goals of the work. Explain the shape of each observed dependence (linear, quadratic) based on the theory and formulas. Describe how the path, velocity, and acceleration change as the weight load increases. Determine the mean velocity on the whole trolley’s path for each weight used.