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Quantum interference via nonradiative transitions between energy levels of atoms in one-dimensional photonic crystals

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Abstract. The self-energy correction to the mass of an atomic electron placed into a void of the photonic crystal medium is investigated. In this study the one-dimensional photonic crystal made from gallium arsenide and vacuum layers is considered. We show that the electron mass change gives rise not only to the shifts of the atomic energies but also to the nonradiative transitions between atomic levels. These transitions add new channels to a decay of an atomic state that leads to a quantum interference between these channels.

1. Introduction

Quantum interference arising from the spontaneous emission of two nearly degenerate excited states to a common ground state leads to a variety of remarkable effects such as coherent population trapping [1–4], ultranarrow spectral lines [5–7], lasing without inversion [8], electromagnetically induced transparency, and spontaneous emission cancellation [5, 9].

However, strong quantum interference requires the presence of near-degenerate atomic transitions with the nonorthogonal transition dipole moments. But this condition is rarely met in real atomic systems [10]. The common approach for achieving strong quantum interference is through two orthogonal dipoles denoted by different decay channels of a Zeeman atom, which was first proposed in 2000 by G. S. Agarwal [11]. It opens the way for controlling coherence in light-matter interactions via modification of the vacuum properties. Since then manifold ways to generate quantum interference have been proposed [12, 13], including placing the atom near the vicinity of metallic surface [13, 14], in the left-handed materials [12, 15, 16], in a cavity by topological insulators [17] or embedded in a photonic crystals [5, 18–21].

Here we want to realize anisotropic vacuum via photonic crystals. In recent decades photonic crystals (PCs) have received considerable attention because of their properties of photonic band gaps (PBGs) [18, 22]. These new artificial structures have been shown to have different density of states in comparison with the case of vacuum. PCs have many practical and theoretical applications for photonics devices and quantum technologies [23]. At the same time, in [24] it has been shown that a strong modification of the interaction of an electron with its own

radiation field in a PC results in the fact that in the PC medium the electron mass changes its value. This effect is important not only from the fundamental point of view but also for applications. In opens new ways for the future possible PC's applications that are not based only on the PBG properties. Actually, the effect is another consequence of the periodic changing of the light-matter interaction in the PC medium. In this Article, we propose a principally new approach to create conditions at which quantum interference becomes possible via nonradiative transitions between atomic levels with breaking the strict selection rules. It becomes possible due to the fact that the operator of the correction to the electron mass in the PC medium has in consequence of the anisotropy nondiagonal matrix elements. At some conditions these matrix elements could be very significant that could provide strong quantum interference.

2. Anisotropy of the electron mass and the energy-level shift of hydrogen atom in the photonic crystal medium

The interaction of an electron with its own radiation field gives rise to a contribution to its physical mass m_e known as the electromagnetic mass of the electron m_{em} that cannot be calculated because of the non-renormalizable ultraviolet divergences [25]. At the same time, the modification of the electromagnetic vacuum in the PC medium gives rise to a correction to the electromagnetic mass m_{em}. This correction δm_{pc} cannot be hidden in the physical mass m_e of the electron and hence is an observable. In [24] it has been shown that the correction to the electron mass propagating with momentum $\mathbf{p}$ in a void of the PC medium is an anisotropic observable and is described by an operator (here and below, we use the natural system of units in which $\hbar = c = 1$):

$$\delta m_{pc}\left(\widehat{\mathbf{I}}_{\mathbf{p}}\right) = \frac{\alpha}{\pi^2}\left[\sum_n \int\limits_{FBZ} \frac{d^3\mathbf{k}}{\omega^2_{\mathbf{k}n}} \sum_{\mathbf{G}} \left|\widehat{\mathbf{I}}_{\mathbf{p}} \cdot \mathbf{E}_{\mathbf{k}n}(\mathbf{G})\right|^2 - \int \frac{d^3\mathbf{k}}{2\mathbf{k}^2} \sum_{\lambda=1}^{2} \left|\widehat{\mathbf{I}}_{\mathbf{p}} \cdot \boldsymbol{\varepsilon}_\lambda(\mathbf{k})\right|^2\right]. \tag{1}$$

with $\widehat{\mathbf{I}}_{\mathbf{p}} = \frac{\widehat{\mathbf{p}}}{|\widehat{\mathbf{p}}|}$ being an operator of the direction of the electron momentum, $\boldsymbol{\varepsilon}_\lambda(\mathbf{k})$ is the unit vector of the field polarization (λ) in free space, n is a band index, the value of $\mathbf{k}$ is limited by the first Brillouin zone (FBZ), $\mathbf{G}$ is the reciprocal lattice vector of the photonic crystal and α is the fine-structure constant. $\mathbf{E}_{\mathbf{k}n}(\mathbf{G})$ are the coefficients in the plane-wave expansion $\mathbf{E}_{\mathbf{k}n}(\mathbf{r}) = \sum_{\mathbf{G}} \mathbf{E}_{\mathbf{k}n}(\mathbf{G}) e^{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}}$ of the Bloch eigenfunctions $\mathbf{E}_{\mathbf{k}n}(\mathbf{r})$ [26, 27].

Let us consider one-dimensional PCs because they are most important for applications of the effect under study. For the case of one-dimensional PC with the selected Z - axis of crystal the coefficients $\mathbf{E}_{\mathbf{k}n}(\mathbf{G})$ have the polarization structure

$$\mathbf{E}_{\mathbf{k}n}(\mathbf{G}) = \sum_{\lambda=1}^{2} E_{\mathbf{k}n\lambda}(G)\boldsymbol{\varepsilon}_\lambda(\mathbf{k}_{\mathbf{G}}), \tag{2}$$

where $\boldsymbol{\varepsilon}_1(\mathbf{k}_{\mathbf{G}})$ and $\boldsymbol{\varepsilon}_2(\mathbf{k}_{\mathbf{G}})$ are unit vectors of the TE (transverse-electric) and TM (transverse-magnetic) polarization, correspondingly, $\mathbf{k}_{\mathbf{G}} = \mathbf{k} + G\mathbf{e}_z$. Thus, for 1D PC the operator of the self-energy correction equation (1) can be rewritten as [28]

$$\delta m_{pc}\left(\widehat{\mathbf{I}}_{\mathbf{p}}\right) = A + \left(\widehat{\mathbf{I}}_{\mathbf{p}} \cdot \widehat{\mathbf{I}}_{pc}\right)^2 B, \tag{3}$$

where $\widehat{\mathbf{I}}_{pc}$ is the unit vector of the 1D PC crystal axis that coincides with vector $\mathbf{e}_z$ and

$$A = \frac{\alpha}{\pi} \sum_{n,\,G} \int k_\rho dk_\rho \int\limits_{FBZ} dk_z \left(\frac{|E_{\mathbf{k}n1}(G)|^2}{\omega^2_{\mathbf{k}n1}} \frac{k^2_{Gz}}{k^2_\rho + k^2_{Gz}} + \frac{|E_{\mathbf{k}n2}(G)|^2}{\omega^2_{\mathbf{k}n2}}\right) - \frac{4\alpha}{3\pi}\int dk,$$

$$B = \frac{\alpha}{\pi}\sum_{n,\,G}\int k_\rho dk_\rho \int\limits_{FBZ} dk_z \left(\frac{|E_{\mathbf{k}n1}(G)|^2}{\omega^2_{\mathbf{k}n1}}\frac{2k_\rho^2 - k_{Gz}^2}{k_\rho^2 + k_{Gz}^2} - \frac{|E_{\mathbf{k}n2}(G)|^2}{\omega^2_{\mathbf{k}n2}}\right).$$

Here $\omega_{\mathbf{k}n1}$ and $\omega_{\mathbf{k}n2}$ are dispersion relations for TE and TM Bloch modes satisfying transcendent equation [29].

Let us now consider the modification of the self-energy correction to the atomic electron with using the example of hydrogen atom. In this case, the electron mass change gives rise to the shift of the energy level of an atomic state $|a\rangle = |n, j, l, m\rangle$ given by

$$\langle \delta m_{pc}\rangle = \int d^3p \Psi^*_{njlm}(\mathbf{p})\delta m_{pc}\left(\widehat{\mathbf{I}}_{\mathbf{p}}\right)\Psi_{njlm}(\mathbf{p}). \tag{4}$$

The transition frequency between the state energy $|a\rangle$ and $|b\rangle$ ($|n', j', l', m'\rangle$) of hydrogen atom placed into a void of the PC medium is given by [24]

$$\omega^{pc}_{ab} = \langle \delta m_{pc}\rangle_a - \langle \delta m_{pc}\rangle_b + \epsilon_a - \epsilon_b \tag{5}$$

with $\epsilon_a = -\frac{1}{2}\frac{\alpha^2 m_e}{n^2} + O(\alpha^4)$ and $\epsilon_b = -\frac{1}{2}\frac{\alpha^2 m_e}{n'^2} + O(\alpha^4)$, respectively.

Assuming that the modification of the interaction of an electron with its own radiation field in the PC medium has not a significant effect on the wave function, the electron wave function can be represented in the form $\Psi_{njlm}(\mathbf{p}) = R_{nl}(p)Y_{lm}(\Omega)$, where $R_{nl}(p)$, $Y_{lm}(\Omega)$ are the radial and spherical functions, and $\Omega = (\Theta, \Phi)$ [30]. Taking into account the normalization conditions $\int\limits_0^\infty R^2_{nl}(p)p^2dp = 1$ and $\int d\Omega Y^*_{lm}(\Omega)Y_{l',m'}(\Omega) = \delta_{l,l'}\delta_{m,m'}$, where $\int d\Omega = \int\int \sin\Theta d\Theta d\Phi$, we get

$$\langle \delta m_{pc}\rangle = \frac{\alpha}{\pi^2}\sum_{n,\mathbf{G}}\int\limits_{FBZ} d^3k \sum_\lambda \frac{|E_{\mathbf{k}n\lambda}(G)|^2}{\omega^2_{\mathbf{k}n\lambda}} I_{lm\lambda} - \frac{4\alpha}{3\pi}\int dk, \tag{6}$$

where $I_{lm\lambda} = \int d\Omega Y^*_{lm}(\Omega)\left|\widehat{\mathbf{I}}_{\mathbf{p}}\cdot\boldsymbol{\varepsilon}_\lambda(\mathbf{k_G})\right|^2 Y_{lm}(\Omega)$ and $\left|\widehat{\mathbf{I}}_{\mathbf{p}}\cdot\boldsymbol{\varepsilon}_\lambda(\mathbf{k_G})\right|^2 = f^2_{\mathbf{p}\cdot\boldsymbol{\varepsilon}_\lambda}$ with $f_{\mathbf{p}\cdot\boldsymbol{\varepsilon}_\lambda} = \sin\Theta_{\boldsymbol{\varepsilon}_\lambda}\sin\Theta\cos(\Phi - \Phi_{\boldsymbol{\varepsilon}_\lambda}) + \cos\Theta_{\boldsymbol{\varepsilon}_\lambda}\cos\Theta$ is the scalar product between the unit vector of electron's momentum with angular coordinates (Θ, Φ) and $\boldsymbol{\varepsilon}_1(\mathbf{k_G})$ and $\boldsymbol{\varepsilon}_2(\mathbf{k_G})$ are the field unit vectors with angular coordinates $(\Theta_{\boldsymbol{\varepsilon}_\lambda}, \Phi_{\boldsymbol{\varepsilon}_\lambda})$. $I_{lm\lambda}$ can be represented in the form:

$$I_{lm\lambda} = \frac{2l+1}{2}\frac{(l-m)!}{(l+m)!}\int\limits_0^\pi \sin\Theta d\Theta \left(P^m_l(\cos\Theta)\right)^2 f^2_{\mathbf{p}\cdot\boldsymbol{\varepsilon}_\lambda}, \tag{7}$$

where $P^m_l(\cos\Theta)$ are associated Legendre polynomials,

$$f_{\mathbf{p}\cdot\boldsymbol{\varepsilon}_1} = -\frac{k_z}{\sqrt{k_\rho^2 + k_z^2}}\sin\Theta\cos(\Phi - \varphi_{\mathbf{k}}) + \frac{k_\rho}{\sqrt{k_\rho^2 + k_z^2}}\cos\Theta, \tag{8}$$

$$f_{\mathbf{p}\cdot\boldsymbol{\varepsilon}_2} = -\sin\Theta\sin(\Phi - \varphi_{\mathbf{k}}). \tag{9}$$

In this study we consider the case of S-states of hydrogen atom and the alkali metals with $l = 0$, $m = 0$, then the value I_{00} is equal $\frac{1}{3}$ [30]. As an optically denser layer of the one-dimensional PC, GaAs with dispersion function (figure 1(a)) [31] was taken. We have neglected the gallium arsenide absorption in our calculation because in our effect only the electron self-interaction is considered. Using equation (6) we have calculated the energy-level structure of a hydrogen atom (figure 1(b)) considered in free space and in a void of the one-dimensional PC on the base of layers of GaAs and vacuum. The corresponding energy shifts of S-states of hydrogen atom and the alkali metals in the case of this PC medium are equal 0.146 eV. From the calculation it follows, that the levels can be strongly shifted and splitted.

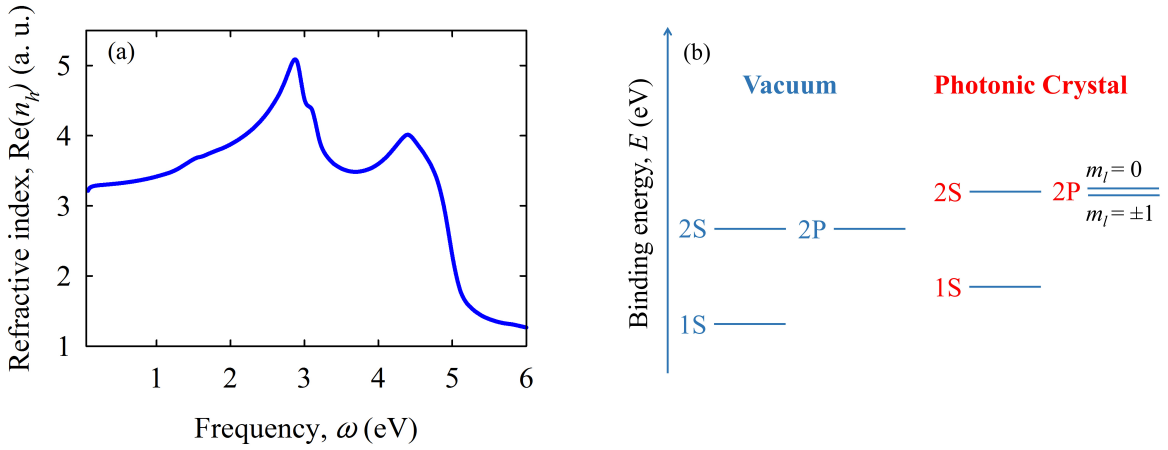

Figure 1. The spectral dependence $n_h(\omega)$ (h means "high") for gallium arsenide that consider as an optically dense layer for the one-dimensional PC (a). The relative layer thickness $d_h/T = 2/3$, as the other medium, a vacuum layer with $n_l = 1$ (l means "low") and with a relative thickness $d_l/T = 1/3$ was taken. The shifts of the energy levels of S-states of hydrogen atom placed in a void of the one-dimensional PC are equal to 0.146 eV. The corresponding energy-level structure of hydrogen atom shown in figure (b). The effect under study does not depend on the principal quantum number n but removes the degeneracy in the absolute value of m_l. The diagram is not drawn to scale, the fine structure and Lamb shifts are neglected.

3. Quantum interference between energy levels of hydrogen atoms in the photonic crystal medium

The electron mass change gives rise not only to the shifts of the atomic energies but also to the nonradiative transitions between atomic levels. This process is possible when nondiagonal matrix elements of the operator (1) and, hence, (3) are not equal zero

$$\langle \delta m_{pc} \rangle_a^b = \langle \Psi_b | \, \delta m_{pc} \left(\widehat{\mathbf{I}}_{\mathbf{p}} \right) | \Psi_a \rangle = \left\langle \Delta m_{em}^{pc} \left(\widehat{\mathbf{I}}_{\mathbf{p}} \right) \right\rangle_a^b . \tag{10}$$

The anisotropy of the PC medium can lead transitions without emitting of a photon which has selection rules different from the familiar ones. These transitions add new channels to a decay of an atomic state that leads to a quantum interference between these channels. The closer are the states between which the transitions takes place, the bigger is the probability amplitude of the quantum interference process. It is important that nondiagonal matrix elements of the self-energy operator lead to mixing of these states. In the simplest case we can consider only two of them, then we get a new pair of dressed states:

$$\begin{aligned} |+\rangle &= c_a^+ |a\rangle + c_b^+ |b\rangle \\ |-\rangle &= c_a^- |a\rangle + c_b^- |b\rangle . \end{aligned} \tag{11}$$

If we consider radiative transitions from dressed states to lower state $|c\rangle$ the probability amplitude of emitting a photon with momentum $\mathbf{k}$ and polarization $\boldsymbol{\varepsilon}_\lambda$ that escapes far enough from the PC medium takes a form

$$\langle c; \mathbf{k}, \boldsymbol{\varepsilon}_\lambda | \, \widehat{H}_I \, | \pm \rangle = c_a^\pm \langle c; \mathbf{k}, \boldsymbol{\varepsilon}_\lambda | \, \widehat{H}_I \, | a \rangle + c_b^\pm \langle c; \mathbf{k}, \boldsymbol{\varepsilon}_\lambda | \, \widehat{H}_I \, | b \rangle ,$$

where $\widehat{H}_I = -\frac{e}{m_e} \widehat{\mathbf{p}} \cdot \widehat{\mathbf{A}}$ is the interaction Hamiltonian. In dipole approximation the total

probability of radiation takes a form [32–34]

$$W_{c,\pm} = \frac{4\omega^3}{3} \left|\mathbf{d}_{c,\pm}\right|^2,$$

where

$$\mathbf{d}_{c,\pm} = c_a^{\pm}\mathbf{d}_{c,a} + c_b^{\pm}\mathbf{d}_{c,b}$$

with $\mathbf{d}_{c,a}$ and $\mathbf{d}_{c,b}$ being the matrix elements of the dipole moment of an electron. Thus, controlling parameters of a PC we can control the magnitude of the effect of the electronic mass change and, as a consequence, manipulate with photon emission rate including turning one of the dressed state into the dark state. It can be easily illustrate with example of the of atomic hydrogen. In this case the nondiagonal matrix element takes the form

$$\langle \delta m_{pc} \rangle_{njlm}^{n'j'l'm'} = \int d^3\mathbf{p} \Psi^*_{n'j'l'm'}(\mathbf{p}) \Delta m^{pc}_{em}(\widehat{\mathbf{I}}_{\mathbf{p}}) \Psi_{njlm}(\mathbf{p}). \tag{12}$$

Taking into account the form of the $\Psi_{njlm}(\mathbf{p})$ we get

$$\langle \delta m_{pc} \rangle_{njlm}^{n'j'l'm'} = \frac{\alpha}{\pi^2} \sum_{n,G} \int_{FBZ} d^3k \sum_{\lambda} \frac{\left|E_{\mathbf{k}n\lambda}(G)\right|^2}{\omega^2_{\mathbf{k}n\lambda}} I^{l'm'}_{lm\lambda} R^{n'l'}_{nl}, \tag{13}$$

where $I_{lm\lambda} = \int d\Omega Y^*_{l'm'}(\Omega) f^2_{\mathbf{p}\cdot\boldsymbol{\varepsilon}_\lambda} Y_{lm}(\Omega)$ (with functions $f_{\mathbf{p}\cdot\boldsymbol{\varepsilon}_\lambda}$ are defined by equations (8) and (9)), and $R^{n'l'}_{nl} = \int\limits_0^\infty R^*_{n'l'}(p) R_{nl}(p) p^2 dp$.

4. Conclusion

We have shown that the self-energy correction to the mass of an electron in the S-state of hydrogen atom placed into a void of the one- dimensional photonic crystal based on the arsenide gallium and vacuum layers could be observed. The effect under study could manifest itself in the energy-level shifts of atoms placed into the PC medium, and in alteration of chemical bounds, in the nonradiative transitions between atomic levels, and, hence, in quantum interference between these channels. This investigation can be used in precise tuning of the atomic energy levels, control of physicochemical processes, and the development of new type line light sources based on PCs.

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