Self-Consistent Wormhole Solutions of Semiclassical Gravity

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We present the first results of a self-consistent solution of the semiclassical Einstein field equations corresponding to a Lorentzian wormhole coupled to a quantum scalar field. The specific solution presented here represents a wormhole connecting two asymptotically spatially flat regions. In general, the diameter of the wormhole throat, in units of the Planck length, can be arbitrarily large, depending on the values of the scalar coupling ξ and the boundary values for the shape and redshift functions. In all cases we have considered, there is a fine structure in the form of Planck-scale oscillations or ripples superimposed on the solutions. [S0031-9007(97)02570-2]

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Wormholes are topological handles in spacetime linking widely separated regions of a single universe or "bridges" joining two different spacetimes. Interest in these configurations dates back at least as far as 1916 [1] with punctuated revivals of activity following both the classic work of Einstein and Rosen in 1935 [2] and the later series of works initiated by Wheeler in 1955 [3]. More recently, a fresh interest in the topic has been rekindled by the work of Morris and Thorne [4], leading to a flurry of activity branching off into diverse directions. A brief resumé of current work devoted to the physics of Minkowski-signature wormholes includes topics addressing fundamental features of traversable wormholes [4,5], explicit modeling of wormhole metrics and the corresponding classical [6] and quantum mechanical stability [7] analyses, wormholes as time machines and the problem of causality violation [8], wormholes in higher-derivative gravity [9], wormholes from the gravitationally squeezed vacuum [10], possible cosmological consequences of early universe wormholes [11,12], and wormholes as gravitational lenses [13]. A thorough and up-to-date survey of the present status of Lorentzian wormholes may be found in the excellent monograph by Visser [14].

There are plausible physical arguments suggesting that Lorentzian wormholes should exist at least at scales of order the Planck length. Most of what is known about them is based on detailed analyses of models, and within the literature devoted to the subject, the existence of wormholes is taken as a working hypothesis. Metrics describing wormholes with desirable traits are written down by fiat, and the properties of the corresponding hypothetical stress-energy tensors needed to support the wormhole spacetime are then worked out and analyzed. In an example of an analysis of this sort, Ford and Roman [15] have derived approximate constraints on the magnitude and duration of the negative energy densities which must be observed by a timelike geodesic observer in static spherically symmetric wormhole spacetimes. More recently, Taylor, Hiscock, and Anderson have argued that stress tensors for massive minimally and/or conformally coupled scalars fail to meet the requirements for maintaining five particular types of static spherically symmetric wormholes, but have not solved the back-reaction problem [16]. In particular, no one up to now has succeeded in writing down a bona fide wormhole *solution* of either the classical or semiclassical Einstein field equations. The reason for this state of affairs is easy to understand. In the first case, it is well known that any stress energy that might give rise to a wormhole must violate one or more of the cherished energy conditions of classical general relativity [4,5]. Hence wormholes cannot arise as solutions of classical relativity and matter. If they exist, they must belong to the realm of semiclassical or perhaps quantum gravity. In the realm of semiclassical gravity, one sets the Einstein tensor equal to the expectation value of the stress-energy tensor operator of the quantized fields present,

$$G_{\mu\nu} = 8\pi\langle T_{\mu\nu}\rangle \,. \tag{1}$$

A primary technical difficulty in semiclassical gravity is that $\langle T_{\mu\nu}\rangle$ depends strongly on the metric and is generally difficult to calculate. Until recently, all calculations of $\langle T_{\mu\nu}\rangle$ have been performed on fixed classical backgrounds. The fixed background in turn, as its name implies, must be a solution of the classical Einstein equation. As there are no classical wormhole backgrounds, no corresponding semiclassical back-reaction problem can be set up meaningfully.

In this Letter we present and summarize the results of the first *self-consistent* wormhole solutions of semiclassical Einstein gravity. Prior to this, a self-consistent wormhole solution had been obtained using a phenomenological stress tensor not derived from quantum field theory [17]. The results of the present calculation may be taken as numerical evidence for the existence of Lorentzian wormholes. For the source term in (1) we employ the stress-energy tensor of Anderson, Hiscock, and Samuel, which is calculated for a quantized scalar field in an arbitrary static and spherically symmetric spacetime [18]. This means that in the field equation (1), both the Einstein tensor as

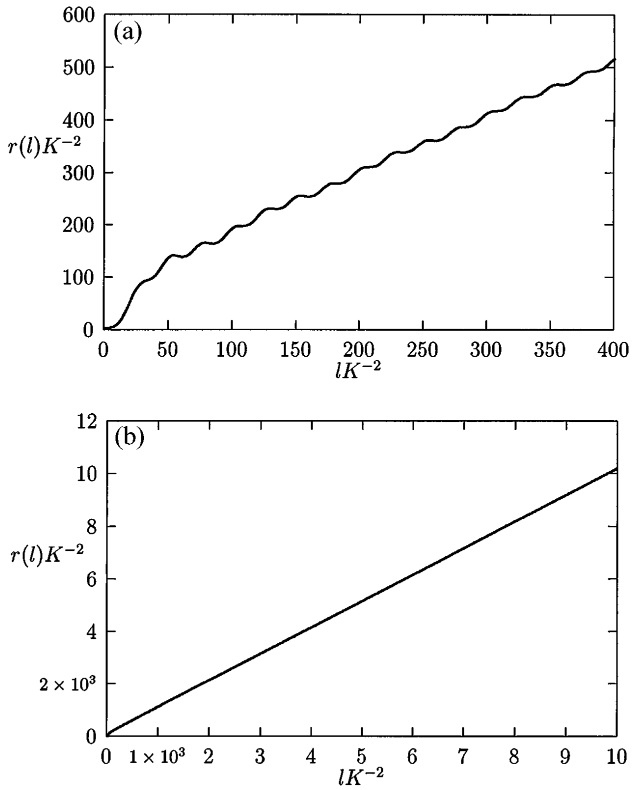

FIG. 2. (a) The wormhole shape function on small scales. (b) The wormhole shape function on large scales.

$R(l)$ are strictly *monotone increasing* and $\phi(l)$ and $\rho(l)$ are bounded *oscillating* functions.

The relative magnitudes of these components and their derivatives may be estimated straightforwardly and then used to expand consistently the coupled Einstein equations. We find that the oscillating modulation is composed of two modes with frequencies $\omega_1^2 = 1/16K^2$ and $\omega_2^2 = 1/16K^2(4 + 3 \ln F)$, respectively, while in the limit of large l, $R(l) \approx l$ and $F(l) \approx (a \ln l - b)^2$ for $a = 5.3$ and $b = 25.5$.

From these combined numeric analytic calculations we see that for the chosen set of boundary conditions, $R(l) \to l$, thus our self-consistent wormhole connects two spatial regions which are asymptotically flat, modulo the Planck-scale wiggles. The redshift function, however, does not approach a constant value as $l \to \infty$, so the metric as a whole is not asymptotically flat. We have found additional self-consistent solutions of (1) by taking different values for the scalar coupling and the boundary data. In this way, we have found local solutions which correspond to large throat $[r(0) \approx 200 - 300 l_P]$ wormholes with horizons located far from the throat and wormholes connecting two bounded spatial regions. A full account of these calculations will appear in a separate publication.

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[20] We have also set the scalar mass and temperature to zero: $m = \kappa = 0$. In this case, the arbitrary renormalization scale parameter μ appearing in $(T_\mu^\nu)_{\log}$ can be absorbed into the definition of f.
[21] This can be checked, for example, by writing the set of three field equations as an equivalent set of (eight) coupled first-order equations plus one algebraic equation. The radial equation reduces to the algebraic transcendental equation in f and r, i.e., it is a constraint.
[22] This follows from evaluating the identity $r'(l) \times [df(r)/dr] = f'(l)$ at $l = 0$.