# About the method of compensation of harmful disturbances in a three-step micromechanical gyroscope


Bogolyubov Vladimir
Kazan National Research Technical University named by A.N. Tupolev (KNRTU-KAI)
Kazan, Russia
bvm200@yandex.ru

Bakhtieva Lyalya
Institute of Computational Mathematics and Information Technology
Kazan Federal University (KFU)
lbakhtie@yandex.ru



***Abstract* — The accuracy of measurements of modern small-sized gyroscopic devices substantially depends on various disturbing factors caused by both imperfect manufacturing techniques and errors in measurement methods and operating conditions. Known methods for compensating for such effects are often technically difficult to implement in microelectromechanical systems. The method proposed by the authors is based on the use of nutation oscillations observed in three-stage micromechanical gyroscopes and the implementation of the disturbance control principle. A modified kinematic diagram of the device is presented, including a valve actuator that revolves the rotor. A mathematical model of the gyroscope motion in a rotating coordinate system is constructed, and numerical calculations are performed. The relations between the amplitudes of nutational vibrations caused by harmful disturbing moments and the angular motion of the base are analyzed. A block diagram of the allocation of gyro information signals is proposed.**



***Keywords* — *gyroscope; fluctuations; nutation***


## I. Introduction

Hybrid micromechanical gyroscopes are well mastered by modern instrument-making enterprises. Despite the fact that such devices are more labor intensive and expensive in comparison with solid-state microelectromechanical systems (MEMS) of mass production and that their dimensions are slightly larger than the dimensions of solid-state MEMS sensors (from 10 to 20 mm), they are quite suitable for use in navigation and orienteering. In turn, hybrid devices are significantly cheaper and smaller than classical navigational electromechanical gyroscopes and do not require a high level of technological support typical of solid-state MEMS technology for their manufacture. In hybrid devices, small time can be spent on the development, manufacture of prototypes and their development by conventional instrument-making enterprises with an existing technological base [1]. It should be noted that a number of foreign companies, for example, Safran, which produces a hybrid solid-state wave gyroscope, are currently moving along the path to creating hybrid MEMS devices [2]. The reason for this is the possibility of obtaining higher accuracy characteristics than in solid-state MEMS devices.

Known methods for compensating for harmful disturbances of gyroscopic devices can be divided into three main groups [3]. The first group includes methods based on structural and technological improvements that reduce disturbing moments acting on the gyroscope. The second are methods that require the availability of external information, when used in various closed and open circuits for controlling gyroscopes, it is possible to reduce their departures. The third group of methods allows to increase the accuracy of devices over a long time of their operation, without the use of external information and without increasing requirements for design and manufacturing technology.

In the domestic and foreign literature, the third group of methods is usually called as methods of autonomous compensation for the departure of gyroscopes (without using external information). From a physical point of view, the methods of auto-compensation that have gained practical application are talking about creating artificial movements of the gyroscope or elements of its suspension (vibration of the suspension supports, reversal of the kinetic moment of the gyroscope, etc.), due to which it is possible to reduce the influence of harmful effects and thereby reduce instrumental errors . However, these methods of auto compensation in some cases cannot be generalized to the class of gyroscopic devices using MEMS technology. This is due to the fact that the basis of such devices is the elastic suspension of the rotor, the mechanical "revitalization" of which is technically difficult to implement and, in addition, changing the parameters of the meter leads to a detuning of the gyroscope, which sharply reduces its sensitivity.

## II. FORMULATION OF THE PROBLEM

This article proposes a method of compensating for harmful disturbances in a three-degree hybrid micromechanical gyroscope (TDMG). The distinguishing feature of this device is the fact that the increment of the amplitude of the oscillations of the rotor caused by the external disturbing moment $M$ does not tend to zero, but remains unchanged, i.e. the stability of the gyroscope with respect to $M$ is not asymptotic.

Since the gyro rotor oscillates with a new (perturbed) amplitude value, nutation vibrations characteristic of three-stage gyroscopes are observed in the device. It will be shown below that by the values of the amplitude and phase of these oscillations one can judge the magnitude and direction of the external disturbing moment that caused them [4].

Since TDMG is a high-quality, resonance-tuned oscillatory system, the nutational oscillations of the rotor of the device, in contrast to the classical three-degree gyroscope, decay slowly. Therefore, they turn out to be observable and can be used when controlling the device as an information signal about external disturbing moments.

## III. CONSTRUCTION OF MATHEMATICAL MODEL

We describe the TDMG motion using the Lagrange equation, where the rotation angles around torsion bars are selected as generalized coordinates $\Theta_x$, $\Theta_y$, and as generalized forces, damping moments and torsion elastic moments [5].

Fig. 1 shows the kinematic diagram of a hybrid TDMG, a feature of which, unlike the known analogues [1], is the presence of a valve electric drive that revolves the rotor.

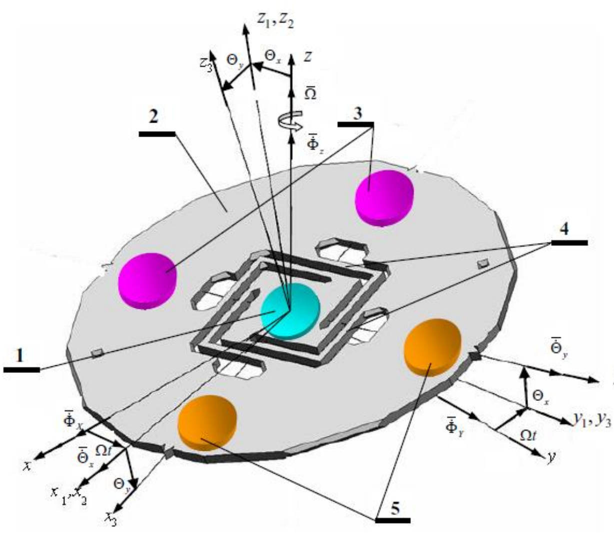


Fig. 1. The kinematic diagram of the hybrid TDMG (1 – electric drive shaft, 2 – rotor-sensitive element, 3 – torque sensors, 4 – spiral-shaped elastic suspension, 5 – angle sensors).

The mathematical model of a TDMG with a symmetric rotor, obtained in a rotating coordinate system and written in a complex form, after performing the factorization procedure and subsequent linearization using the Jacobi matrix has the form

$$J\,\Theta''_{xy} + (\mu + j\,(2J - C)\,\Omega)\,\Theta'_{xy} + (k + (C - J)\,\Omega^2)\,\Theta_{xy} = \\ = (Q_1(t) + j\,M\,V'_{XY} + M_{XY})\,e^{-j\Omega t} - M'V'_Z), \quad (1)$$

$$Q_1(t) = -J\,\Phi''_{XY} + j\,C\,\Omega\,\Phi'_{XY}, \quad J = (A+B)/2,$$
$$M = m\,(\Delta Z_1 + \Delta Z_2)/2, \quad M' = m\,\Delta y,$$

$A$, $B$, $C$ – respectively the equatorial and polar moments of inertia of the rotor, $k$ – the rotor suspension elastic coefficient, $\Omega$ – the angular speed of rotation of the shaft, $\mu$ – the suspension structural damping coefficient, $m$ – the rotor mass, $\Delta y$ – the radial unbalance value, $\Delta Z_1 = \Delta Z_2 = \Delta Z$ – the axial unbalance value (symmetric non-intersection of torsion axes relative to the rotor center of mass), $\Theta_{xy} = \Theta_x + j\,\Theta_y$ – the rotor angular deviation around the axes of the coordinate system associated with the shaft, $V'_{XY} = V'_X + j\,V'_Y$ – the linear overload acting in the plane of the gyro sensitivity, $V'_Z$ is – the linear acceleration of the base, acting along the axis of rotation of the shaft, $\Phi_{XY} = \Phi_X + j\,\Phi_Y$ – the angular deviation of the base of the device around the corresponding axes, $M_{XY} = M_X + j\,M_Y$ – the external disturbing moment.

From the equation (1), it follows that the TDMG, responding to the angular movement of the base and external "leading" moments acting in the plane of its sensitivity modulates them. Thus, the rotor movement is amplitude-modulated oscillations, the envelope of which contains information about both the movement of the base and the effect of harmful moments, leading to a change in the amplitude of these oscillations, i.e. to measurement errors.

## IV. THE DISCUSSION OF THE RESULTS

After presenting (1) in the operator form and introducing the detuning of the device $\Delta\Omega$ due to the presence of residual rigidity, we solve (1) with respect to $\Theta_{xy}$

$$\Theta_{xy}(p) = G_{xy}(p) / \tau(p), \quad (2)$$

$$\tau(p) = J\,(p + \mu/2j - j\,(\Omega - \Delta\Omega))\,(p + \mu/2j + j\,(\omega + \Delta\Omega));$$

$$G_{xy}(p) = Q_1(p + j\Omega) + M_{XY}(p + j\Omega) + j\,M\,V'_{XY}(p + j\Omega) - M'\,V'_Z(p)$$

$$Q_1(p + j\,\Omega) = -J\,(p + j\,\Omega\,(1 - C/J))\,\Phi'_{XY}(p + j\,\Omega);$$

$$\omega = \Omega\,(3 - C/J); \quad \Delta\Omega = (k - k_c)/(C\Omega) = \Delta k / H;$$

$k_c$ – the calculated torsion stiffness.

To ensure the resonance tuning of the TDMG, it is necessary that one of its natural frequencies be equal to the frequency of the forced oscillations, i.e. angular shaft rotation velocity $\Omega$. This condition corresponds to the relation

$$\Omega^2 = k/(2(2J - C)), \quad (3)$$

defined as nutational dynamic tuning of TDMG.

It is known that in engineering practice it is customary to describe the dynamic characteristics of sensitive elements by transfer functions, which are the operator of converting a signal at the input of an instrument into an output signal. For TDMG the input effects can be considered the vector of the angular velocity of the base in the sensitivity plane of the device $\Phi'_{XY}$ and the vector of its linear displacement with acceleration $V'_{XY}$ as well as in the general case, the vector of the external disturbing moment $M_{XY}$. And the output – the signal vector of the information acquisition and conversion system $U_{XY}$.

Consider the dynamic characteristics of TDMG for various input influences. We assume that the information on the rotor motion is collected in a coordinate system rotating with the shaft and that the mathematical model of the TMMG is linear. Then, on the basis of the principle of superposition, the transfer functions of the device with respect to the input actions will have the form:

— in relation to the angular velocity of the base

$$W^{(\Phi')}(p) = \Theta_{xy}(p)/(\Phi'^{*}_{XY}(p - j\,\Omega)) = -k'_n (p + j\,\Omega)\ \mathrm{x}\ (1 - C/A))/(p + d - j(\Omega - \Delta\Omega))(p + d + j(\omega + \Delta\Omega)), \quad (4)$$

— in relation to the linear overload

$$W^{(V')}(p) = \Theta_{xy}(p)/(V'^{*}_{XY}(p - j\,\Omega)) = j\,k'_n M/J(p + d - j(\Omega - \Delta\Omega))(p + d + j(\omega + \Delta\Omega)), \quad (5)$$

— in relation to the disturbing moment

$$W^{(M)}(p) = \Theta_{xy}(p)/(M^{*}_{XY}(p - j\,\Omega)) = k'_n/J(p + d - j(\Omega - \Delta\Omega))(p + d + j(\omega + \Delta\Omega)), \quad (6)$$

where $k'_n$ – the transmission coefficient of the system of information retrieval and conversion, $d = \mu/(2j)$ – the gyro damping coefficient, * – the complex conjugation mark.

Expressions (4) – (6) are written for the envelope of oscillations of the gyroscope rotor, which contains information about both the projections of the angular velocities of the instrument body and the projections of disturbing moments on the corresponding sensitivity axes of the TDMG. To separate the information into appropriate components, it is necessary to demodulate the gyro rotor oscillations. If we take into account that in the output signals of demodulators, components with frequencies $2\,\Omega$ and $(2\,\Omega + \omega_n)$, then the expressions for the output signals of the demodulators can be written in the form of the following relations

$$U_{XY}^{(\Phi')}(p) = k_{dm} W^{(\Phi')}(p + j\,\Omega)\,\Phi'_{XY}{}^{*}(p) = -k_n (p + j\,\omega_n)\,\Phi'_{XY}{}^{*}(p)/(p + d + j\,\Delta\Omega)(p + d + j(\omega_n + \Delta\Omega)), \quad (7)$$

$$U_{XY}^{(V')}(p) = k_{dm} W^{(V')}(p + j\,\Omega)\,V'_{XY}{}^{*}(p) = j\,k_n M\,V'_{XY}{}^{*}(p)/J(p + d + j\,\Delta\Omega)(p + d + j(\omega_n + \Delta\Omega)), \quad (8)$$

$$U_{XY}^{(M)}(p) = k_{dm} W^{(M)}(p + j\,\Omega)\,M_{XY}{}^{*}(p) = k_n M_{XY}{}^{*}(p)/J(p + d + j\,\Delta\Omega)(p + d + j(\omega_n + \Delta\Omega)), \quad (9)$$

where $k_n = k'_n k_{dm}$, $k_{dm}$ – the transmission coefficient of demodulators, $\omega_n = \Omega(2 - C/J)$ – the gyro nutation frequency.

We define the output signals of the device under conditions of movement of the base with a constant angular velocity in accordance with expression (7). Assuming that $\Phi'_{XY}{}^{*} = \Phi_0'_{XY}{}^{*}/p$ and the detuning magnitude $\Delta\Omega = 0$, we get

$$U_X^{(\Phi')}(t) = k_n((1 - \exp(-d\,t))\,\Phi_0'_X/d + d\exp(-d\,t)(\Phi_0'_X\cos(\omega_n t) + \Phi_0'_Y\sin(\omega_n t)/\omega^2_n), \quad (10)$$

$$U_Y^{(\Phi')}(t) = -k_n((1 - \exp(-d\,t))\,\Phi_0'_Y/d + d\exp(-d\,t)(\Phi_0'_Y\cos(\omega_n t) - \Phi_0'_X\sin(\omega_n t)/\omega^2_n). \quad (11)$$

It follows from the above expressions that the tuned gyro performs precessional and vibrational (nutational) motion with a frequency $\omega_n$. The movement of the device in this case is due only to the effects of the angular velocity vector $\Phi'_{XY}{}^{*}$. However, under real conditions, external perturbations act on the sensitive element, leading to measurement errors. It is known that TDMG errors can be methodical, instrumental, and operational. The first ones are related to the features of their work on an arbitrarily moving base subject to angular and linear vibrations and overloads, the second to the effect of disturbing moments on the sensitive element due to inaccurate manufacturing and tuning, and the third to changes in the values of their main parameters during operation.

The following are received in accordance with (7) – (9) the values of the amplitude of nutational vibrations of TDMG for a moment in time $t = 0$ on one of the channels

$$A^{\Phi'} = k_n d\,\Phi_0'_X/\omega^2_n, \quad A^{V'} = k_n M\,V_0'_X/(J\,\omega^2_n),$$

$$A^{2\Omega} = k_n d\,\Omega\,\Phi_{0X}/\omega^2_n,\ A^M = k_n M_0\mathrm{x}/(J\,\omega^2_n), \quad (12)$$

$$A^{M_{2\Omega}} = \frac{k_n \cos(\phi)\,M^o}{J\,\omega_n^2},\ A^{V'_\Omega} = \frac{k_n M'\,a_z^o\,\Omega^2\sin(\alpha)}{J\,\omega_n^2},$$

where $\phi$ – the phase angle in space, $a_z^0$ – the linear displacement of the gyroscope shaft along its rotation axis, $\alpha$ – the phase angle between rotor vibrations and linear shaft movement with frequency $\Omega$.

Formulas (12) reflect the following effects:

— the movement of the base with a constant angular velocity $\Phi_0'_X$;

— the radial linear overload in the direction perpendicular to the shaft axis in the presence of axial imbalance and non-intersection of the torsion axes;

— the angular vibration of the base with a frequency 2Ω (the synchronous interference);

— the constant momentum (associated, for example, with the tension of the angle sensor);

— the axial and radial vibration with a frequency 2Ω in the presence of axial and radial unbalance;

— the axial vibration with shaft speed Ω in the presence of radial unbalance.

Assessment of the relationship between the amplitude of nutational vibrations caused by harmful disturbing moments and the amplitude of nutational vibrations due to the angular movement of the base showed their significant discrepancy (by several orders of magnitude)

$$a_1 = \left(\frac{A^{(2\Omega)}}{A^{\Phi}}\right) = \left(\frac{\Omega\, \Phi_x^{\,o}}{(\Phi_x^{\,o})'_{min}}\right), \quad a_2 = \left(\frac{A^{M}}{A^{\Phi}}\right) = \left(\frac{M_x^{\,o}}{J\,d\,(\Phi_x^{\,o})'_{min}}\right),$$

$$a_3 = \left(\frac{A^{V'}}{A^{\Phi}}\right) = \left(\frac{\mathrm{M}(V_x^{\,o})'}{J\,d\,(\Phi_x^{\,o})'_{min}}\right), \quad a_4 = A^{M_{2\Omega}} = \left(\frac{M^{o}}{2\,J\,d\,(\Phi_x^{\,o})'_{min}}\right),$$

$$a_5 = \left(\frac{A^{V'_{\Omega}}}{A^{\Phi}}\right) = \left(\frac{\Omega^2\, a_z^{\,o}\, M'}{J\,d\,(\Phi_x^{\,o})'_{min}}\right). \tag{13}$$

It should be noted that a comparative assessment was carried out for a precision meter having a sufficiently low level of the threshold value of the measured angular velocity.

## V. IMPLEMENTATION OF THE METHOD

For the realization of the invariance conditions [6] in a dynamic system, it is necessary that there exist at least two channels of the action of external perturbations on the sensitive element. In a tuned TDMG with a symmetrical rotor, there are two information channels: in the first precession channel there is information about the measured angular movement of the base and about harmful disturbances generated by methodological and instrumental errors, in the second - nutational channel, there is information only about harmful disturbances leading to the departure of the gyroscope (fig. 2).

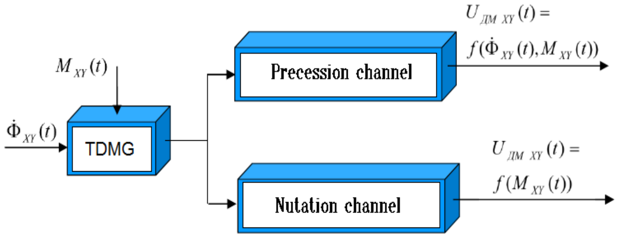


Fig. 2. Block diagram of the allocation of information signals TDMG.

The creation of the compensation moment can be carried out either using moment sensors or by changing the angular velocity of the gyro rotor relative to its resonance value. The limiting factor for the second case is the fact that at significant values of harmful disturbances the detuning of the device will lead to a decrease in the gyro time constant *T*g (and, consequently, to a decrease in its sensitivity) and to a decrease in the amplitude of nutation oscillations and, correspondingly, to a reduction in their time observation. However, in this case, there is no need to supply control signals of significant power to the rotating part of the device.

## VI. CONCLUSION

The analysis of the TDMG reaction to various types of external disturbances showed, that the selection of observed nutational vibrations with information ability (the presence of a second information processing channel) allows us to implement the invariance condition in the system and, therefore, to compensate for harmful effects on the device by implementing the principle of control perturbation.

The proposed method for compensating for harmful effects in a hybrid TMMG can significantly improve its accuracy parameters and ensure the effective use of the device in solving problems of inertial navigation and orientation of moving objects.

## REFERENCES

[1] S. Konovalov, Yu. Ponomarev, D. Maiorov, V. Podchezercev, A. Sidorov, Hybrid microelectromechanical gyroscopes and acceleration gages, Science and education, Electronic scientific and technical periodical, 2011, 77-30569/219257.

[2] F. Delhaye, VTG Safran. Breakthrough Technology, 5th IEEE International Symposium on Inertial Sensors and Systems, 2018, in press.

[3] S. M. Zel'dovich, M.I. Maltinsky, I.M. Okon, Y.G. Ostromukhov, Auto compensation of instrumental errors of gyrosystems, L .: Shipbuilding, 1976, 255 p.

[4] V.A. Pavlov, I.V. Pavlov, On the Use of Additional Information to Improve the Accuracy of Gyroscopic Instruments, Leningrad University Press, 1974, 207 p.

[5] L. Bakhtieva, V. Bogolyubov, Parametrically Excited Microelectro-mechanical System in Navigation Problems, Proceedings of IEEE East-West Design & Test Symposium (EWDTS'2018), Kazan, 2018, pp. 897-900.

[6] B.N. Petrov, On the feasibility of invariance conditions, The theory of invariance and its application in automatic devices: Proceedings of the meeting, Kiev: AN USSR, 1958, p. 59-80.