Comparison of kinematic and dynamic leg trajectory optimization techniques for biped robot locomotion

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Comparison of kinematic and dynamic leg trajectory optimization techniques for biped robot locomotion

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Abstract. The paper presents comparison analysis of two approaches in defining leg trajectories for biped locomotion. The first one operates only with kinematic limitations of leg joints and finds the maximum possible locomotion speed for given limits. The second approach defines leg trajectories from the dynamic stability point of view and utilizes ZMP criteria. We show that two methods give different trajectories and demonstrate that trajectories based on pure dynamic optimization cannot be realized due to joint limits. Kinematic optimization provides unstable solution which can be balanced by upper body movement.

1. Introduction

In the past three decades, an increasing interest in developing humanoid robots is observed [1-3]. The main objective of research community is to create a robot which will be able to operate in different environments, such as homes, plants, offices, and to perform operations originally designed for humans. Among various types of humanoid locomotion bipedal walking is the most natural and interesting, since bipedal robots have more potential to move in rugged terrains or complex environments, where wheeled or tracked robots cannot move. On the other hand, bipedal robots are less stable and can easily fall down. Therefore, the research community considers bipedal walking stability, and different algorithms were proposed to control robot movement and to prevent it from falling down [4-5].

Besides the problem of stable robot locomotion, leg trajectory optimization plays an important role. This problem was in the focus of many works ([6] - [9]). In particular, in paper [10] the authors propose a method to generate swing foot trajectory by kinematic optimization considering joint speed. However, a supporting foot was not considered, since its motion is often defined by the balance controller. Similar analysis was performed in [11], where a dynamic programming approach was used to calculate an optimal swing leg trajectory. In these works, the authors considered either kinematic or dynamic behaviour and took into account only swing leg optimization. In our work, we analyse both supporting and swing leg trajectories and show their advantages and disadvantages.

To address this problem, the remainder of the paper is organized as follows. Section 2 formulates the problem statement, section 3 gives the kinematic optimization method, section 4 presents the dynamic optimization approach and section 5 concludes the paper.

2. Problem statement

Majority of the algorithms for stable walking of a bipedal robot usually focus on balance control and do not take into account joint constraints. Therefore, calculated trajectories may be not reachable in practice because of velocity/acceleration/jerk limits in joints and lead to wrong foot positioning. On the other hand, there are works which study trajectory optimality with joints limits, but without stability analysis of such trajectories.

Anthropomorphic robot AR-601M (Figure 1), which is in the focus of our study, has 41 DoF in total, although during walking only 12 joints are used (6 in each pedipulator). For simplicity, we take into account that the robot motion lies in a sagittal plane. In this case, the problem of optimal trajectory is reduced to the 4 DoF system. So, a swing leg and a supporting leg can be presented as simple two-link manipulators with hip and knee joints. The link lengths correspond to AR601-M robot leg parameters, which are equal to 280 mm each. The trajectory of the swing leg and principle model parameters are shown in Figure 2. We consider one step motion with a symmetrical position at the start and end. The hip is located at the middle between two feet, i.e. $x_0=L_0/2$. L_0 in our calculations is equal to 0.3 m, the hip height at start y_0 is equal to 0.5. These assumptions are formulated from analysing human's natural walk; they are widely applied in experimental works for biped walking [12].

The problem, which we analyse in our research can be formulated as following: compare kinematic and dynamic parameters of the motion in two cases. The first is the motion that is optimal from the kinematic point of view and fully utilizes joint kinematic limits. The second is the motion that is optimal from the stability point of view. Let us consider them sequentially.

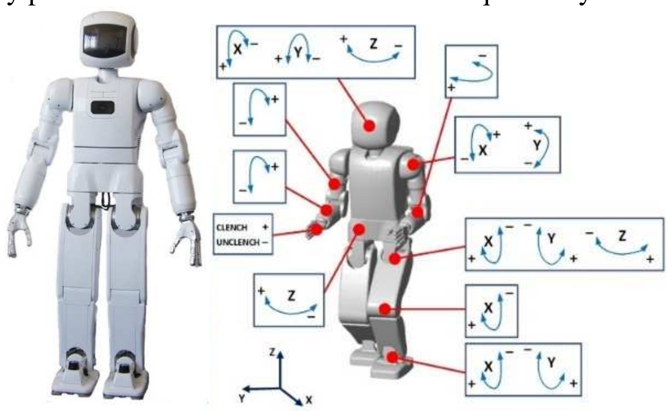

Figure 1. Biped robot AR601-M and its kinematic structure.

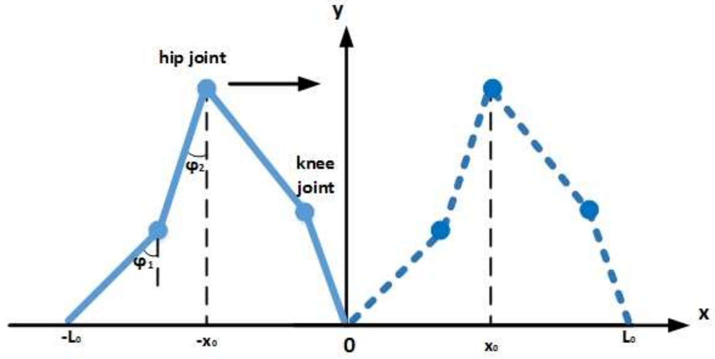

Figure 2. Step motion of the swing and supporting legs in the sagittal plane.

3. Kinematic optimization

In the kinematic optimization approach, we find leg trajectories of the robot, which maximally use capabilities of the joints in terms of angular velocity and angular acceleration. In the simulation study,

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the acceleration and velocity limits were assigned to 1 rad/s2 and 1 rad/s for each joint. These values correspond to characteristics of the motors, which are used in the AR601-M robot. In addition, we calculated trajectories with maximum acceleration of 10 rad/s2.

Firstly, we calculated initial angles in joints using an inverse kinematics problem for a two link manipulator. Hip (q1) and knee (q2) angles are found according to following formulas:

$$q_2 = arccos\left(\frac{(x_{foot} - x_{hip})^2 + (y_{foot} - y_{hip})^2 - l_1^2 - l_2^2}{2l_1 \mathrm{l}_2}\right) \quad (1)$$

$$q_1 = \mathrm{atan2}\left(x_{foot}\text{-}\mathrm{x}_{hip}, y_{foot}\text{-}\mathrm{y}_{hip}\right)\text{-}\mathrm{atan2}\left(\mathrm{l}_2\sin(\mathrm{q}_2), \mathrm{l}_1+\mathrm{l}_2\cos(\mathrm{q}_2)\right), \quad (2)$$

where l_1 and l_2 are upper and lower link lengths accordingly, x_{foot}, y_{foot}, x_{hip}, y_{hip} are foot and hip coordinates. For given initial and final parameters, hip and knee joint angles in the starting position are -0.08 and 0.74 rad, correspondingly, and at the end of the trajectory, they are -0.66 and 0.74 rad. As we see, the hip angle decreases, when knee angles are the same at the final position. Supposing that initial angular velocities are zero and that joint velocity profiles are symmetrically trapezoidal or triangular (the motion with maximum acceleration or speed), we can find the time needed for hip joint angle change. After that, for the given time we define a velocity profile for the knee joint, again taking into consideration zero initial and final velocities, a symmetrical trapezoidal or triangular function with a total zero integral (angle change). Joint angle functions are found from velocity functions by integration with known initial values. Figures 3 and 4 show velocity and angle functions for acceleration limits 1 rad/s2 and 10 rad/s2 correspondingly. The motion time is 1.53 sec for lower and 0.68 sec for higher acceleration. We see that in the first case, the maximum velocities are not reached, when in the second case, profiles are trapezoidal. Since the motion is symmetrical, which means that final angles of the swing leg are initial angles of the supporting foot, we define velocity profiles of the supporting leg as profiles of the swing leg with an opposite sign.

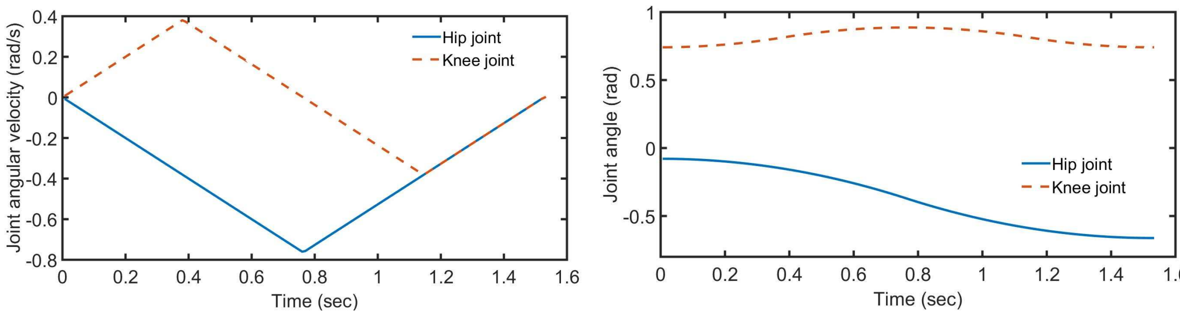

Figure 3. Angular velocity profiles and joint angle profiles of the swing leg. A triangular case.

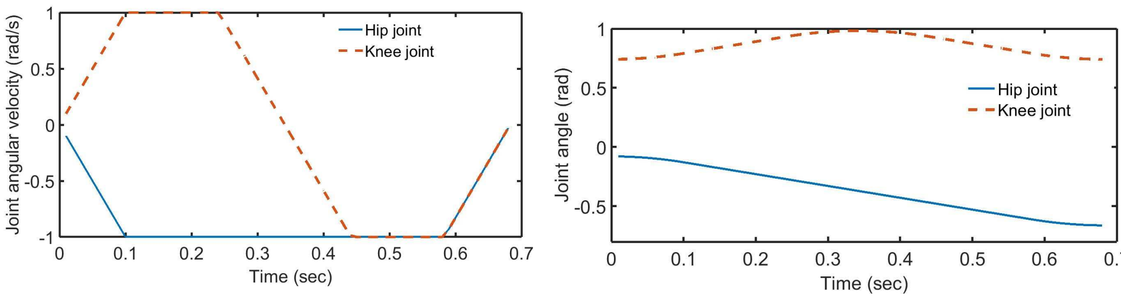

Figure 4. Angular velocity profiles and joint angle profiles of the swing leg. A trapezoidal case.

After joint angle functions are found, we use the forward kinematics solution to find the foot trajectory in Cartesian space. Figure 5 shows foot trajectories for two acceleration limits. We see that in both cases, the foot is always above the ground, which is a necessary requirement.

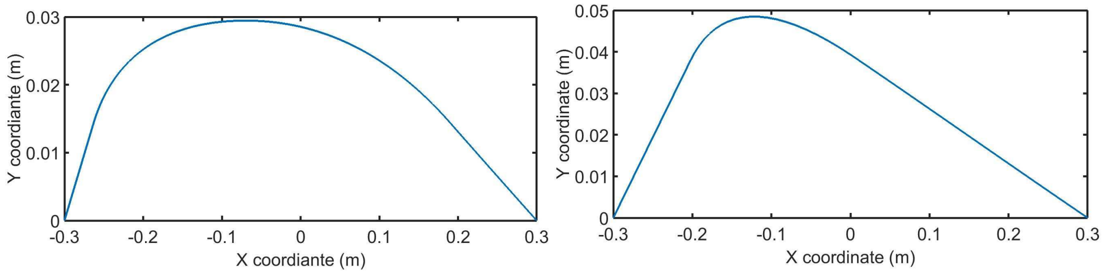

Figure 5. The trajectory of the swing foot in Cartesian space for the lower (left) and higher (right) acceleration limit.

Finally, we estimate the dynamic stabilty of such trajectories using a ZMP approach. An inverted pendulum model of the robot, with point mass at the hip, gives us the following equation for the ZMP x coordinate (see [13]):

$$x_{zmp} = \frac{x_{hip}(g+\ddot{y}_{hip}) - \ddot{x}_{hip}\, y_{hip}}{g+\ddot{y}_{hip}} \tag{3}$$

where g – gravity acceleration, $\ddot{x}_{hip}, \ddot{x}_{hip}$ - hip acceleration. Substitution of hip trajectory functions into the above-mentioned equation gives results, shown in Figure 6. Maximum stability is reached when $x_{zmp}=0$, that is, when a ZMP coordinate is at the supporting foot. We see that ZMP significantly differs from the zero value at the beginning and end of the motion, when the hip is maximally away from the supporting foot. It is more important that hip acceleration and decelleration at the start and end only increase a ZMP difference. These results mean that such trajectories, when applied without additional stability correction, are unstable and are not suitable for robot motion.

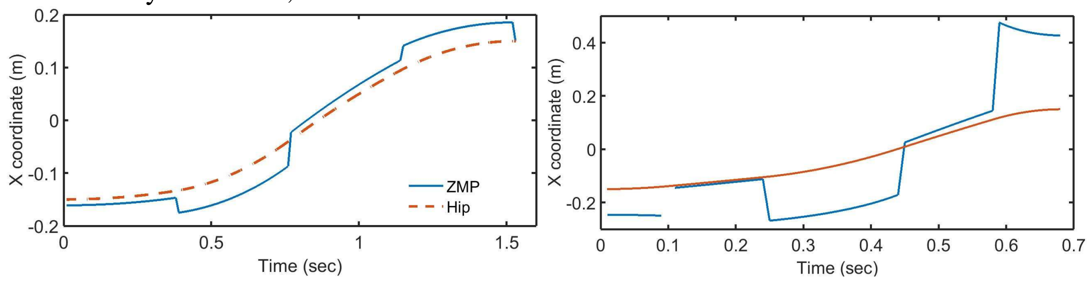

Figure 6. ZMP and a hip x coordinate for the lower (left) and higher (right) acceleration limit.

4. Dynamic optimization

In the dynamic optimization approach, we, first, calculate the optimal trajectory of the hip in Cartesian space. To do that, we assume that the hip is moving at constant height y_0. If so, a ZMP equation for the x coordinate and its solution can be written as follows:

$$\ddot{x} - \frac{g}{y_{hip}} x_{hip} = 0, \tag{4}$$

and its solution is

$$x(t) = C_1 e^{-wt} + C_2 e^{wt}, \quad w = \sqrt{\frac{g}{y_{hip}}}, \tag{5}$$

where coefficients C_1 and C_2 are found from coordinate boundary conditions. The trajectory time is equal to 1.5 seconds, which corresponds to the maximal speed with kinematic limits of 1 rad/s and 1 rad/s2 in the previous approach. Figure 7 shows the solution of the above-mentioned equation with $x(0) = -0.15$ m and $x(1.5) = 0.15$ m. A full definition of legs motion needs a swing foot trajectory to be defined. We model the swing foot motion with cycloid according to equations below:

$$\begin{aligned} x(t) &= -L_0 \cos(\pi t / t_0) \\ y(t) &= 0.5h(1-\cos(2\pi t / t_0)) \end{aligned}, \tag{6}$$

where t_0 – step time, h – maximum height at the middle of the trajectory, L_0 – step length.

After the hip and swing foot motion are defined in Cartesian space, we apply inverse kinematics equations to find trajectories in a joint space. Differentiation of the joint angles for swing and supporting legs gives angular velocities and accelerations, which are shown in Figures 8 and 9. We see that velocity values exceed limit (1 rad/s) only at the start and the end of the motion, when accelerations are significantly above the limit during the whole motion. It means that such trajectories are not possible due to limitations in motor joints.

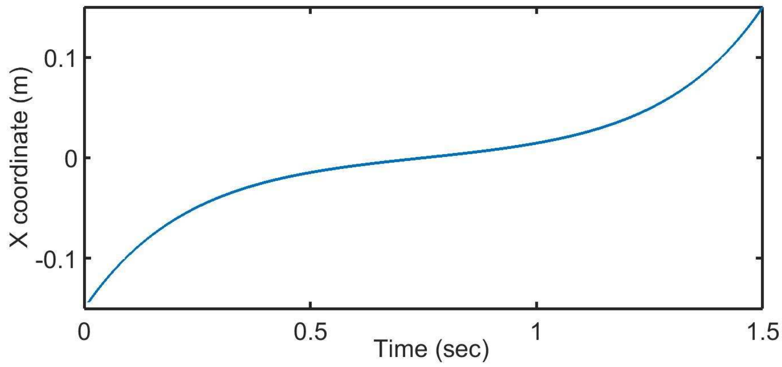

Figure 7. Solution of the equation for the hip x coordinate.

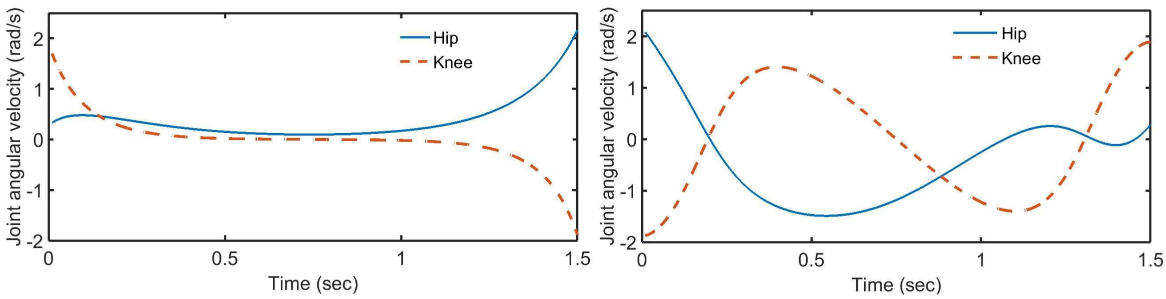

Figure 8. Joint angular velocities for supporting (left) and swing (right) legs.

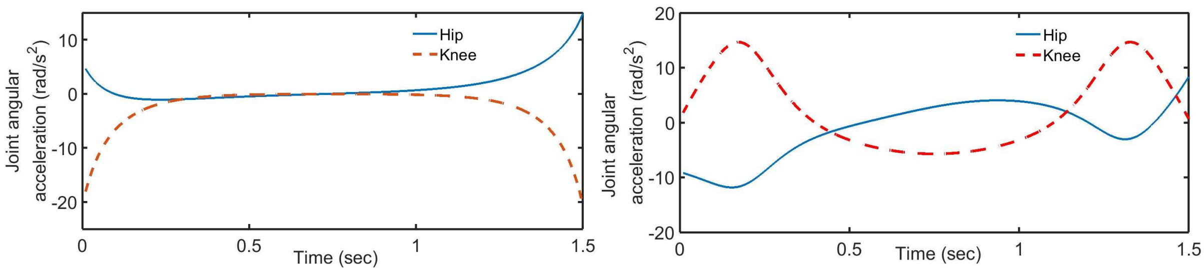

Figure 9. Joint angular accelerations for supporting (left) and swing (right) legs.

5. Conclusion

In this paper, we compared two approaches of defining the optimal leg trajectory for biped locomotion. The first one uses kinematic limits of the joints and finds trajectories where joints are rotated with the maximum allowed speed or acceleration. However, the ZMP analysis of such trajectories shows that this kind of motion is unstable if there is no additional compensation of ZMP deviation from the stable one, corresponding to the center of the supporting foot. Therefore, such optimal trajectories are applicable only if we use the upper body motion to balance the robot dynamics during walking.

The second approach considers a ZMP value to find an optimal trajectory of the hip in Cartesian space. After that, inverse kinematics is used to find trajectories in the joint space. The results show that the robot motion with the same speed as in the first approach requires joint velocities and especially accelerations to be much higher than the limits, which indicates that such motion is not possible for a given robot.

To ensure stable locomotion with high speed, it is proposed to use kinematic-based optimization with further dynamic optimization, using the robot body and the redundant actuation. In future, this approach will be advanced to the case of multi-criterial optimization taking into account both kinematic and dynamic constraints. These results will be applied for dynamically stable locomotion of the AR601-M robot.

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