COMPUTING COMPLEX PROPAGATION CONSTANTS OF DIELECTRIC WAVEGUIDES

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ABSTRACT
An approach for computing propagation constants of dielectric waveguides with arbitrary circuit of cross section is presented. The method is demonstrated by a numerical example of waveguides with circular and square cross sections. The convergence of this method is analyzed.

We shall now consider the propagation of electromagnetic waves in a cylindrical dielectric waveguide with constant permittivity ε_1 embedded into a medium with constant permittivity $\varepsilon_2<\varepsilon_1$. The cross section of the waveguide S_1 is an area bounded with twice continuously differentiable circuit C. The permeability μ_0 is equal to 1 everywhere. Electromagnetic waves in such a structure satisfy the source-free Maxwell equations

$$rotH = \varepsilon\frac{\partial E}{\partial t},\ rotE = -\mu_0\frac{\partial H}{\partial t}.$$

We represent unknown functions in the form of $E(x,y,z,t) = \mathrm{Re}\left(\overline{E}(x,y)e^{i(\omega t-\beta z)}\right)$, $H(x,y,z,t) = \mathrm{Re}\left(\overline{H}(x,y)e^{i(\omega t-\beta z)}\right)$. Expressing vector fields in terms of longitudinal magnetic and electric components $u=\overline{E}_z$, $v=\overline{H}_z$, we reduce the initial problem to the spectral problem (see [1]) of finding such values of parameter β which allow to obtain nontrivial solutions of system of the Helmholtz equations $\Delta u + \chi_j^2 u = 0,\quad \Delta v + \chi_j^2 v = 0,\quad M=(x,y)\in S_j,\ j=1,2$ that satisfy the condition

$$u^+ - u^- = 0,\qquad v^+ - v^- = 0,\qquad M\in C,$$

$$\frac{1}{\chi_1^2}\left(\beta\frac{\partial u^-}{\partial\tau} - \omega\mu_0\frac{\partial v^-}{\partial\nu}\right) - \frac{1}{\chi_2^2}\left(\beta\frac{\partial u^+}{\partial\tau} - \omega\mu_0\frac{\partial v^+}{\partial\nu}\right) = 0,\quad M\in C,$$

$$\frac{1}{\chi_1^2}\left(\beta\frac{\partial v^-}{\partial\tau} + \omega\varepsilon_1\frac{\partial u^-}{\partial\nu}\right) - \frac{1}{\chi_2^2}\left(\beta\frac{\partial v^+}{\partial\tau} + \omega\varepsilon_2\frac{\partial u^+}{\partial\nu}\right) = 0,\quad M\in C,$$

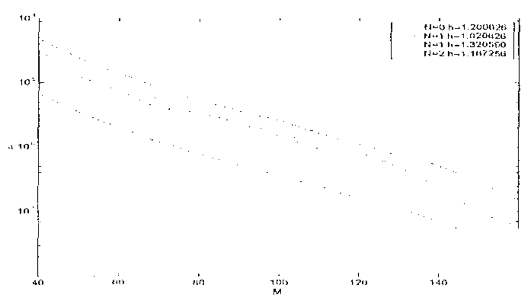
and Reichard condition at infinity $\begin{pmatrix}u\\v\end{pmatrix} = \sum_{n=-\infty}^{\infty}\begin{pmatrix}A_n\\B_n\end{pmatrix}H_n^{(1)}(\chi_2 r)e^{in\varphi},\ r\ge R$. Here, $\chi_j=\sqrt{k_0^2n_j^2-\beta^2}$, $k_0=\omega^2\varepsilon_0\mu_0$, $\varepsilon_j=\varepsilon_0 n_j$, $j=1,2$, $\partial/\partial\tau(\partial/\partial\nu)$ is tangential (normal) derivative. We search for β in the Riemann surface Λ of the function $\ln\chi_2(\beta)$. If β lies in the main "physical" sheet Λ_0^1 of this surface, which satisfy the conditions $Im\chi_2\ge 0$ and $-\frac{\pi}{2}\le\arg\chi_2\le\frac{3\pi}{2}$, then functions $\overline{E}_z,\overline{H}_z$ decrease exponentially in infinity. If β lies in the sheet Λ_0^2, which satisfy the conditions $Im\chi_2\ge 0$ and $-\frac{\pi}{2}\le\arg\chi_2\le\frac{3\pi}{2}$, then functions $\overline{E}_z,\overline{H}_z$ increase exponentially in infinity.

The numerical method of computing the propagation constants β is presented in [1]. We use the expressing unknown functions in terms of potential theory basic problem reduced to an equivalent system of singular integral equations with nonlinear introducing of spectral parameter β in kernels. The Galerkin method with trigonometric basis was suggested for numerical solution of this system. This method was used only for finding real dispersion characteristics.

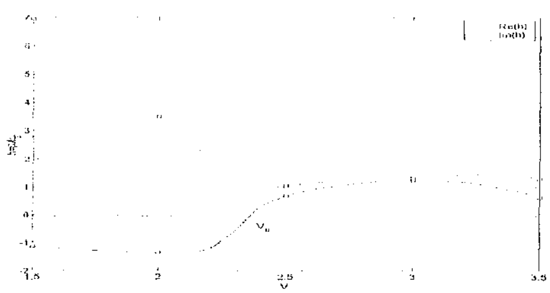
The aim of present work was to test this method for finding complex propagation constants. First, we analyze singularities of kernels of integral operators. It was proved that these singularities can be extracted analytically. Integral operators with singularities were represented as a sum of operator without singularities and operator with known eigenfunctions and eigenvalues. We use the Galerkin method based on eigenfunctions of singular operators. A program complex for numerical solution of this system was developed.

The method has two parameters, i.e. M-number of the integration point and N-number of basis functions in the Galerkin method. Our experiments show that the method has an interior convergence by M and N.

First, the circular cross section with radius R example was considered. In this case it is possible to formulate characteristic equation rigorously for the propagation constant β (see [2]). The calculations was done with $\varepsilon_1=2$, $\varepsilon_2=1$, $k_0R=4$. The differences Λ between the exact solution and approximating solution and the number of points of integration M for four roots are presented in Fig1. Here $h=\beta/k_0$. Increasing N does not make effect in this case because Galerkin's method resulting system is equivalent to characteristic equation. Besides, we can see that it is not necessary to take large M to obtain a good accuracy.



To analyze such complex modes we applied the method of [2], see Fig.2. The solid curves are for characteristic equation, the squares denote values computed by our method. Here $V=k_0R(\varepsilon_1-\varepsilon_2)^{1/2}$, $\varepsilon_1=61$, $\varepsilon_2=1$.



As seen (Fig 2), Re(β) changes its sign for $V=V_0$. It means that β moves from Λ_0^1 to Λ_0^2. Numerical experiments for waveguide with square cross section were carried out. Their result coincidences with [1] are presented in Fig3. We specify the square as

$$r(\varphi)=\left(\left(\frac{\cos\varphi}{d}\right)^{2m}+\left(\frac{\sin\varphi}{d}\right)^{2m}\right)^{-\frac{1}{2m}}.$$

Here $p=\dfrac{d}{\pi\omega}$, d is the length of square side. The dependence of calculation accuracy on the number of basis functions N for p=0.7 is shown in Fig4. As one can see, the method is convergent by N.

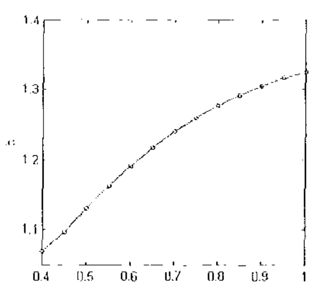


Fig 3

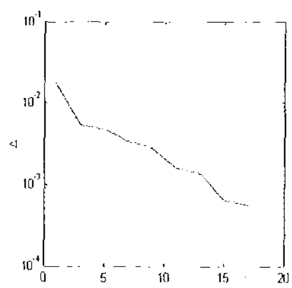


Fig 4

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[2] Tomas F. Jablonski, "Complex modes in open lossless dielectric waveguides", J. Opt. Soc. Am. A/Vol. 11, 4/April 1994