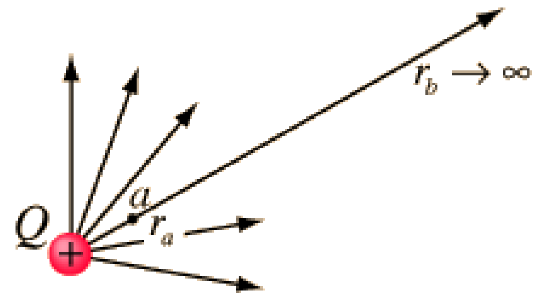
Potential Reference at Infinity

The general expression for the electric potential as a result of a point charge Q can be obtained by referencing to a zero of potential at infinity. The expression for the potential difference is:

$$V_a - V_b = kQ\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$



Taking the limit as $r_b \rightarrow \infty$ gives simply

$$V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

for any arbitrary value of *r*.

The choice of potential equal to zero at infinity is an arbitrary one, but is logical in this case because the electric field and force approach zero there. The electric potential energy for a charge *q* at *r* is then

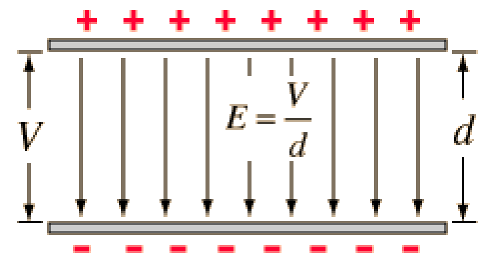
$$U = \frac{kQq}{r}$$

where k is Coulomb's constant.

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/elewor.html

Work and Voltage: Constant Electric Field

The case of a constant electric field, as between charged parallel plate conductors, is a good example of the relationship between work and voltage.



The electric field is by definition the force per unit charge, so that multiplying the field times the plate separation gives the *work* per unit charge, which is by definition the change in voltage.

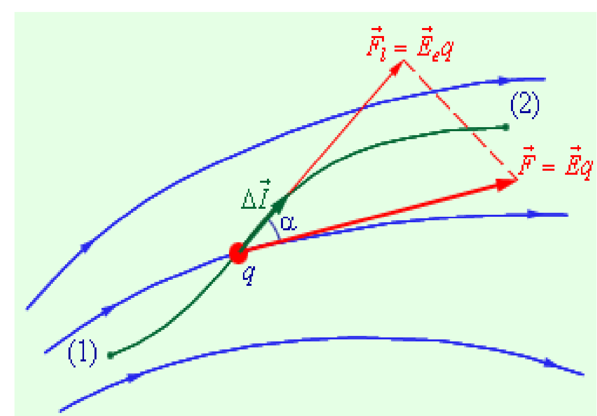
$$Ed = \frac{Fd}{q} = \frac{W}{q} = \Delta V$$ (*for constant electric field*)

$$\text{Units: } \frac{\text{N}}{\text{C}}\text{m} = \frac{\text{Nm}}{\text{C}} = \frac{\text{Joule}}{\text{C}} = \text{Volts}$$

This association is the reminder of many often-used relationships:

$$E = \frac{F}{q}, \qquad W = q\Delta V, \qquad E = \frac{V}{d}, \qquad V = Ed$$

When you move a test charge q in an electric field of electrical forces perform *work*:



$$\Delta A = F \cdot \Delta l \cdot \cos\alpha = Eq\Delta l \cos\alpha = E_1 q \Delta l$$

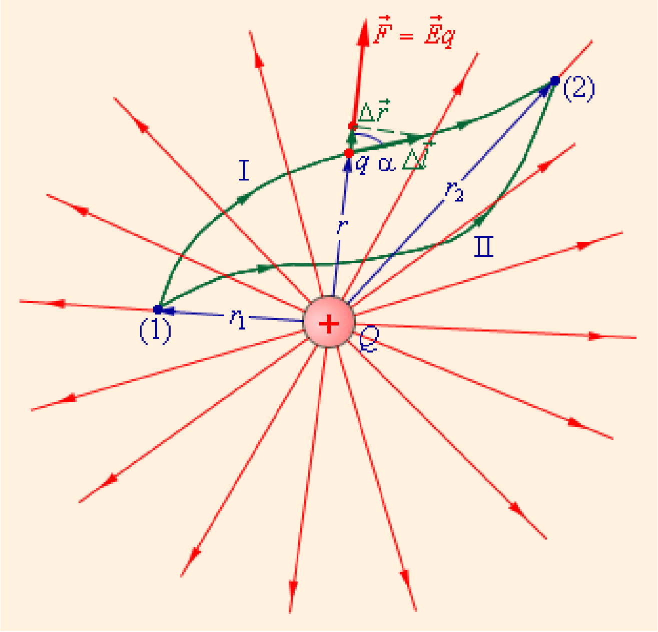
Work and Voltage: Constant Electric Field

The work of electrostatic forces for moving the charge from one point to the other is independent of the shape of the trajectory, and is determined only by the position of the start and end points and charge quantity.

The work of electrostatic forces for moving the charge for any closed path is zero.

The work of Coulomb forces:

$$A = \int_{r_1}^{r_2} E \cdot q \cdot dr = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

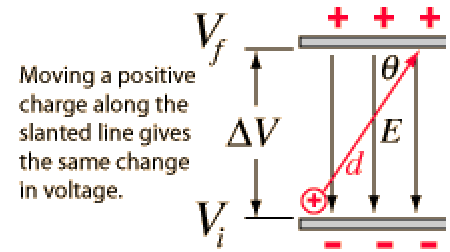


Voltage Difference and Electric Field

The change in voltage is defined as the work done per unit charge against the electric field. In the case of constant electric field when the movement is directly against the field, this can be written

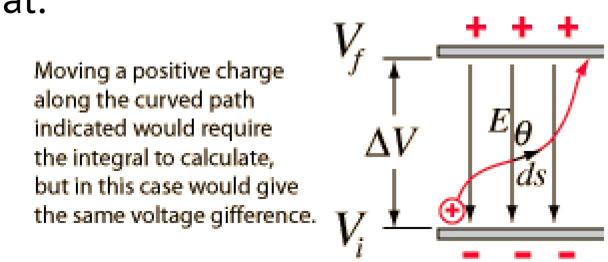
If the distance moved, d, is not in the direction of the electric field, the work expression involves the scalar product:

$$V_f - V_i = \frac{\vec{F}\cdot\vec{d}}{q} = -\vec{E}\cdot\vec{d} = -Ed\cos\theta$$



In the more general case where the electric field and angle can be changing, the expression must be generalized to a line integral:

$$V_f - V_i = -\int \vec{E}\cdot\vec{d}s$$



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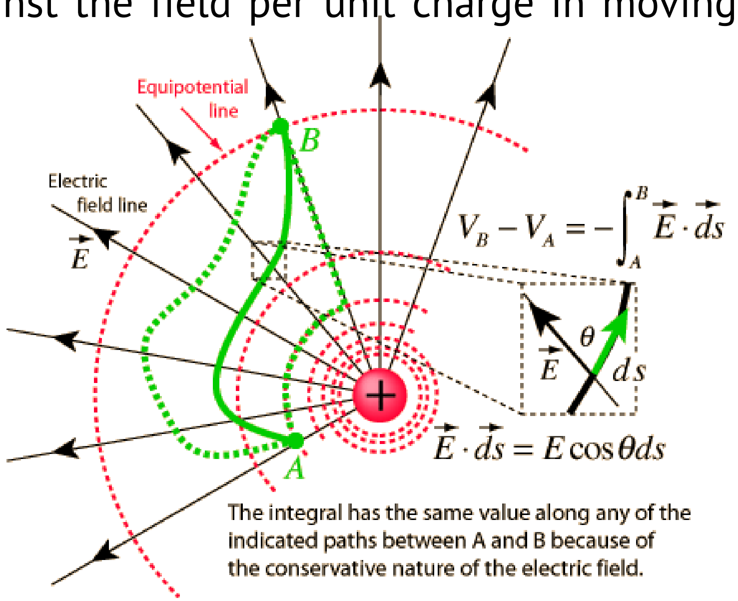
Voltage from Electric Field

The change in voltage is defined as the work done per unit charge, so it can be in general calculated from the electric field by calculating the work done against the electric field. The work per unit charge done by the electric field along an infinite small path length ds is given by the scalar product

$$\frac{dW}{q} = \frac{\vec{F} \cdot \vec{d}s}{q} = \vec{E} \cdot \vec{d}s$$

Then the work done against the field per unit charge in moving from A to B is given by the line integral:



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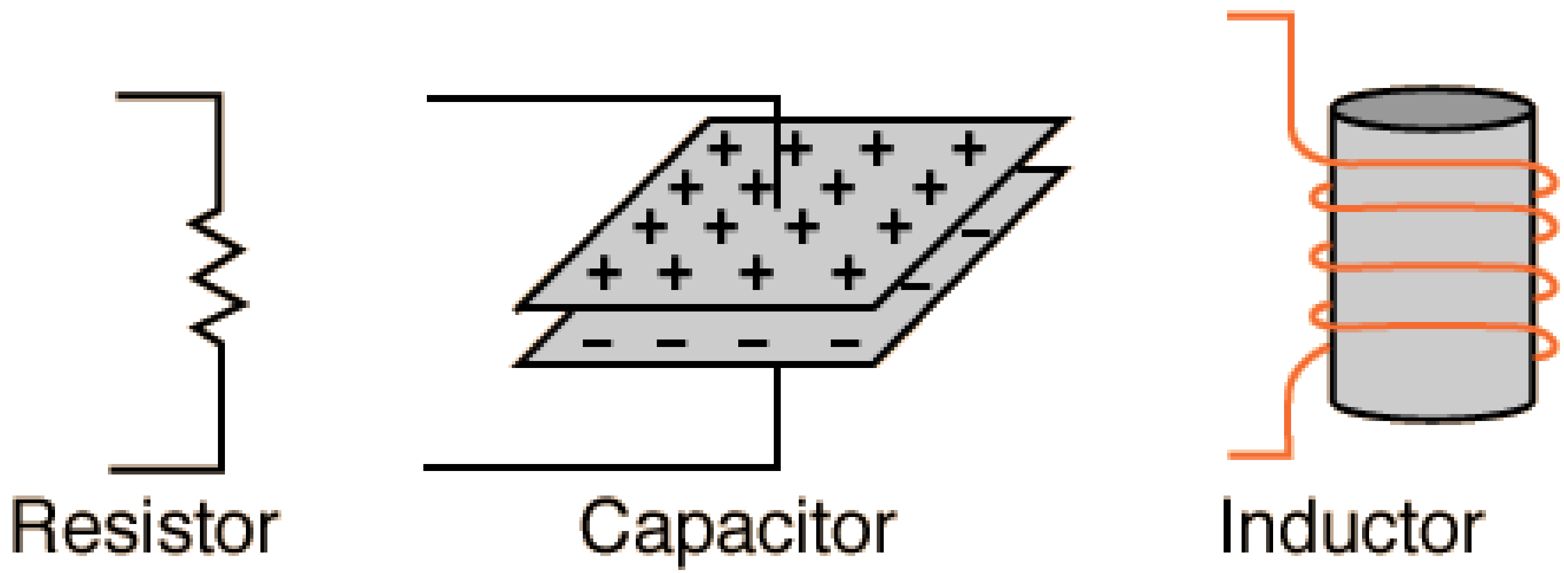
Circuit Elements

Electric circuits are considered to be made up of localized circuit elements connected by wires which have essentially negligible resistance.

The three basic circuit elements are ***resistors***, ***capacitors***, and ***inductors***. Only these passive elements will be considered here; active circuit elements are the subject of electronics.



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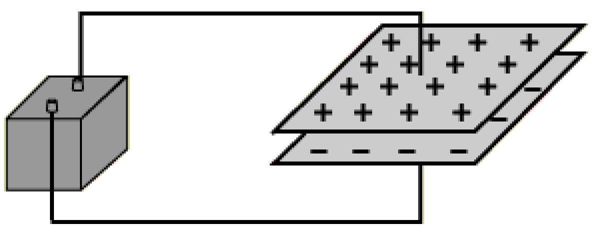
Capacitors

Capacitance is typified by a parallel plate arrangement and is defined in terms of charge storage:



A battery will transport charge from one plate to the other until the voltage produced by the charge buildup is equal to the battery voltage.

$$C = \frac{Q}{V}$$

$$Unit = \frac{coulomb}{volt} = Farad$$

where

- Q = magnitude of charge stored on each plate.
- V = voltage applied to the plates.

Electrical capacitance magnitude depends on the shape and size of conductors, and the dielectric properties

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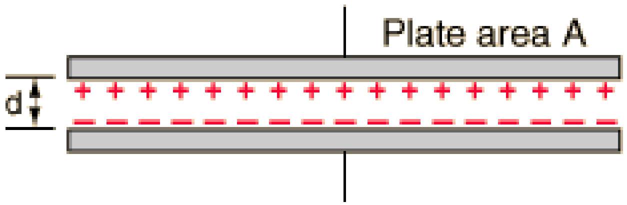
Parallel Plate Capacitor



$$C = \frac{\varepsilon A}{d} = \frac{k\varepsilon_0 A}{d}$$

The capacitance of flat, parallel metallic plates of area A and separation d is given by the expression above where: $\varepsilon_0 = 8.854 \cdot 10^{-12} \frac{F}{m}$ = permittivity of space and k = relative permittivity of the dielectric material between the plates.

k=1 for free space, k>1 for all media, approximately =1 for air.

Electrical capacitance magnitude depends on the shape and size of conductors, and the dielectric properties

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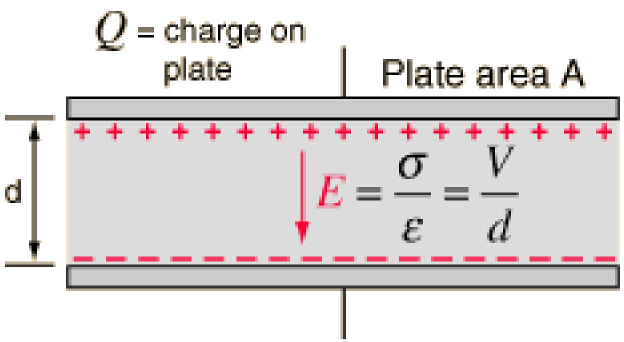
Parallel Plate Capacitor



The electric field between two large parallel plates is given by

$E = \frac{\sigma}{\varepsilon}$ where σ – charge density, ε – permittivity and $\sigma = \frac{Q}{A}$

The voltage difference between the two plates can be expressed in terms of the work done on a positive test charge q when it moves from the positive to the negative plate.

$$V = \frac{work\ done}{charge} = \frac{Fd}{q} = Ed$$

It then follows from the definition of capacitance that

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{Q\varepsilon}{\sigma d} = \frac{QA\varepsilon}{Qd} = \frac{A\varepsilon}{d}$$

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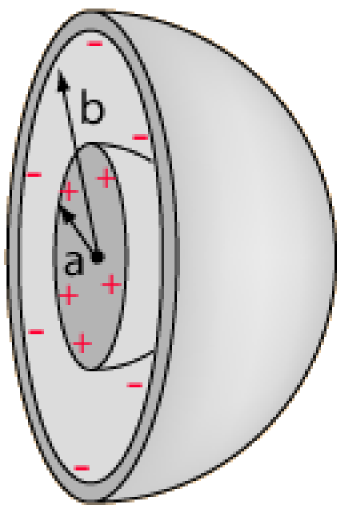
Spherical Capacitor



The capacitance for spherical or cylindrical conductors can be obtained by evaluating the voltage difference between the conductors for a given charge on each. By applying Gauss' law to an charged conducting sphere, the electric field outside it is found to be

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The voltage between the spheres can be found by integrating the electric field along a radial line:

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

From the definition of capacitance, the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$

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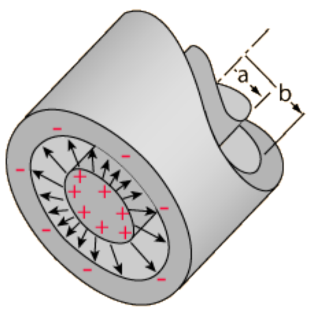
Cylindrical Capacitor



For a cylindrical geometry like a coaxial cable, the capacitance is usually stated as a capacitance per unit length. The charge resides on the outer surface of the inner conductor and the inner wall of the outer conductor. The capacitance expression is

$$\frac{C}{L} = \frac{2\pi k\varepsilon_0}{\ln\left[\frac{b}{a}\right]}$$

The capacitance for cylindrical or spherical conductors can be obtained by evaluating the voltage difference between the conductors for a given charge on each. By applying Gauss' law to an infinite cylinder in a vacuum, the electric field outside a charged cylinder is found to be

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The voltage between the cylinders can be found by integrating the electric field along a radial line:

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{1}{r}\,dr = \frac{\lambda}{2\pi\varepsilon_0}\ln\left[\frac{b}{a}\right]$$

From the definition of capacitance and including the case where the volume is filled by a dielectric of dielectric constant k, the capacitance per unit length is defined as

$$\frac{C}{L} = \frac{\lambda}{\Delta V} = \frac{2\pi k\varepsilon_0}{\ln\left[\frac{b}{a}\right]}$$

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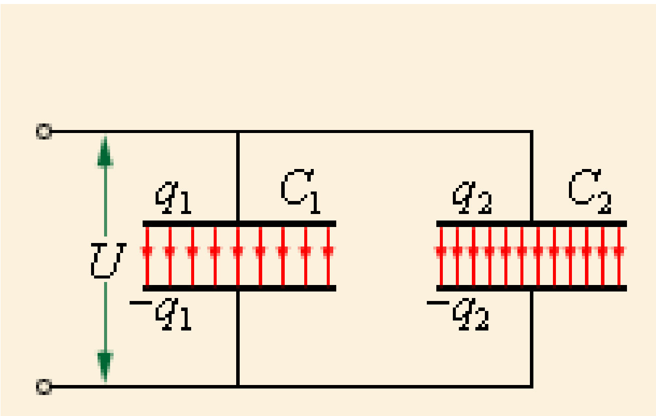
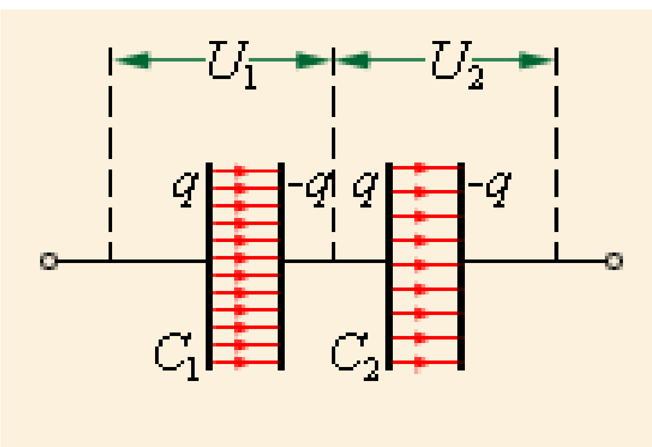
Capacitor Combinations



Capacitors in parallel add ...

$$C = \frac{q_1+q_2}{V} \quad \text{or} \quad C = C_1 + C_2$$



Capacitors in series combine as reciprocals ...

$$C = \frac{q}{V_1+V_2} \quad \text{or} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

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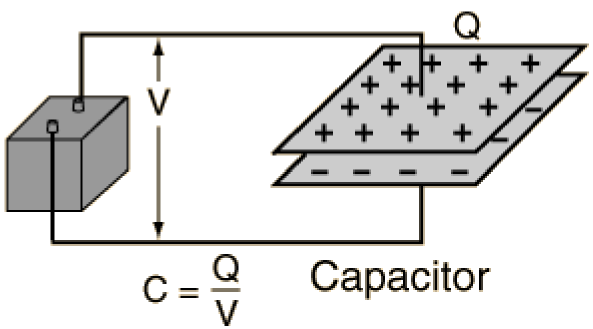
Energy Stored on a Capacitor

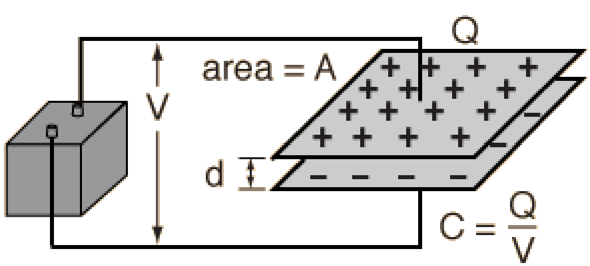


The energy stored on a capacitor can be calculated from the equivalent expressions:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

This energy is stored in the electric field.

The energy stored on a capacitor is in the form of energy density in an electric field is given by



$$\eta_E = \frac{energy}{volume} = \frac{1}{2}\varepsilon E^2$$

This can be shown to be consistent with the energy stored in a charged parallel plate capacitor

$$Energy = \eta V = \frac{1}{2}\varepsilon E^2 Ad = \frac{1}{2}\varepsilon\frac{V^2}{d^2}Ad = \frac{1}{2}\frac{\varepsilon A}{d}V^2 = \frac{1}{2}CV^2$$

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Resistance

Resistor

The electrical resistance of a circuit component or device is defined as the ratio of the voltage applied to the electric current which flows through it:

$$R = \frac{V}{I}$$

If the resistance is constant over a considerable range of voltage, then Ohm's law, I = V/R, can be used to predict the behavior of the material. Although the definition above involves DC current and voltage, the same definition holds for the AC application of resistors.

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Resistivity and Conductivity

The electrical resistance of a wire would be expected to be greater for a longer wire, less for a wire of larger cross sectional area, and would be expected to depend upon the material out of which the wire is made. Experimentally, the dependence upon these properties is a straightforward one for a wide range of conditions, and the resistance of a wire can be expressed as

$$R = \frac{\rho L}{A}$$

ρ – density, L – length, A – cross section area

The factor in the resistance which takes into account the nature of the material is the resistivity . Although it is temperature dependent, it can be used at a given temperature to calculate the resistance of a wire of given geometry.

It should be noted that it is being presumed that the current is uniform across the cross-section of the wire, which is true only for Direct Current. For Alternating Current there is the phenomenon of "skin effect" in which the current density is maximum at the maximum radius of the wire and drops for smaller radii within the wire. At radio frequencies, this becomes a major factor in design because the outer part of a wire or cable carries most of the current.

$$Electrical\ conductivity = \sigma = 1/\rho$$

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Ohm's Law

For many conductors of electricity, the electric current which will flow through them is directly proportional to the voltage applied to them. When a microscopic view of Ohm's law is taken, it is found to depend upon the fact that the drift velocity of charges through the material is proportional to the electric field in the conductor. The ratio of voltage to current is called the resistance, and if the ratio is constant over a wide range of voltages, the material is said to be an "ohmic" material. If the material can be characterized by such a resistance, then the current can be predicted from the relationship:

$$I = \frac{V}{R}$$

Electric current = Voltage / Resistance

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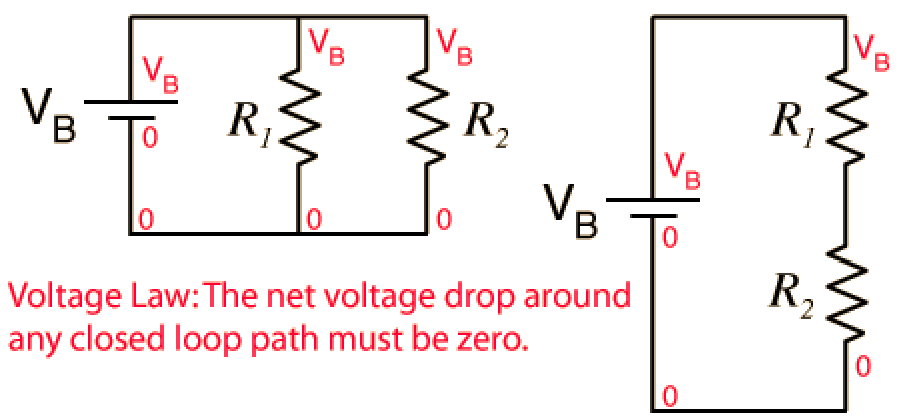
Voltage Law

The voltage changes around any closed loop must sum to zero. No matter what path you take through an electric circuit, if you return to your starting point you must measure the same voltage, constraining the net change around the loop to be zero. Since voltage is electric potential energy per unit charge, the voltage law can be seen to be a consequence of conservation of energy.

The voltage law has great practical utility in the analysis of electric circuits. It is used in conjunction with the current law in many circuit analysis tasks.



For any path you follow around the circuit, the sum of the voltages rises (like batteries) must equal the sum of the voltage drops. Voltage represents energy per unit charge, and conservation of energy demands that energy is neither created nor destroyed.

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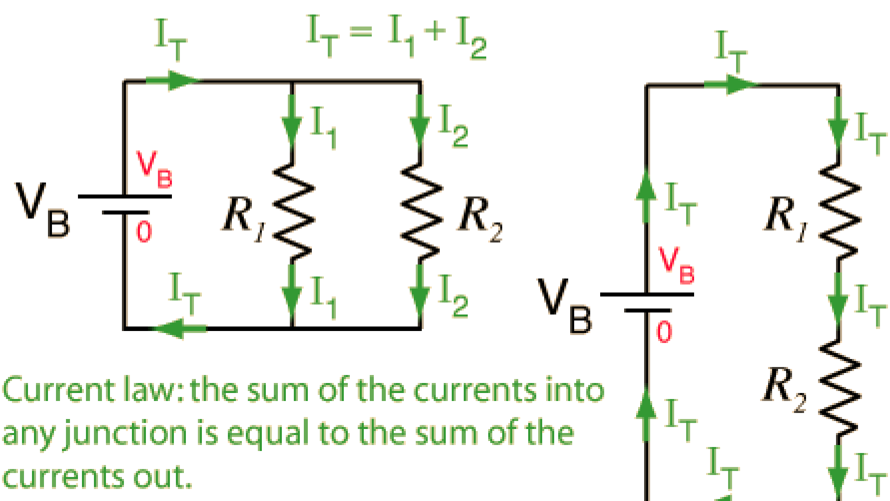
Current Law

The electric current in amperes that flows into any junction in an electric circuit is equal to the current which flows out. This can be seen to be just a statement of conservation of charge. Since you do not lose any charge during the flow process around the circuit, the total current in any cross-section of the circuit is the same. Along with the voltage law, this law is a powerful tool for the analysis of electric circuits.



For any branch of the circuit, the current out of the branch must be equal to the current into the branch. This is required by the conservation of electric charge. Any cross-section of the circuit must carry the total current. For a series circuit, the current is the same at any point in the circuit.

Resistor Combinations

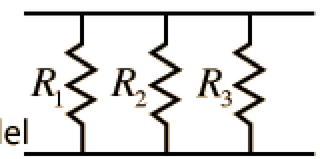
The combination rules for any number of resistors in series or parallel can be derived with the use of Ohm's Law, the voltage law, and the current law.

Series

$$R_{equivalent} = R_1 + R_2 + R_3 + \ldots$$

$$R_{equivalent} = \frac{V}{I} = \frac{V_1 + V_2 + V_3 + \ldots}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2} + \frac{V_3}{I_3} + \ldots = R_1 + R_2 + R_3 + \ldots$$

Series key idea: The current is the same in each resistor by the current law.



Parallel

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Parallel:
$$\frac{V}{R_{equivalent}} = I = I_1 + I_2 + I_2 + \ldots = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \ldots$$

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Parallel key idea: The voltage is the same across each resistor by the voltage law.

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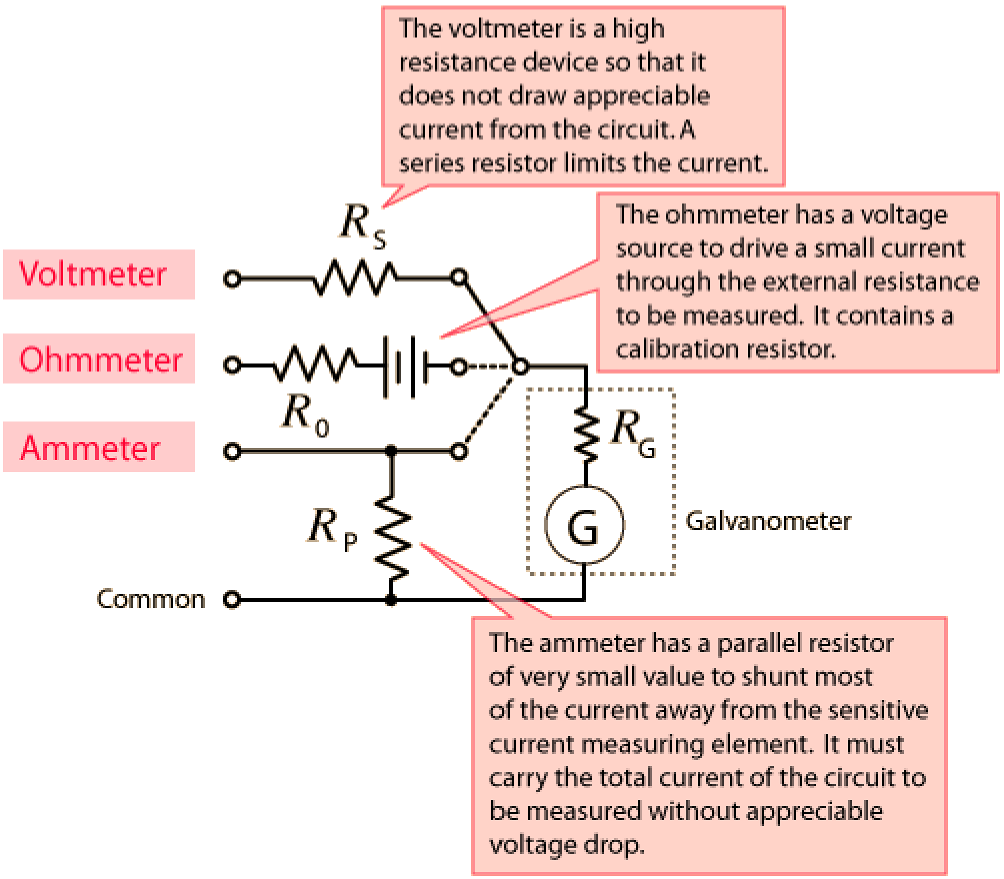
Moving Coil Meters



The design of a voltmeter, ammeter or ohmmeter begins with a current-sensitive element. Though most modern meters have solid state digital readouts, the physics is more readily demonstrated with a moving coil current detector called a galvanometer. Since the modifications of the current sensor are compact, it is practical to have all three functions in a single instrument with multiple ranges of sensitivity. Schematically, a single range "multimeter" might be designed as illustrated.

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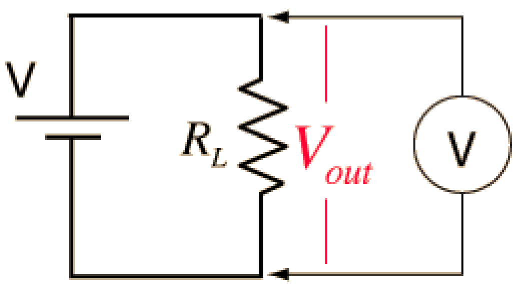
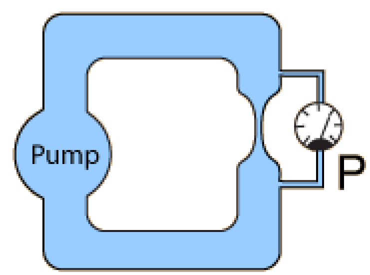
Voltmeter

A voltmeter measures the change in voltage between two points in an electric circuit and therefore must be connected in parallel with the portion of the circuit on which the measurement is made. By contrast, an ammeter must be connected in series. In analogy with a water circuit, a voltmeter is like a meter designed to measure pressure difference. It is necessary for the voltmeter to have a very high resistance so that it does not have an appreciable affect on the current or voltage associated with the measured circuit. Modern solid-state meters have digital readouts, but the principles of operation can be better appreciated by examining the older moving coil meters based on galvanometer sensors.



A voltmeter is connected in parallel to measure the voltage change across a circuit element.



A pressure gauge is connected in parallel to measure the pressure drop across a region of resistance to flow.

A voltmeter is always connected in parallel with the part of the circuit for which you wish to measure voltage.

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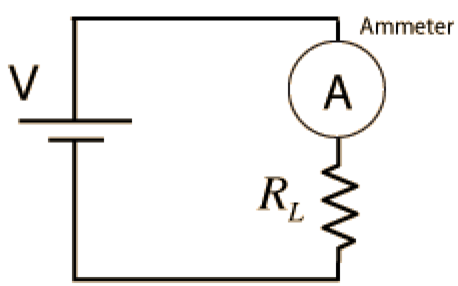
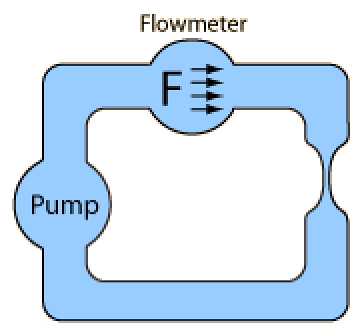
Ammeter

An ammeter is an instrument for measuring the electric current in amperes in a branch of an electric circuit. It must be placed in series with the measured branch, and must have very low resistance to avoid significant alteration of the current it is to measure. By contrast, an voltmeter must be connected in parallel. The analogy with an in-line flowmeter in a water circuit can help visualize why an ammeter must have a low resistance, and why connecting an ammeter in parallel can damage the meter. Modern solid-state meters have digital readouts, but the principles of operation can be better appreciated by examining the older moving coil meters based on galvanometer sensors.



An ammeter is connected in series with a resistor to measure the current through the resistor.



A meter for volume flowrate must be in series to measure the flow, but must not appreciably affect the flow.

An ammeter is always connected in series with the part of the circuit in which you wish to measure current.

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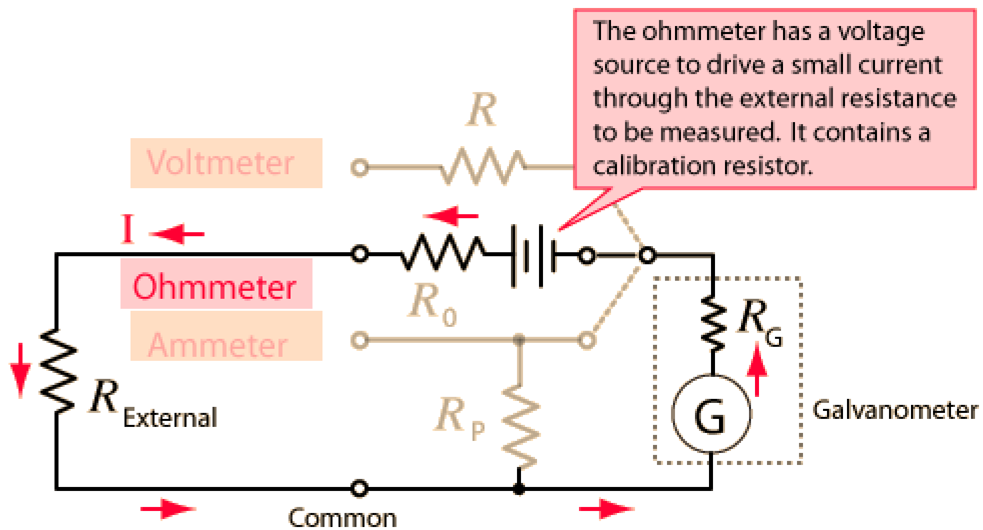
Ohmmeter

The standard way to measure resistance in ohms is to supply a constant voltage to the resistance and measure the current through it. That current is of course inversely proportional to the resistance according to Ohm's law, so that you have a non-linear scale. The current registered by the current sensing element is proportional to 1/R, so that a large current implies a small resistance. Modern solid-state meters have digital readouts, but the principles of operation can be better appreciated by examining the older moving coil meters based on galvanometer sensors.



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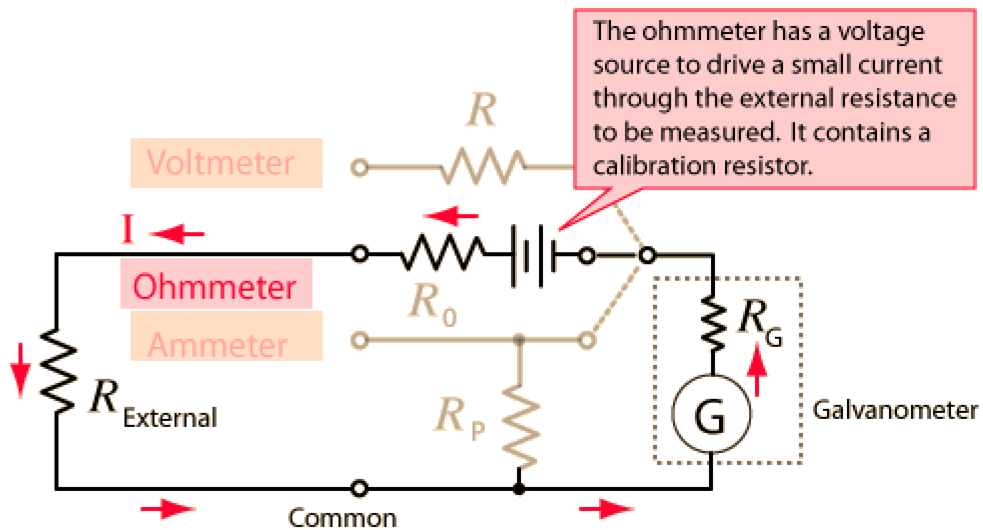

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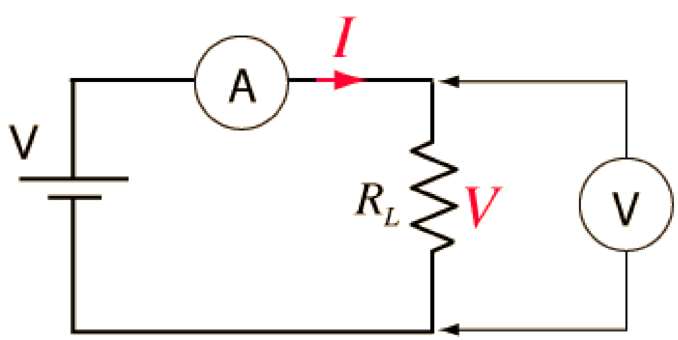
Voltmeter/Ammeter Measurements

The value of electrical resistance associated with a circuit element or appliance can be determined by measuring the voltage across it with a voltmeter and the current through it with an ammeter and then dividing the measured voltage by the current. This is an application of Ohm's law, but this method works even for non-ohmic resistances where the resistance might depend upon the current. At least in those cases it gives you the effective resistance in ohms under that specific combination of voltage and current.

An ammeter is placed in series with the circuit element of interest, and it measures the current through the element with minimal change in that current.



$$R_L = \frac{V}{I}$$

A voltmeter is connected in parallel to measure the voltage change across a circuit element. Its resistance is very high, so it diverts a minimal amount of current away from the intended path through the circuit element.

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Kirchhoff's circuit laws

Kirchhoff's circuit laws are two equalities that deal with the current and potential difference (commonly known as voltage) in the lumped element model of electrical circuits. They were first described in 1845 by German physicist Gustav Kirchhoff.

Kirchhoff's current law (KCL)

This law is also called ***Kirchhoff's first law***, ***Kirchhoff's point rule***, or ***Kirchhoff's junction rule***.

The principle of conservation of electric charge implies that:

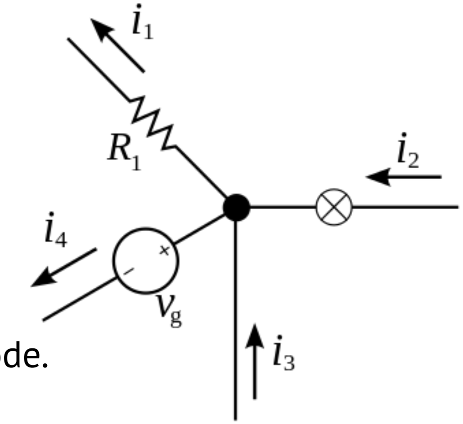
At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node

or equivalently

The algebraic sum of currents in a network of conductors meeting at a point is zero.

$$\sum_{k=1}^{n} I_k = 0$$

n is the total number of branches with currents flowing towards or away from the node.



The current entering any junction is equal to the current leaving that junction. $i_2 + i_3 = i_1 + i_4$

https://en.wikipedia.org/

Kirchhoff's circuit laws

Kirchhoff's voltage law (KVL)

This law is also called ***Kirchhoff's second law***, ***Kirchhoff's loop (or mesh) rule***, and ***Kirchhoff's second rule***.

The principle of conservation of energy implies that

The directed sum of the electrical potential differences (voltage) around any closed network is zero, or:

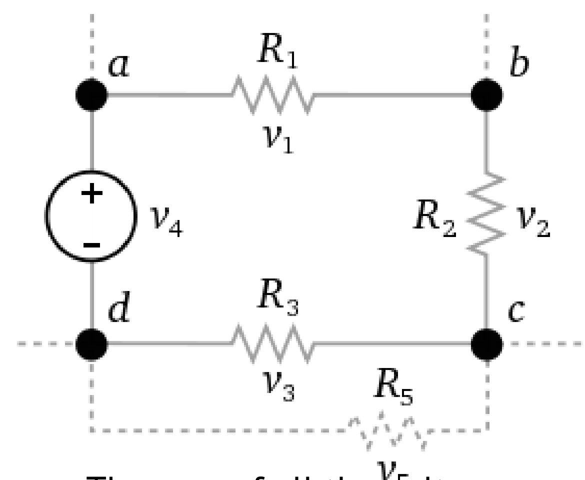
More simply, ***the sum of the emfs in any closed loop is equivalent to the sum of the potential drops in that loop***, or:

The algebraic sum of the products of the resistances of the conductors and the currents in them in a closed loop is equal to the total emf available in that loop.

Similarly to KCL, it can be stated as:

$$\sum_{k=1}^{n} V_k = 0$$

Here, n is the total number of voltages measured



The sum of all the voltages around a loop is equal to zero.
$v_1 + v_2 + v_3 - v_4 = 0$

https://en.wikipedia.org/

DC Electric Power

The electric power in watts associated with a complete electric circuit or a circuit component represents the rate at which energy is converted from the electrical energy of the moving charges to some other form, e.g., heat, mechanical energy, or energy stored in electric fields or magnetic fields. For a resistor in a D C Circuit the power is given by the product of applied voltage and the electric current:

$$P = VI$$

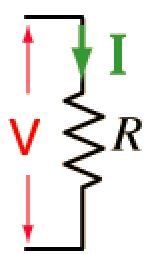
Power = Voltage x Current

The details of the units are as follows:

$$Power = volts \cdot amperes = \frac{joule}{coulomb} \cdot \frac{coulomb}{second} = \frac{joule}{second} = watt$$

Power Dissipated in Resistor

Convenient expressions for the power dissipated in a resistor can be obtained by the use of Ohm's Law.



$$P = VI = \frac{V^2}{R} = I^2R$$