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Examination of Lower Secondary Mathematics Teachers' Content Knowledge and Its Connection to Students' Performance

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Abstract This mixed methods study examined an association between cognitive types of teachers' mathematical content knowledge and students' performance in lower secondary schools (grades 5 through 9). Teachers ($N = 90$) completed the Teacher Content Knowledge Survey (TCKS), which consisted of items measuring different cognitive types of teacher knowledge. The first cognitive type (T1) assessed participants' knowledge of basic facts and procedures. The second cognitive type (T2) measured teachers' understanding of concepts and connections. The third cognitive type (T3) gauged teachers' knowledge of mathematical models and generalizations. The study comprised two levels of quantitative data analysis. First, we explored each cognitive type of teachers' content knowledge and the overall TCKS score as they related to student performance. Second, we studied the correlation between each cognitive type of teacher content knowledge to deepen the understanding of content associations. Results of the study show a statistically significant correlation between cognitive types T1 and T2 of teacher content knowledge and student performance ($p < .05$). The correlation between cognitive type T3 and student performance was not significant ($p = .0678$). The most substantial finding was the correlation between teachers' total score on the TCKS and student performance (Pearson's $r = .2903$, $p = .0055 < .01$). These results suggest that teachers' content knowledge plays an important role in student performance at the lower secondary school. The qualitative phase included structured interviews with two of the teacher participants in order to further elaborate on the nature of the quantitative results of the study.

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Introduction and Purpose of the Study

Although there are numerous studies that focus on understanding teachers' knowledge, there is a dearth of research that provides an in-depth examination on the various facets of such knowledge and its relationship to student performance. Studies conducted by Hill, Schilling and Ball (2004), Rowland, Huckstep and Thwaites (2005), Davis and Simmt (2006), and Stylianides and Stylianides (2014) emphasize the importance of different aspects of teacher knowledge. Most of those studies relate to examination of teacher knowledge in the USA, Europe, and Asia. This study is intended to contribute to the field of inquiry related to teacher knowledge with participants selected from Russian lower secondary schools.

The motivation for this study is grounded in the following perspectives. There are a limited number of research studies on mathematics teacher education in Russia available to an international audience. At the same time, Russia sustains considerably high level of mathematics education. It is evident by Russia's active participation in international assessments and strong performance both at the teachers' and students' level. For example, the TEDS-M (International Association for the Evaluation of Educational Achievement, 2012) study of teachers' mathematics content knowledge ranked Russian secondary teachers in the top 3 (Chinese Taipei, Russia, and Singapore) among 12 countries participating in the study. Russian lower secondary students (8th grade) according to TIMSS-2011 (Mullis, Martin, Foy & Arora, 2012) are also ranked in the top 6 (Korea, Singapore, Taipei, Hong Kong, Japan, and Russia) among 60 countries participating in the study.

For the purpose of this study, we will focus on the examination of Russian lower secondary teachers' content knowledge. Bransford, Brown and Cocking (2000) state that content knowledge requires "a deep foundation of factual knowledge, understanding of the facts and ideas in the context of a conceptual framework, and organization of the knowledge in ways that facilitate retrieval and application" (p. 16). Hill, Ball and Schilling (2008) consider a special kind of content knowledge—mathematical knowledge for teaching (MKT)—"that allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (p. 378). To know what kind of teacher content knowledge has an impact on student achievement is an important issue worth of studying. Due to the multifaceted nature of student achievement, few scholars have analyzed how different types of teacher content knowledge are associated with student learning. Stein, Smith, Henningsen and Silver (2000) claim that student learning depends on the level of cognitive demand established in the mathematics classroom. Following Henningsen and Stein (1997), we emphasize the importance of teacher knowledge in sustaining high cognitive demand in the classroom. More specifically, the kind of knowledge a teacher possesses has an impact on his or her teaching (Steinberg, Haymore & Marks, 1985). From this perspective, the cognitive demand is a function of the cognitive type of knowledge a teacher has (Tchoshanov, Lesser & Salazar, 2008). Therefore, the study considered the following cognitive types of teacher content knowledge: knowledge of facts and procedures (T1), knowledge of concepts and connections (T2), and knowledge of models and generalizations (T3). The study was guided by the following *research questions*: (1) To what extent are different cognitive types of lower secondary school mathematics teachers' content knowledge associated with student performance in Russia? (2) What is the nature of a relationship between different cognitive types of Russian lower secondary school mathematics teachers' content knowledge?

The Russian educational system does not apply state-mandated standardized test at the lower secondary school level. In this study, we refer to the term *student performance* as the grade obtained in a common cumulative assessment composed by a set of open-ended multi-step problems at the end of an academic year. More specifically, the variable of "student performance" is based on the percentage of students who receive highest grades "4" and "5".

The paper is organized in four main sections. First, we present the theoretical background of the study focusing on the categories of teacher knowledge. Second, we outline the cognitive framework for teacher content knowledge. Then, we present the methodology of the study and its major results. Finally, we discuss the results and make concluding statements.

Theoretical Background

This section discusses major theoretical components of the study. We start with addressing the construct of teacher knowledge. Then, we delve further into teacher content knowledge including a literature review on the relationship between teacher content knowledge and student achievement. We end the section with discussion of the major construct of the study—cognitive types of teacher content knowledge.

Teacher Knowledge

In the past three decades, a growing number of studies focusing on teacher knowledge have been conducted (e.g. Bishop, Mellin-Olsen & van Dormolen, 1991; Davis & Simmt, 2006; Hill et al., 2008; Mason & Spence, 1999; Shulman, 1986). However, the notion of teacher knowledge is broad and includes different kinds of knowledge that need to be classified. The problem of classification of different types of knowledge that a teacher should possess is relevant for a broad audience including teachers, teacher education, scholars, and policymakers. Synthesizing the existing research on teacher knowledge, we consider two major dimensions for the classification: (1) classifications based on the discipline-specific domain such as subject matter, pedagogy, curriculum, teaching, learning, etc. and (2) classifications based on the cognitive domain that includes types of knowledge such as facts and procedures, concepts and connections, models, and generalizations.

Classifications in the discipline-specific domain include but are not limited to categories of content knowledge (Shulman, 1986), common content knowledge, horizon content knowledge, specialized content knowledge (Ball, Thames & Phelps, 2008), pedagogical content knowledge (Shulman, 1986), knowledge of content and curriculum (Ball et al., 2008), knowledge of learners' cognition (Fennema & Loef-Franke, 1992), knowledge of content and students (Ball et al., 2008), mathematical knowledge for teaching (Hill, Rowan, & Ball, 2005), mathematics for teaching (Davis & Simmt, 2006), the knowledge quartet (Rowland et al., 2005), and epistemological knowledge (Steinbring, 1998).

Each kind of teacher knowledge in discipline-specific domain could be considered through the lens of cognitive domain. There are different frameworks to study cognitive types of knowledge in the literature proposed by different scholars: Skemp (1978), Stein et al. (2000), and Porter (2002), to name a few. Skemp (1978) focused on the distinction between procedural knowledge (instrumental understanding) and conceptual knowledge (relational understanding). Stein et al. (2000) extended the frame to study four different types: (1) memorization, (2) procedures without connections, (3) procedures with connections, and (4) doing mathematics. Porter (2002) further granulized the upper level of the construct: (1) memorize; (2) perform procedures; (3) demonstrate understanding; (4) conjecture, generalize, and prove; and (5) solve non-routine problems. Teacher knowledge is critical factor in sustaining high cognitive demand in the classroom (Henningsen & Stein, 1997; Steinberg et al., 1985). Based on the work of Skemp (1978), Stein et al. (2000), Porter (2002), and others, we conceptualized the distinction across the cognitive domain of teacher knowledge using the following types: low (knowledge of facts and procedures), medium (knowledge of concept and connections), and high (knowledge of models and generalizations).

Teacher Content Knowledge

Shulman (1986) defined teacher content knowledge as "the amount and organization of knowledge per se in the mind of teachers" (p. 9). Leinhardt and Smith (1985) state that teacher subject-matter knowledge is the knowledge of "concepts, algorithmic operations, and the connections among different algorithmic procedures…" (p. 247). Similarly, Fennema and Loef-Franke (1992) identified features of the content knowledge needed for teaching mathematics, which implies "…teacher knowledge of the concepts, procedures, and problem solving processes…" (p. 162). It also includes the organization of mathematical topics and the connections between them.

Several studies have focused on different kinds of subject matter knowledge such as mathematical knowledge, common content knowledge, and specialized content knowledge (Bishop et al., 1991; Hill et al., 2005, 2008). With regard to measures of teacher knowledge, the US-Based Task Group on teachers and teacher education of the National Advisory Panel (2008) indicated that teacher knowledge has been mostly assessed through teacher certification, mathematics coursework, and a content knowledge test for teachers addressing general mathematical domains. Considering challenges in assessing teacher knowledge (Schoenfeld, 2007), scholars (McCrory, Floden, Ferrini-Mundy, Reckase & Senk, 2012; Saderholm, et al., 2010) have developed measures and ways of assessment sensitive to teacher knowledge of specific content domains (e.g. number sense, algebra, geometry, and probability).

Overall, existing research on teacher knowledge and student performance across different grade levels shows a promising trend. Results from a study by Goldhaber and Brewer (2000) show that high school teachers' scores on content-specific certification test are a significant predictor of higher student achievement. Hill et al. (2008) designed an instrument to assess elementary and middle school teachers' mathematical knowledge for teaching and reported that the MKT is significantly associated with student success in mathematics.

Cognitive Types of Teacher Content Knowledge

The construct of cognitive demand is rooted in the work of Stein et al. (2000) and others referring to the kind of knowledge and thinking processes required to successfully accomplish a task. Resnick and Zuravsky (2006) attempted to broaden the ways the notion of cognitive demand is interpreted by scholars: "The term 'cognitive demand' is used in two ways to describe learning opportunities. The first way is linked with curriculum policy and students' course-taking options—how much math and which courses. The second way relates to how much thinking is called for in the classroom" (p. 1). Following this interpretation, we use the term cognitive demand in the second way to describe both learning and teaching opportunities: (a) "how much thinking is called for in the classroom"—the learning opportunity aspect; (b) "the kind of teacher knowledge needed to sustain students' thinking in the classroom"—the teaching opportunity aspect (Tchoshanov, 2011). In the context of the teaching opportunity, we emphasize the teacher knowledge as a key factor in sustaining high cognitive demand in the classroom. To this end, we echo previous studies (Fennema & Loef-Franke, 1992; Grossman, Schoenfeld, & Lee, 2005; Steinberg et al., 1985) which claimed that teachers whose mathematical knowledge is connected and conceptual were also more conceptual in their teaching, while teachers with procedural knowledge were more rule-based. To support this claim, we refer to the term *cognitive types of teacher content knowledge*. This study capitalized on the following cognitive types: knowledge of facts and procedures (type 1), knowledge of concepts and connections (type 2), and knowledge of models and generalizations (type 3). Memorization and application of basic mathematical facts, rules, and algorithms to solve routine problems are required for cognitive type 1 knowledge. For example, if a teacher is able to find a fraction between two given fractions (e.g. a teacher finds out that fraction 4/6 is located between 1/2 and 3/4), then one could say that he or she has procedural knowledge of determining a fraction between two given fractions. This cognitive type 1 of teacher knowledge (procedural knowledge) has been analyzed in studies conducted by Skemp (1978), Stein et al. (2000), and others.

The quantity and quality of connections between mathematical procedures and ideas are part of the mathematical conceptual understanding in which the cognitive type 2 is grounded in. For instance, the teacher, who was able to identify that fraction 4/6 is located between 1/2 and 3/4, noticed that 4/6 could be represented as $(1+3)/(2+4)$ and, thus, concludes that $1/2 < (1+3)/(2+4) < 3/4$ (as discussed by Stylianides & Stylianides, 2014). Then, the teacher is asked to illustrate and interpret whether a general inequality $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$ is true using pictorial or graphical representation. In order to complete this task, he or she needs to possess conceptual understanding of meaning of fraction and ratio and establish its connection to the concept of slope and graphical interpretation (example is illustrated in Fig. 1).

Type 3 knowledge focuses on models and generalizations. It includes conjecturing, generalizing, proving theorems, etc. For example, using the same concept represented in Fig. 1, the teacher could be asked to extend his or her knowledge to the next cognitive level and formally prove the inequality: $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$.

Cognitive types of teacher content knowledge are a focal point of the study. There are several aspects of teacher knowledge that undoubtedly have an impact on student

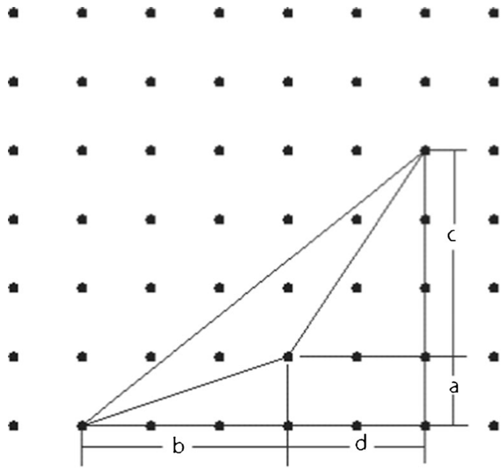

Fig. 1 Graphical illustration/ interpretation of the inequality $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$

learning. Therefore, analyzing teacher knowledge will contribute to the field of mathematics education and provide tools to improve teacher professional development in order to influence students' performance.

Methodology

In the "Methodology" section, we will start with describing the participants' sample and data sources. Then, we will discuss the research design of the study following up with the instrument development and validation. We will end up the section with data analysis methods for both quantitative and qualitative phases of the study.

Participants

The study sample consisted of $N = 90$ lower secondary school teachers from different regions of Russia. Participating teachers had attained a secondary mathematics teacher preparation degree, which allowed them to teach in secondary school (grades 5–11). This sample was composed of 78 % of teachers who have more than 10 years of teaching experience. Approximately 64 % of surveyed teachers had students who participated in Mathematics Olympiads, which is considered as one of the indicators of teacher quality in Russia.

Participants' teaching assignments included grades 5 through 9 with content addressing the following main topics: arithmetic, algebra and functions, probability and statistics, and geometry. Majority of the participants (67 %) had multiple grades teaching assignment. A third of participants (33 %) had 5th grade teaching assignment, 35 % for 6th grade teaching assignment, 29 % for 7th grade, 27 % for 8th grade, and 46 % for 9th grade teaching assignment. The spiral distribution of the content in Russian lower secondary school mathematics curriculum is presented in Table 1 below.

The cluster sample of $N = 6,478$ students of participating teachers was used for collecting teacher-reported student performance data.

Data Sources

The study used two data sources:

(1) Teacher Content Knowledge Survey (TCKS) to collect data on cognitive types of teacher content knowledge.
(2) Student performance data self-reported by participating teachers and checked for accuracy by department chairs.

Research Design

The study implemented mixed methods (*QUAN-qual*) design. The quantitative phase of the study consisted of descriptive statistics of teacher content knowledge as well as regression and correlation analysis performed to address the research questions. The analysis encompassed two levels of investigation. At the first level, we explored each cognitive type of teachers' content knowledge and the overall Teacher Content Knowledge Survey (TCKS) score as they related to student performance. At the second level, we studied the correlation between each cognitive type of teacher content knowledge to deepen the understanding of mathematical content associations.

In the qualitative phase of the study, we conducted a structured interview with two of the participating teachers—Irina and Marina—whose results on the TCKS represented low and upper quartiles of mean scores (names of the teachers are changed to keep the data anonymous). Both of them are experienced lower secondary mathematics teachers and both of them are females. Irina has 33 years of teaching experience and Marina has 21 years of teaching experience. We purposefully selected two contrasting cases with regard to teachers' mean scores on different cognitive types of content knowledge to closely examine the impact of teacher knowledge on student performance while solving a set of problems related to selected items on the TCKS instrument.

Instrument–Teacher Content Knowledge Survey

Teacher Content Knowledge Survey is the instrument that was designed to assess teacher content knowledge based on the three cognitive types identified previously in the section on "Cognitive Types of Teacher Content Knowledge." The survey consisted of 33 multiple choice items addressing main topics of lower secondary mathematics curriculum presented in Table 1: arithmetic, algebra and functions, probability and statistics, and geometry (including measurement). The distribution of the TCKS items across the topics of the lower secondary school curriculum is presented in the specification table (Table 2).

As indicated in Table 2, there were 10 items that measured cognitive type 1 of teacher content knowledge. Cognitive type 2 was measured by 13 items. The rest of the 10 items measured cognitive type 3. The survey does not have any division per cognitive types. All the items are located randomly throughout the survey. The instrument was developed by interdisciplinary faculty with expertise in the following domains: mathematics, mathematics education, statistics, and statistics education, representing various institutions such as university, community college, and local

Table 1 Example of lower secondary school mathematics curriculum in Russia

#	Topics	Lower secondary school	
		Grades 5–6	Grades 7–9
1.	Arithmetic		
	• Natural numbers	+	
	• Fractions and decimals	+	
	• Rational numbers	+	
	• Real numbers		+
	• Numeric measurement and approximation		+
2.	Algebra and functions		
	• Algebraic expressions	+	+
	• Equations		+
	• Inequalities		+
	• Basic concepts of function		+
	• Linear functions		+
	• Quadratic functions		+
	• Number sequences		+
3.	Probability and statistics		
	• Descriptive statistics	+	+
	• Probability and chance	+	+
	• Combinatorics	+	+
4.	Geometry		
	• Visual geometry	+	+
	• Geometric figures		+
	• Measurement		+
	• Coordinate system		+
	• Vectors		+

schools. In order to assist the faculty with item development process, we created a *list of descriptors for each cognitive type*. The list for the type 1 included, but was not limited, to the following descriptors: recognize basic terminology and notation, recall

Table 2 Distribution of the TCKS items across content topics

#	Topics	Number of TCKS items			Total
		Type 1	Type 2	Type 3	
1.	Arithmetic	3	3	3	9
2.	Algebra and functions	3	3	3	9
3.	Probability and statistics	2	2	2	6
4.	Geometry and measurement	2	5	2	9
	Total	10	13	10	33

facts, state definitions, name properties and rules, do computations, make observations, conduct measurements, and simplify and evaluate numerical expressions. The type 2 list consisted of the following descriptors: select and use appropriate representation, translate between representations, transform within the same representation, transfer knowledge to a new situation, connect two or more concepts, explain and justify solutions, communicate mathematical ideas, and explain findings and results from analysis of data. The list for the type 3 included, but was not limited, to the following descriptors: generalize patterns, formulate mathematical problems, generate mathematical statements, derive mathematical formulas, make predictions and hypothesize, design mathematical models, extrapolate findings from data analysis, test conjectures, prove statements and theorems, and solve non-routine problems. Main steps in the item development process were the selection of items for the survey, classification of items by cognitive type, and modifying an item in other cognitive types. In order to establish *content validity*, the specification Table 2 was constructed to guide the process of test development. The table included major content topics and objectives for teachers closely aligned with corresponding objectives in lower secondary content standards. Aside from the specification table, the item analysis table was used to further ensure *construct validity*. The item analysis table included samples of competencies and items from the Teacher Content Knowledge Survey (TCKS) mapped and aligned with competencies from the lower secondary mathematics standards for students (Ministry of Education & Science of Russian Federation, 2004). The instrument was piloted with a small sample of in-service lower secondary mathematics teachers ($n = 22$) enrolled in a graduate mathematics education course. After the pilot, the TCKS items were revised by a group of experts in mathematics, statistics, and mathematics education to finalize the instrument. Sample items of the TCKS instrument are presented in the Appendix. The alpha coefficient technique (Cronbach, 1951) was utilized to evaluate the *reliability* of the teacher content knowledge survey by each cognitive type. The mean value of the coefficient of *.839* suggests that the items comprising the TCKS by cognitive types are internally consistent (Tchoshanov, 2011).

Data Analysis

The following variables were considered as independent: teacher score on cognitive type 1 items of the survey (T1), teacher score on cognitive type 2 (T2), teacher score on cognitive type 3 (T3), and the total teacher score on TCKS. Student performance (SP) was identified as a dependent variable. In order to respond to the first research question, we conducted correlation analysis between teacher scores on different cognitive types and student performance.

To address the second research question, we performed three correlational considerations. The first consideration made a comparison using the cognitive type T2 as dependent variable and cognitive type T1 as independent variable. For the second correlation, we used cognitive type T3 as dependent variable and cognitive type T1 as independent variable. Lastly, we calculated a correlation between cognitive type T3 and cognitive type T2, using the latter one as independent variable. Data analysis was performed using regression analysis to determine how are the cognitive types of teacher content knowledge scores related to student performance as well as the relationship among the cognitive types.

For the qualitative phase of the study, the teacher interviews were audio-recorded and transcribed. Following the grounded theory approach, we used open coding followed by axial coding technique (Strauss & Corbin, 1998) applied to the transcribed narratives as a primary method of analysis.

Results

In this section, we will present major findings of the study starting with descriptive statistics on teacher knowledge by content objectives and cognitive types. Then, we will report data on correlations between cognitive types of teacher content knowledge and student performance. Finally, we will present results of the qualitative phase of the study including the teachers' interviews and students' performance on selected tasks.

Before we discuss major results of the study, we briefly present the descriptive statistics of Russian lower secondary school teachers' content knowledge by objectives and cognitive types. Table 3 depicts descriptive statistics by content objectives of the lower secondary school mathematics curriculum.

As it captured in the table, the highest performing objective for Russian lower secondary teachers was algebra and functions (72.76 %). The lowest performing objective was statistics and probability (38.71 %). One of the explanations for this outcome could be the fact that statistics and probability were not included in the Russian secondary school mathematics curriculum until recently (in the late 1990s).

Table 4 includes descriptive statistics of Russian lower secondary teachers' content knowledge by cognitive types.

As evident from the table, the highest mean score belongs to cognitive type T1 of teacher content knowledge—knowledge of facts and procedures (mean score 77.1 %), the lowest score—to cognitive type T2 (51.0 %), and medium score—to cognitive type T3 (59.4 %). It is worth mentioning that the total average TCKS score obtained in this study (61.5 %) is notably close to the result of the international TEDS-M study (International Association for the Evaluation of Educational Achievement, 2012), according to which the mean score of lower secondary Russian teachers on the content knowledge test was reported at 59.4 %.

As we mentioned earlier, we refer to the term *student performance* as the grade obtained in a common cumulative assessment at the end of an academic year, which is composed of 5–6 open-ended problems reflecting major concepts addressed in the course. It is based on the percentage of students who receive grades "4" and "5" ("5"

Table 3 Teacher knowledge by content objectives

Content objectives	Mean (%)	SE	SD	Conf. level (95 %)
Arithmetic	65.59	1.066	3.197	2.458
Algebra and functions	72.76	0.829	2.487	1.911
Probability and statistics	38.71	1.251	3.064	3.216
Geometry and measurement	58.56	0.727	2.181	1.676

Table 4 Teacher knowledge by cognitive types

Cognitive types	Mean (%)	SE	SD	Conf. Level (95 %)
Facts and procedures (T1)	77.07	1.487	13.470	2.959
Concepts and connections (T2)	51.03	1.338	12.118	2.663
Models and generalizations (T3)	59.39	1.887	17.094	3.756
Total TCKS Score	61.46	1.159	10.500	2.307

being the highest mark in Russian system). This criterion of student performance is used as one of the teacher quality indicators in Russian schools. For example, teacher Marina who was selected for the qualitative part of the study (see below) has student performance at 78 % level, which means that 78 % of her students obtained grades "5" and "4" in the examination whereas 22 % of her students received grades of "3" or below. Student performance data ($N = 6{,}478$) self-reported by teachers ranged from as low as 33 % to as high as 87 % with a mean value of 56.08 %.

The following results are representative of the main findings, which examined the relationship between cognitive types of teacher content knowledge (T1, T2, T3, and total TCKS score) and student performance (SP). Results of the study show statistically significant correlation ($p < .05$) between cognitive types T1 and T2 of teacher content knowledge and student performance: for T1 Pearson's $r = .2076$, $p = .0496$; T2 Pearson's $r = .2295$, $p = .0296$. Thus, it implies that cognitive types T1 and T2 of teachers' content knowledge have a significant association with student performance. We also examined the association between combinations of cognitive types of teacher content knowledge (T1 + T2, T1 + T3, T2 + T3) and student performance (SP). Analysis showed statistically significant associations across all combinations with the strongest correlation for T1 + T2 (which reflects a combination of teacher's procedural and conceptual knowledge) and SP: Pearson's $r = .2751$, $p = .0086 < .01$. However, the correlation between cognitive type T3 and SP was only approaching significance: Pearson's $r = .1904$, $p = .0678$.

The most substantial finding was the correlation between teachers' total score on the TCKS and student performance presented in Table 5. An analysis of variance (ANOVA) showed a significant correlation between teacher content knowledge measured as the total score on the TCKS and student performance (Pearson's $r = .2903$, $p = .0055 < .01$). The corresponding F value was $F(1, 88) = 8.0968$. In other words, teachers' content knowledge measured by the TCKS is significantly related to students'

Table 5 Total teacher content knowledge score and student performance

	df	*SS*	*MS*	*F*	*Significance*
Regression	1	972.97	972.97	8.0968	0.0055
Residual	88	10574.73	120.1674		
Total	89	11547.7			

performance, implying that teachers who performed better in the content test overall had a higher student success rate.

It is apparent that, when separated, a particular cognitive type of teacher content knowledge (e.g. T3) has only a certain, yet non-significant, influence on students' performance. At the same time, teacher's overall mastery of all cognitive types of content knowledge can significantly influence the students' success in attaining the highest grades in mathematics classes. Therefore, it is suggested that teachers' content knowledge plays an important role in teaching and learning lower secondary school mathematics in Russia.

Furthermore, analysis of the relationships between the cognitive types of Russian lower secondary mathematics teachers' content knowledge (T1-T2, T1-T3, T2-T3) produced significant results. We conducted three correlational analyses. The first analysis examined T1 and T2. The results for this comparison showed Pearson's correlation $r = .2670$, $p = .0122$. Therefore, we can deduce that a teacher who possesses the knowledge of concepts and connections has a sound foundational knowledge of facts and procedures. The second analysis focused on comparing T1 and T3. For this comparison, the relationship between cognitive types was also significant at a level of $p < .01$ ($r = .3151$, $p = .0033$). This finding implies that a teacher, who is able to make generalizations and apply mathematical models, has knowledge of mathematical procedures and facts. Finally, the most significant finding showed a strong correlation between cognitive types T2 and T3 ($r = .4658$, $p < .0018$). Thus, we can presume that a teacher who knows mathematical models and generalizations is expected to also know the concepts and connections among mathematical procedures. Results of the correlational analysis between cognitive types of teacher content knowledge are summarized in Table 6 below.

The results of the correlation analysis suggest that each cognitive type of content knowledge is a kind of building block for the other two. In other words, for a teacher to be able to develop connections in mathematics, he or she needs to understand basic facts and procedures. Congruently, a teacher who has knowledge of models and generalizations must be able to understand the connections between mathematical procedures and concepts.

In order to further elaborate on quantitative findings of the study, we conducted structured interviews with two of the study participants—Irina and Marina. Both of them are experienced teachers. Irina is a female secondary school mathematics teacher with 33 years of teaching experience. Her mean student performance is 50 %. Irina's own mean scores on the TCKS items are as following: T1: 80 %, T2: 46 %, T3: 30 %, and total score: 51 %. Marina is also a female secondary school mathematics teacher with 21 years of teaching experience. Marina's mean student performance is 78 %. Her own mean scores on the TCKS: T1: 90 %, T2: 69 %, T3: 70 %, and total score: 75 %. As indicated by teachers' mean scores, they had similar performance on the TCKS items measuring knowledge of facts and procedures (T1)—80 and 90 % accordingly. However, Marina's mean score on items related to knowledge of concepts and

Table 6 Correlations between cognitive types of teacher content knowledge

	T1	T2	T3
T1	1		
T2	.2670*	1	
T3	.3151**	.4658**	1

$^{*}p < .05$, $^{**}p < .01$

connections (T2) was higher than Irina's performance on the same items: 69 and 46 % correspondingly. Most dramatically, Marina's mean score on knowledge of models and generalizations (T3) was higher than Irina's: 70 and 30 %, respectively. We purposefully selected two contrasting cases with regard to teachers' mean scores on different cognitive types of content knowledge to closely examine the impact of teacher knowledge on student performance while solving a set of problems related to selected items on the TCKS (items 1–3 in the Appendix).

This phase of the study included two stages: (1) teacher interview and (2) student problem solving. The teacher interview consisted of the following set of questions aimed at teachers' general understanding of learning objectives for the topic of fraction division, as well as their possession of cognitive types of content knowledge:

(1) When you teach fraction division, what are important procedures and concepts your students should learn?
(2) What is the fraction division rule?
(3) Divide two given fractions $1\frac{3}{4}$ and $\frac{1}{2}$.
(4) Construct a word problem for the fraction division from the previous question; and
(5) Is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ (*a, b, c, and d* are positive integers) ever true?

Responses were audio recorded, and teachers were provided with scratch paper. We used open coding followed by an axial coding technique applied to the transcribed narratives to analyze the meaning in teacher responses with regard to different cognitive types of content knowledge. Below, we present teachers' responses to the five questions.

Irina's response to the question 1 is transcribed below.

IRINA: Before introducing the fraction division, I would like my students to recall the topic on factoring a polynomial, recall the rule of fraction multiplication, and recall reciprocals. After the lesson on fraction division I expect my students to know fraction multiplication and division rules, acquire skills to use these rules in standard situations, as well as apply factorization of polynomials.

Based on Irina's response, it is evident that she capitalizes on her procedural knowledge with little or no attention to concept development. There is a slight indication of applying the rule in "standard situations" (line 4 of the interview excerpt) with no further clarification on the nature of this application. Also, we thought that reference to "factorization of polynomials" in teaching fraction division was not further elaborated by Irina and, therefore, was confusing.

Irina's response to the question 2 further confirmed that she has well-established procedural knowledge of the fraction division rule.

IRINA: The rule of fraction division is reduced to the rule of fraction multiplication. Therefore, you need to multiply the first fraction by the reciprocal of the second one.

INT: What do you mean by reduced to fraction multiplication?

IRINA: As students say, cross multiply fractions.

Irina's response to the question 3 consisted of the solution only (she wrote it on a scratch paper) without any commentary: $1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1} = \frac{7}{2} = 3.5$.

With regard to the question 4, it took a while for Irina to think about the problem. Then, Irina clarified whether she can write down the word problem she came up with on a scratch paper.

IRINA:	May I write down the problem on the paper?
INT:	Yes, of course.
IRINA (writes on a scratch paper):	Area of a rectangle is equal to 1 3/4 cm^2, its length is equal to 1/2 cm. Find width of the rectangle.

In Irina's answer to question 5, she basically repeated her response to the question 2.

IRINA: The given statement is not correct. In order to divide fractions you need to multiply the first one by a reciprocal of the second one.

Now, we present Marina's responses to the questions starting with her answer to question 1 below.

MARINA: When I teach fraction division, first of all, I expect students to learn fraction division rule as it applies to the case of common fractions. Then, I expect them to know how to apply the rule to mixed fractions. Further, students need to understand how to use the fraction division in routine and non-routine problem solving situations. Pedagogy wise, I always support students' motivation through engaging students in small group work and classroom discussion.

Marina's response to the question 2 is depicted below. Surprisingly, Marina used a similar conclusion connecting fraction division to multiplication as Irina did in her response to the same question.

MARINA: In order to divide fractions, you need to multiply dividend by the reciprocal of the divisor. For example, $\frac{15}{4} \div \frac{3}{10} = \frac{15}{4} \times \frac{10}{3} = \frac{25}{2}$ (writes on a scratch paper). Generally speaking, fraction division "boils down" to multiplication.

Unlike Irina, Marina supported her response to the question 3 with step-by-step comments.

MARINA: First, we convert given mixed fraction 1 3/4 to common one 7/4. Notice, here the numerator is larger than denominator. Then, we replace division by multiplication reversing the divisor. Hence, $1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1} = \frac{7}{2}$ (writes on a scratch paper).

After some thinking, Marina offered two word problems in her response to the question 4.

MARINA:	Here is my word problem: an automated machine packs butter in 1/2 kg bricks. How many bricks can one make out of 1 3/4 kg of butter?
	I have another one. May I!?
INT:	Sure.
MARINA:	Let's change the context to a ribbon. Here we go: a ribbon needs to be cut into pieces of 1/2 m in length each. How many pieces can you cut out of 1 3/4 m of ribbon?

Question 5 was the most challenging to Marina. Nonetheless, she confessed that she liked it.

MARINA:	I like this question. It makes me think.
INT:	Good.
MARINA:	Alright, notice that in order to solve this problem ac/bd should be equal to ad/bc. Right?
INT:	So…
MARINA:	Therefore, c/d = d/c. This is possible only if c = d.

At the stage of student problem solving, we asked groups of 6th grade students of participating teachers (Irina's group had $n = 29$ students and Marina's group $n = 26$) to solve a subset of questions (questions 3–5) corresponding to different cognitive types of content knowledge:

(3) Divide two given fractions $1\frac{3}{4}$ and $\frac{1}{2}$.
(4) Construct a word problem for fraction division from the previous question.
(5) Is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ (*a, b, c, and d* are positive integers) ever true?

Question 3 meant to tap into students' procedural knowledge. Question 4 aimed to measure conceptual knowledge whereas question 5 generalized knowledge of fraction division. In selecting questions for students for the qualitative phase of the study, we closely followed the federal standards (Ministry of Education & Science of Russian Federation, 2000, 2004), which encourage teachers to use different types of tasks including tasks to create a story or word problems for a given operation. Considering this, both teachers Marina and Irina, as well as their students, were treated equally with the questions assigned during the interview.

Questions were presented to students after they completed a chapter on the basic operations with fractions. Students wrote their responses on a paper, and student work was collected for further analysis. The number of correct students' responses on questions 3–5, along with the chi-square statistic comparing student performance between groups on each question, is presented in Table 7. Since expected frequency for one of the groups was less than 5, Yates' chi-square correction was employed.

The data in Table 7 show that students in both groups had the same level of procedural knowledge: about 96 % of students in each group were able to correctly perform division of the two given fractions, and therefore, the difference was not significant ($p = .522$). Two other questions revealed differences in student performance. Only 41.3 % of students in Irina's group were able to construct correct word problems to the given fraction division task, whereas in Marina's group, 57.7 % of students made up correct word problems. However, the difference between the groups was still not significant ($p = .348$).

In the following, we present examples of students' incorrect and correct responses to question 4. An example of students' incorrect response: "Turtle travelled 1 3/4 km and her friend Snail travelled half less than Turtle. What distance was travelled by Snail?" An example of students' correct response: "Distance between points A and B is equal 1 3/4 km. Misha walked the distance in 1/2 h. Find Misha's speed."

The difference in student performance between the two groups further increased for question 5: only one student (comprising 3.4 %) in Irina's group correctly solved it, whereas in Marina's group, the percentage was significantly higher, 38.5 %. We were surprised by the correct student's response from Irina's group considering the fact that Irina herself was not able to correctly solve question 5. Examples of students' incorrect and correct responses to question 5 are presented next. Incorrect response: "It is never true because when you divide fractions you have to flip the second fraction". An example of students' correct response: "It could be true if $c/d = 1$ *or* $c = d$".

Even though the difference between student performance in the two groups on all three questions was only at the level of a practical significance, p value approaching *.05* ($\chi^2 = 5.402$, $p = .067$), nonetheless, the difference in students' performance on question 5 was statistically significant ($\chi^2 = 8.43$, $p = .0037$). Overall, students' responses were reflective of their teachers' knowledge: student performance in Marina's group was stronger than in Irina's group, particularly in solving questions 4 and 5.

Discussion and Conclusion

The most important finding of the study was the correlation between teachers' content knowledge measured as the total score on the TCKS and student performance measured as a grade obtained in a common cumulative assessment at the end of an academic year in lower secondary schools in Russia. This result suggests that teachers who performed better in the content test had a higher student success rate. More specifically, teacher's overall mastery of cognitive types of content knowledge is significantly associated with

Table 7 Chi-square analysis' results of student responses on the fraction division questions

	Question 3	Question 4	Question 5
Irina's group ($n = 29$)	28	12	1
Marina's group ($n = 26$)	25	15	10
Chi-square and p value ($df = 1$)	$\chi^2 = .41$	$\chi^2 = .88$	$\chi^2 = 8.43$
	$p = .522$	$p = .348$	$p = .0037$

the students attaining higher grades in mathematics classes. Thus, findings of this study contribute to the body of research claiming that teacher content knowledge is critical for student learning (Hill et al., 2004; Rowland et al., 2005; Stylianides & Stylianides, 2014).

Moreover, the results of the correlation analysis showed that each cognitive type of teacher content knowledge plays a role of a stepping stone for the higher cognitive type. In a sense, in order to make connections in mathematics, a teacher needs to know facts and procedures. Similarly, in order to work with mathematical models and make generalizations, a teacher needs to understand connections between mathematical procedures and concepts.

Follow-up teacher interviews and students' problem solving helped us to look closely at the nature of the relationship between teacher knowledge and student performance. Several observations deserve further discussion. First observation: teacher inclination toward higher cognitive demand (Henningsen & Stein, 1997; Stein et al., 2000). While answering question 1, both teachers mentioned the importance of developing students' skills in applying a fraction division rule. However, the difference in responses was clear: Irina talked about "standard situations" only whereas Marina took it further to "non-routine problem solving situations." Porter (2002) emphasized the importance of non-routine problem solving as the highest level of cognitive demand in his model. Second observation: connecting content and pedagogy (Hill et al. 2008). Responding to the same question 1, Irina did not explicitly mention pedagogy whereas Marina made a clear connection between content (what she would teach) and pedagogy (how she would teach it), "supporting students' motivation through engaging students in small group work and classroom discussion" (lines 4–5 in Marina's response to question 1). Third observation: teacher use of mathematical terminology (Murray, 2004). We noticed that despite the fact that Irina and Marina had similar mean scores on items measuring cognitive type 1, the use of mathematical terminology was more accurate in the case of Marina. For instance, in answering question 2, she used terms "dividend" and "divisor" whereas Irina used "first fraction" and "second fraction" instead. As Murray (2004) asserts: "The accurate use of the vocabulary is an effective measure of conceptual understanding" (p. 5).

Another revealing observation: teacher disposition toward challenge (Valverde & Tchoshanov, 2013). It was obvious that both teachers—Irina and Marina—faced challenges with the cognitively demanding questions 4 and 5. It took both teachers some time to think about question 4. Nonetheless, both of them constructed correct word problems. The difference is: while Irina offered only one word problem, Marina was open and excited to offer a second one (line 3 in Marina's response to the question 4). This difference in teacher disposition escalated further as they were challenged by question 5: Irina did not express any affection (she simply provided a wrong answer) while Marina explicitly did so: "I like this question. It makes me think" (line 1 in Marina's response to the question 5). Synthesizing observations on question 5, we attest that there was a distinct difference in the behavior of two teachers: Marina recognized and accepted the challenge whereas Irina ignored and avoided it. The observation is also revealed Irina's insecurity with questions tapping into her conceptual understanding of the topic. This qualitative piece of data supports previously reported quantitative data on the difference between Irina's and Marina's mean scores on the cognitive types T2 and T3 of content knowledge (see "Results" section). Most importantly, it was reflected in their students' performance while solving questions 3–5 (see Table 7). Overall, results of the teacher interview and students problem-solving helped us to elaborate qualitatively on quantitative data obtained from the TCKS.

We are cognizant that the study had its limitations, such as teacher sample size, multiple choice format of the teacher content knowledge survey, and non-uniform assessment (e.g. cumulative examination) used in Russia as an indicator of student performance. Following on the discussion about complexities of assessing teacher knowledge (Schoenfeld, 2007), we are aware of the limitations of the multiple choice format in test construction and assessment of teacher knowledge (p. 201). Therefore, we included the qualitative phase of the study to delve further into teacher knowledge and understanding. Considering these limitations, we are sensitive enough not to overgeneralize the results obtained in the study. The findings of this study open an opportunity to discuss the importance of different cognitive types of teacher knowledge and its potential impact on student performance. The study showed that cognitive types T1 (facts and procedures) and T2 (concepts and connections) of teacher content knowledge are significantly correlated with student performance in lower secondary school mathematics in Russia. However, cognitive type T3 (knowledge of models and generalizations) did not have the same impact. One possible interpretation of this finding might be rooted in the shift that currently takes place in Russia toward standardized assessment—Unified State Examination—with more emphasis on procedural knowledge and less attention given to reasoning and proof. This interpretation is supported by lower secondary (8th grade) students' performance on TIMSS-2011 (Mullis et al., 2012) where Russian students obtained high average scale score (548) on the low cognitive domain, *knowing*, midlevel score (538) on *applying*, and low scale score (531) on the high cognitive domain, *reasoning* (p. 150).

Another promising finding of the study in lower secondary schools in Russia. There is a significant correlation between cognitive types of teacher knowledge: high cognitive type requires the low type of content knowledge as a foundation. Overall, teachers with a strong background in content knowledge will be able to prepare students more effectively. Correspondingly, a teacher with poor content knowledge might be limited in providing a broader spectrum of learning opportunities to improve student performance.

Appendix

Sample Items from the Teacher Content Knowledge Survey

(1) What is the rule for fraction division?

A. $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$
B. $\frac{a}{b} \div \frac{c}{d} = \frac{ab}{cd}$
C. $\frac{a}{b} \div \frac{c}{d} = \frac{cd}{ab}$
D. $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$

(2) Which of the following word problems below represents the given fraction division $1\frac{3}{4} \div \frac{1}{2} = ?$

A. Juan has a piece of rope $1\frac{3}{4}$ feet long and cuts it in half. At what length should he cut the rope?
B. Maria has $1\frac{3}{4}$ liters of juice. How many $\frac{1}{2}$ liter containers can she fill?

C. A boat in a river moves $1\frac{3}{4}$ miles in 2 h. What is the boat's speed?
D. Daniel divides $1\frac{3}{4}$ pounds of coffee evenly between 2 customers. How many pounds of coffee will each customer get?

(3) Is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ (*a, b, c, d* – positive integers) ever true?

A. The statement is always true
B. The statement true if $c = d$
C. The statement is never true
D. The statement is true if $ad = bc$

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