SOLUTIONS OF INVERSE PROBLEM OF GRAVIMETRY ON THE SPHERE USING "NATIVE" WAVELET TRANSFORM

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ABSTRACT

In this study we present a method for interpretation of gravimetric data obtained on spherical surface based on wavelet transform with the so-called "native" wavelet basis. We show that this apporach has several distinct advantages over commonly used methods, including a simple way of obtaining formal solutions of the inverse problem and easy identification of the causative sources.

Keywords: gravimetry, inverse problem, wavelet transform

1. Introduction

Developing of new systems for processing and interpretation geophysical data is one of important direction in contemporary geophysics. For this purpose many researchers have been applying the wavelet transform – the modern mathematical tool which has proven itself to be useful for solving various geophysical problems, including processing and interpretation of gravity and magnetic data.

In the works [1] and [2] were used wavelet transform with wavelets based on vertical [1] or horizontal [2] derivatives of the gravitational potential of a two-dimensional or three-dimensional point mass. Such approach allow to receive information about parameters of causative sources. Close approach was applied in work [3], where was offered to use wavelet functions built on the basis of the Poisson kernel of the integral transformation that is used in geophysics for the analytic continuation of harmonic fields. In [4], the authors described separating regional gravity anomalies into components and studying of geological structures using mother wavelet of Halo. Gibert and Pessel [5] have showed that the continuous wavelet transform can be used for localization of point sources of potential anomalies.

In order to solve of inverse problems and interpret of gravimetric data in work [1] was offered to use a 2D and 3D wavelet transform with the special basis called by authors "native". Principal advantages of this kind of wavelet transform is that:

1. "Native" inverse wavelet transform is also a formal solution of inverse problem of gravimetry.

2. This transform facilitates search of parameters of causative sources.

This advantages, from our point of view, are important for solution of general gravimetric problems, therefore we have decided to take this approach as a basis to create of similar transforms for processing gravimetric data obtained on spherical surface. The main troubles and their solutions on the way from planar case across 2D-spherical case toward 3D spherical case will described below.

2. Mathematical preliminaries

2D and 3D planar case

Above-mentioned family of "native" wavelets has the form:

$$\psi_{(n)}(h,x) = \frac{2^{n-3}h^{n-2}}{(n-2)!\,\pi^2 f} V_{(n)}(h,x), n > 1 \qquad (1)$$

where x is horizontal coordinates, h is vertical coordinate (depth), f is gravitational constant, function $V_{(n)}(x,h)$ is n-th vertical derivative of the gravitational potential of a two-dimensional point mass.

Figure 1 shows wavelet transform of gravity field $V_{(1)}$ for a single two-dimensional point source at a depth of 5 km with the basis functions $\psi_{(n)}$ of (1) for n equal to 4. In this case maximum of wavelet coefficients has the same coordinates that causal source.

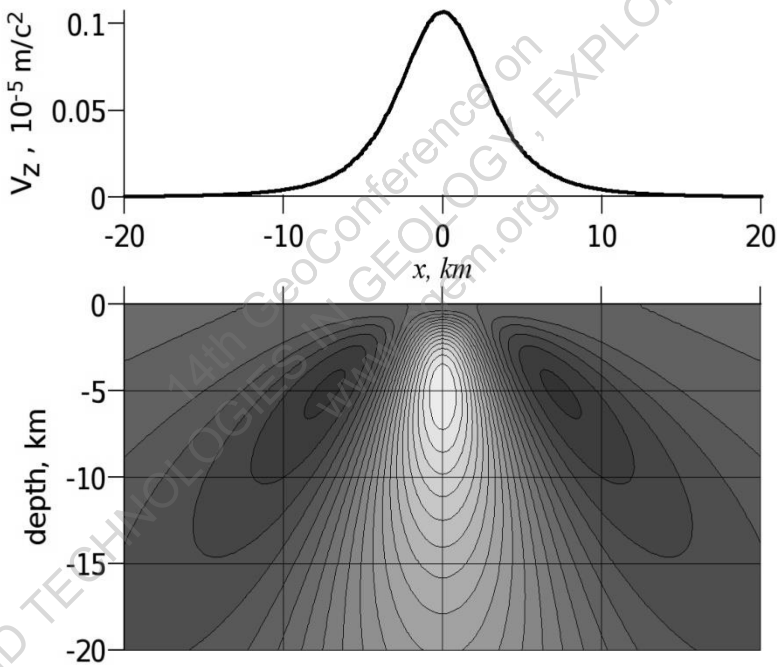

Figure 1. Gravity field of a two-dimensional single point source located 5 km deep and solutions of inverse problem with basis functions $\psi_{(n)}$ from formula (1) for n, equal to 4. (1 mGal = 10^{-5} m/c^2).

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In three-dimensional case this formula turns into

$$\psi_{(n)}(x,y,h) = \frac{2^{n-1}h^{n-2}}{(n-2)!\,f} V_{(n)}^{3D}(x,y,h), n > 1 \qquad (2)$$

where superscript «3D» emphasizes the three-dimensional of the model.

2D sphere

The first radial derivative of the gravitational potential of a two-dimensional point mass on 2D-sphere has the follow form

$$V_r(\gamma) = 2f\,\frac{R - Hcos(\gamma)}{H^2 + R^2 - 2HRcos(\gamma)} \quad (3)$$

where R is radius of the sphere, γ – zenith angle, h is depth of point mass, $H = R - h$. Function (2) is even and $2\pi R$-periodic, therefore its Fourier transform is

$$S_{V_r}(\omega) = \frac{2G}{R\pi}\int_0^{2\pi R}\frac{\left[1-qcos\left(\frac{t}{R}\right)\right]e^{i\omega t}}{q^2+1-2qcos\left(\frac{t}{R}\right)}dt = \frac{2G}{\pi}\int_0^{2\pi}\frac{[1-qcos(y)]e^{iR\omega y}}{q^2+1-2qcos(y)}dy, \quad (4)$$

here $q = (R - h)\,/\,R$. In order to calculate this integral is need the theory of residues. Finally, there was found following ultimate result:

$$S_{V_r}(\omega) = 2fe^{-R|\omega|ln\left(\frac{1}{q}\right)} = 2fe^{-h_k|\omega|}, \quad (5)$$

where function

$$h_k = -Rln\left(\frac{1}{q}\right) = -Rln\left(1 - \frac{h}{R}\right) \quad (6)$$

might be named "seeming" depth. Here we should compare the result (6) with formula of Fourier spectrum of gravity anomaly of two-dimensional point source in planar case:

$$S_{V_z}(\omega) = 2Ge^{-h|\omega|} \quad (7)$$

Comparison of (5) and (7) shows that radial derivative of the gravitational potential of a two-dimensional point mass on 2D-sphere and its planar analogue have the same formula of Fourier spectrum taking but different parameters of depth into account formula (6). On figure 2 was shown gravity anomaly on plane for $2\pi R$-periodic set of gravity sources and the same anomaly on the sphere for single point mass.

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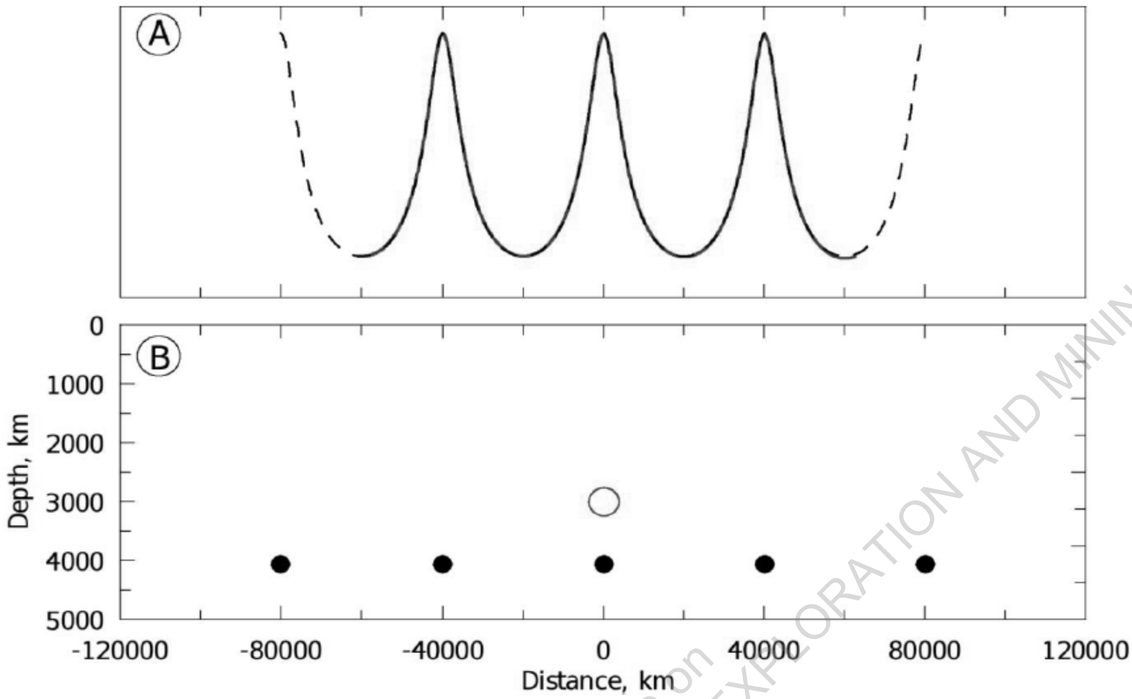

Figure 2. Comparison of the potential field on a plane of 2πR-periodic set of sources and potential field on the sphere of single source. The depths of sources of both models (3000 and approximately 4055 km) are linked by formula (6).

A: dotted black line corresponds potential field of planar model, solid red line corresponds field of spherical model.

B: red circle corresponds the source of spherical model, black filled circles correspond $2\pi R$ -periodic set of sources of planar model.

This result allowed to consider a data obtained on 2D-spherical surface as a data on the plane but with other depth parameters of causative sources, consequently if we perform wavelet transform with "native" basis we will get solution of inverse problem on 2D-sphere as well as on plane.

3D sphere

When we tried to transfer the results described above from 2D-sphere to 3D-sphere we had encountered some problems.

At first, there is problem with integration of data on the sphere. At second, formula (6) do not working in 3D case for first radial derivative of potential. Fortunately we have proved that the same formula (6) is working for data of potential of point mass measured on the sphere.

Detailed description our approaches of solution this problems exceed the limits of this article so we going to describe it in our next paper.

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3. Synthetic samples

Figure 3 shows the consecution of actions to building "native" wavelet transform on the 2D-sphere. At first, is required to construct regular "native" wavelet transform for sweep of function of radial derivative of potential (see figures (A) and (B)). Next, is required to correct depth (scale parameter of wavelet transform) on the strength of formula (6) (figure (C)). Finally, the wavelet coefficients were converted to spherical form (figure D).

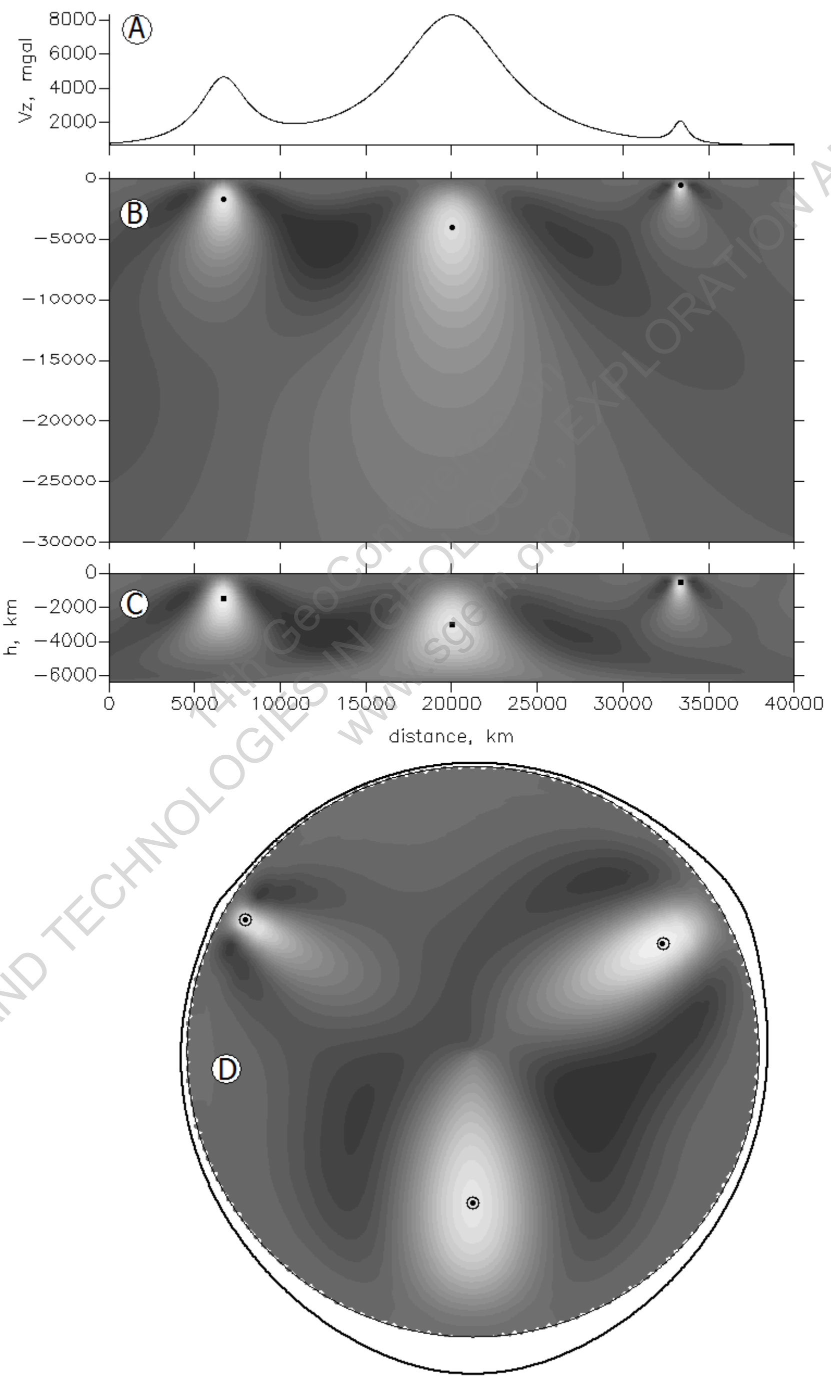

Figure 3. The wavelet transform on the 2D-sphere on an example of three causative sources.

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A: The sweep of function of radial derivative of potential.
B: The “native” continuous wavelet transform.
C: field of wavelet coefficients after correction of depth.
D: recalculated field of wavelet coefficients on the 2D-sphere.

Circles indicate location of the sources in the model, and black dots – positions from the inverse solution.

Note, that initial locations of the sources in the model, and positions from the inverse solution is almost identical.

The similar example for gravimetric data on 3D-sphere was presented on figure 4.

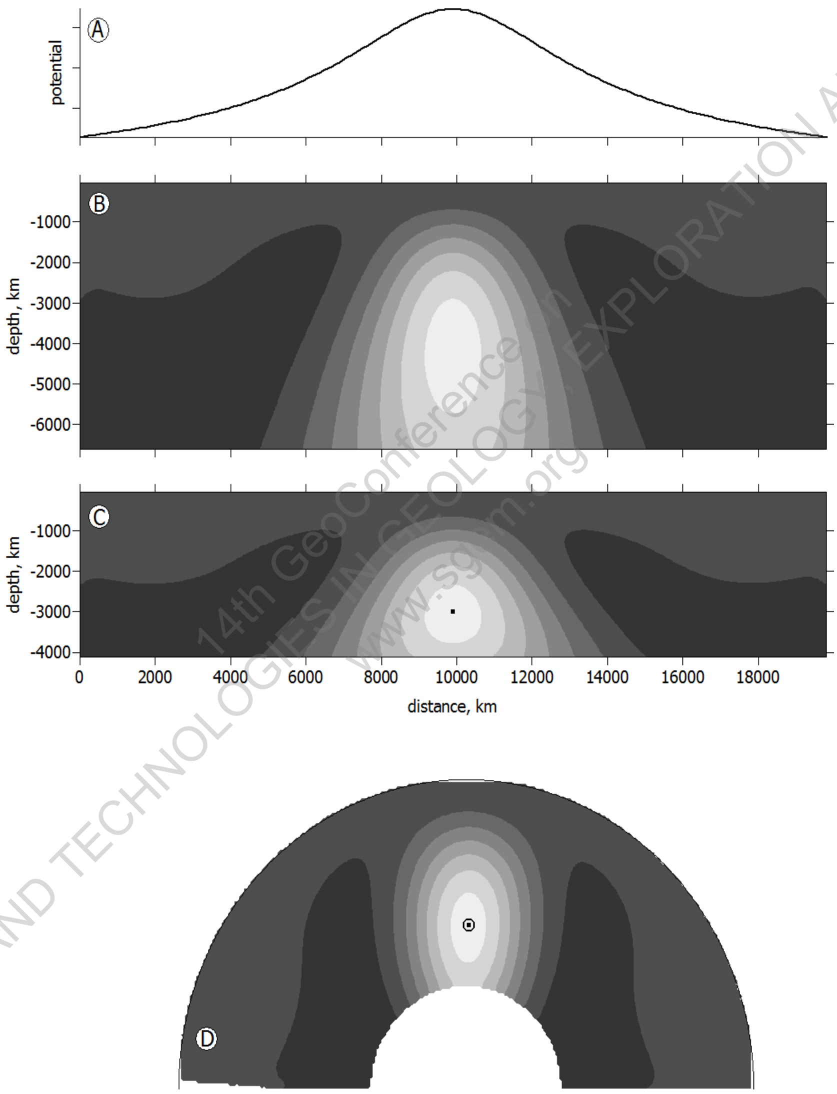

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Figure 4. The wavelet transform on the sphere on an example of single causative source located 3000 km deep.

A: The sweep of function of potential.
B: The “native” continuous wavelet transform.
C: field of wavelet coefficients after correction of depth.
D: recalculated field of wavelet coefficients on the sphere.

Circle indicates location of the source in the model, and black dot position from the inverse solution.

4. Practical sample

As a practical example on figure 5 was shown a slice of “native” wavelet transform of geopotential function obtained on spherical Earth's surface, that corresponds to depth 150 km.

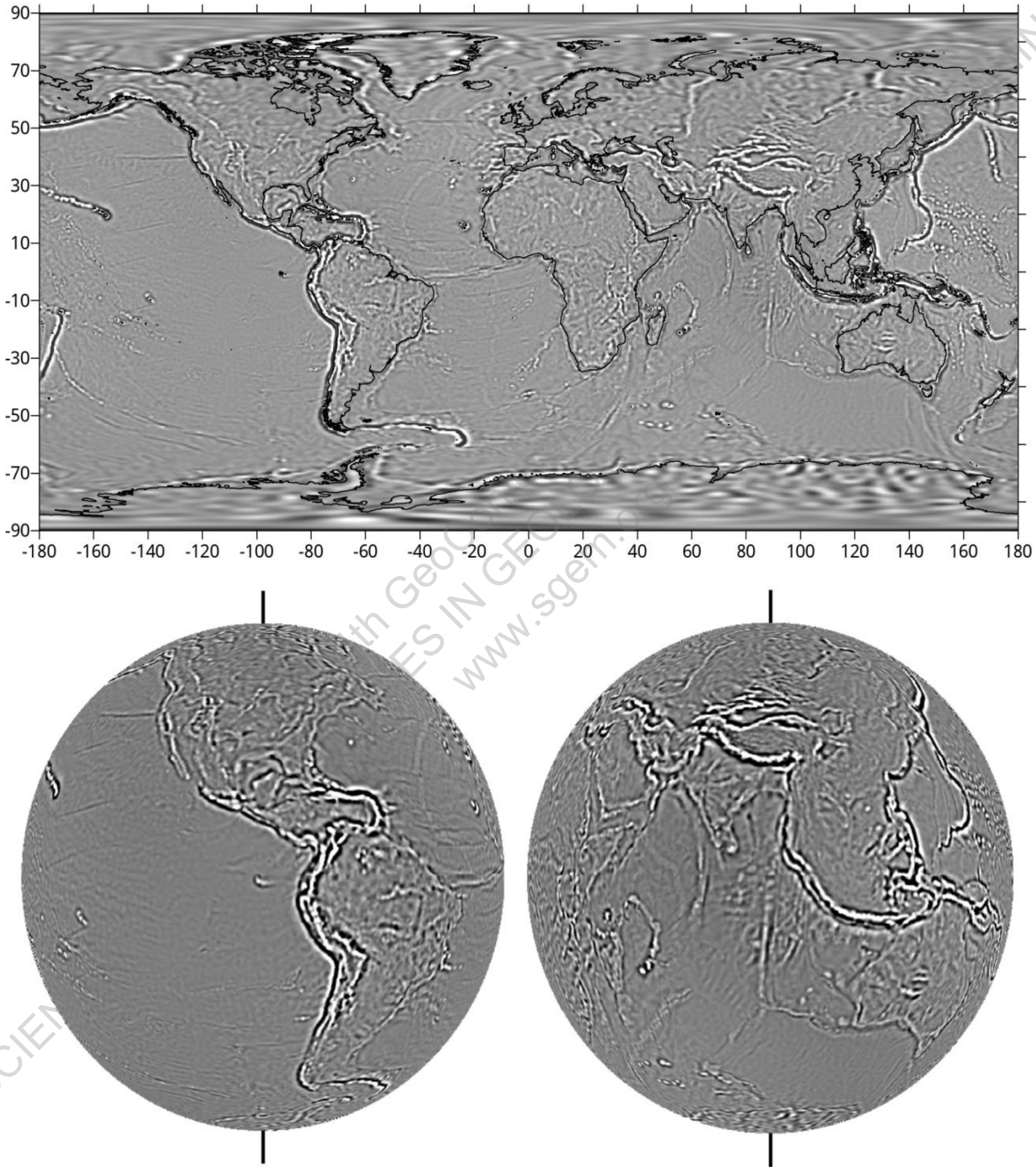

Figure 5. A slice of “native” wavelet transform of potential obtained on Earth's surface, that corresponds to depth 150 km.
For calculating were used gravimetric data of International Centre for Global Earth Models (ICGEM) (http://icgem.gfz-potsdam.de/ICGEM).

5. Conclusion

The technique of consistent creation of wavelet-transform on the sphere presented in this study is based on the use of "native" wavelet basis. Its most distinctive feature is that this wavelet transform provides a formal solution of inverse problem of gravimetry. Besides it is important the fact that transform facilitates search of parameters of causative sources.

It is also worth pointing out that nearly all of the techniques used in this study can be directly applied to other potential-field geophysical problem.

Acknowledgments

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