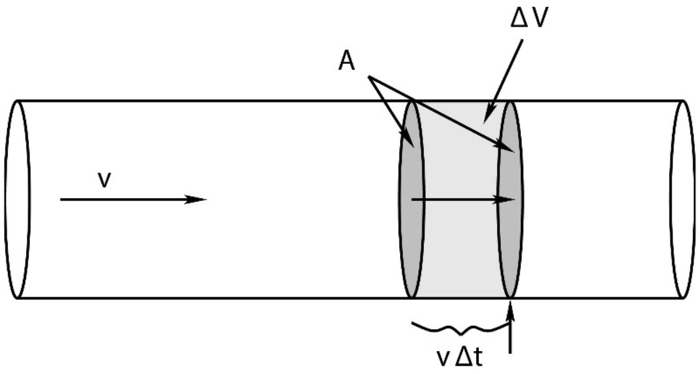
Lecture 6. Fluids

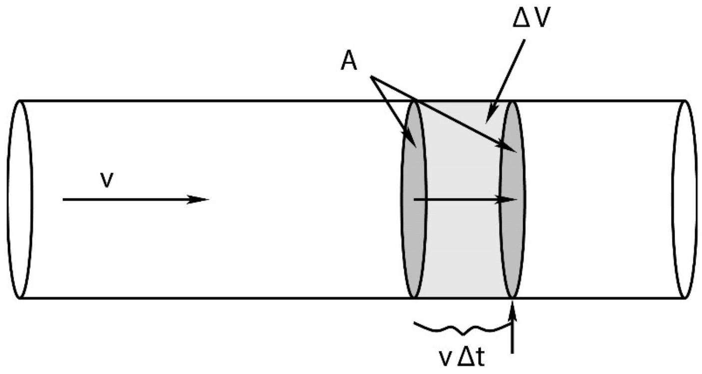
Fluid Flow



In figure we see fluid flowing from left to right in a circular pipe. The pipe is assumed to be "frictionless" for the time being – to exert no drag force on the fluid flowing within – and hence all of the fluid is moving *uniformly* (at the same speed *v* with no relative *internal* motion) in a state of ***dynamic equilibrium***.

- We are interested in understanding the ***flow*** or ***current*** of water carried by the pipe, which we will define to be the ***volume per unit time*** that passes any given point in the pipe.
- We would like to understand the relationship between area, speed and flow

Fluid Flow



In a time Δ*t*, all of the water within a distance *v*Δ*t* to the left of the second shaded surface will pass *through* this surface and hence past the point indicated by the arrow underneath. The volume of this fluid is just the area of the surface times the height of the cylinder of water:

$$\Delta V = Av\Delta t$$

If we divide out the Δ*t*, we get:

$$I = \frac{\Delta V}{\Delta t} = Av$$

This, then is the ***flow***, or ***volumetric current*** of fluid in the pipe.

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Conservation of Flow

Fluid does not, of course, only flow in smooth pipes with a single cross-sectional area. Sometimes it flows from large pipes into smaller ones or vice versa.

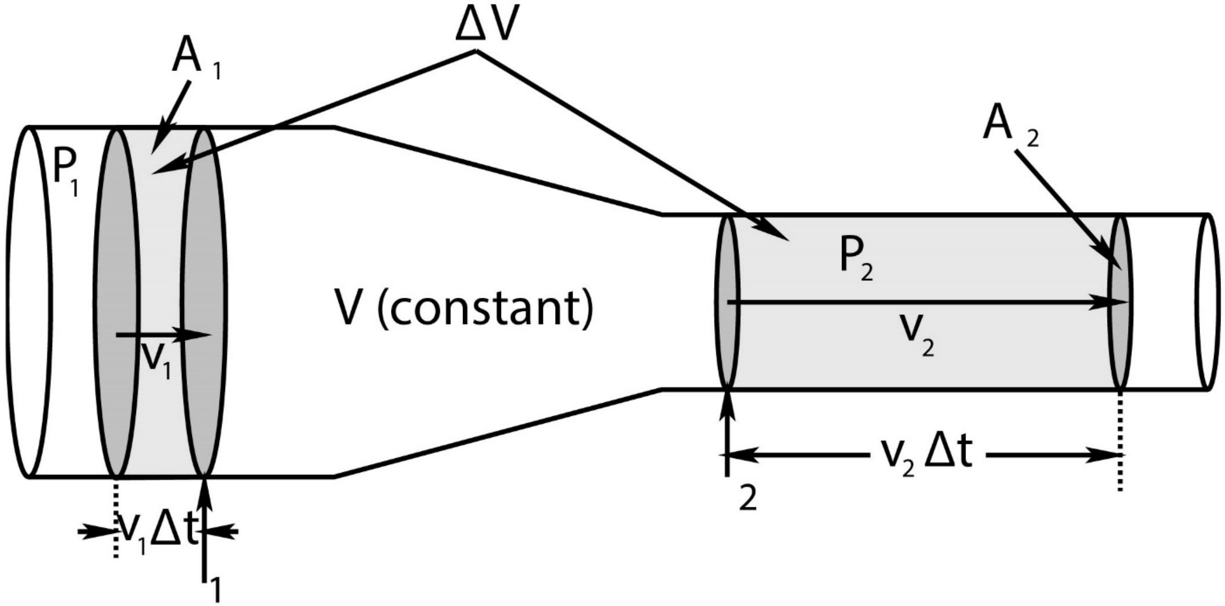

Figure shows a fluid as it flows from just such a wider pipe down a gently sloping neck into a narrower one. As before, we will ignore drag forces and assume that the flow is as uniform as possible. The pressure, speed of the (presumed incompressible) fluid, and cross sectional area for either pipe are P_1, v_1, and A_1 in the wider one and P_2, v_2, and A_2 in the narrower one.

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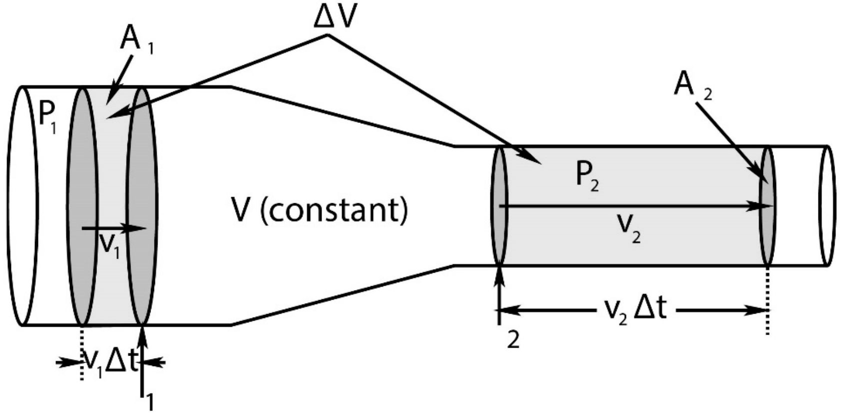
Conservation of Flow



In a time Δt a volume of fluid $\Delta V = A_1 v_1 \,\Delta t$ passes through the surface/past the point 1 marked with an arrow in the figure. In the volume between this surface and the next grey surface at the point 2 marked with an arrow ***no fluid can build up*** so actual quantity of mass in this volume must be a *constant*.

This is a kind of *conservation law* which, for a continuous fluid or similar medium, is called a *continuity equation*.

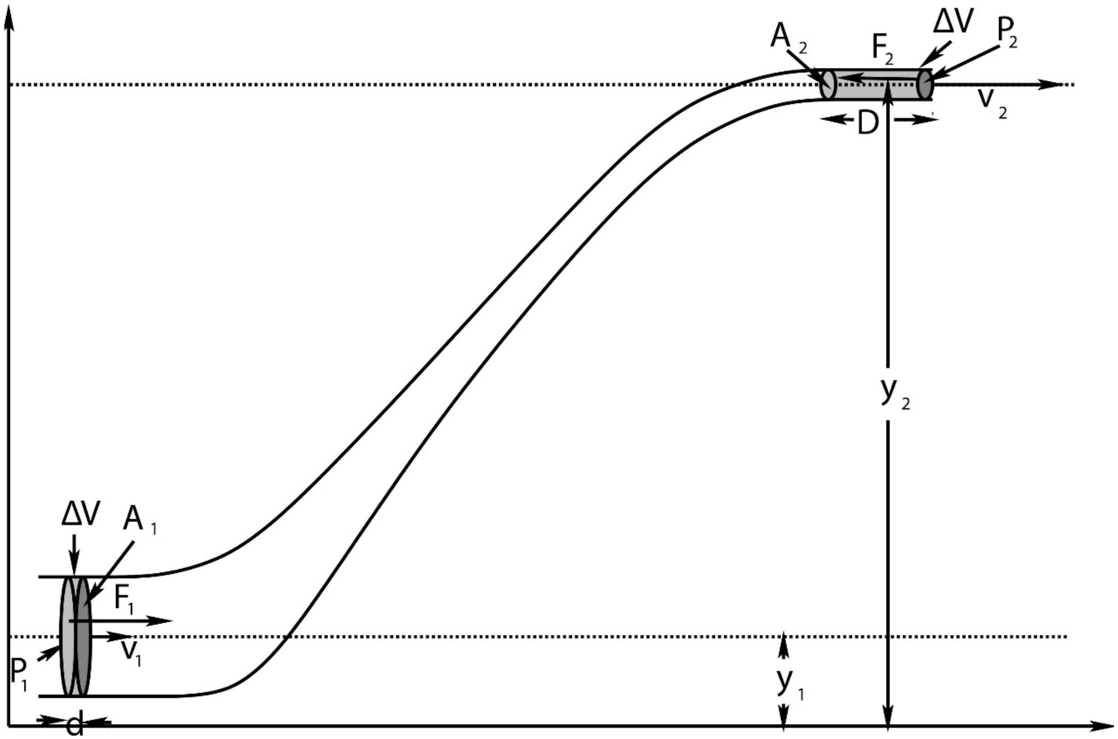
$$\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$$I = \frac{\Delta V}{\Delta t} = A_1 v_1 = A_2 v_2$$

Thus the current or flow through the two surfaces marked 1 and 2 must be the same:

$$A_1 v_1 = A_2 v_2$$

Work-Mechanical Energy in Fluids: Bernoulli's Equation



A circular cross-sectional necked pipe is arranged so that the pipe *changes height* between the larger and smaller sections. We will assume that both pipe segments are narrow compared to the height change, so that we don't have to account for a potential energy difference (per unit volume) between water flowing at the top of a pipe compared to the bottom, but for ease of viewing we do not draw the *picture* that way.

Work-Mechanical Energy in Fluids: Bernoulli's Equation

The fluid is incompressible and the pipe itself does not leak, so fluid cannot build up between the bottom and the top. As the fluid on the bottom moves to the left a distance d (which might be $v_1\Delta t$ but we don't insist on it as rates will not be important in our result) exactly the same amount fluid must move to the left a distance *D* up at the top so that fluid is conserved.

The total *mechanical* consequence of this movement is thus the disappearance of a chunk of fluid mass:

$$\Delta m = \rho \Delta V = \rho A_1 d = \rho A_2 D$$

that is moving at speed v_1 and at height y_1 at the bottom and it's appearance moving at speed v_2 and at height y_2 at the top. Clearly ***both*** the kinetic energy ***and*** the potential energy of this chunk of mass have changed.

Work-Mechanical Energy in Fluids: Bernoulli's Equation

What caused this change in mechanical energy?

Well, it can only be work.

What does the work?

The walls of the (frictionless, drag free) pipe can do no work as the only force it exerts is perpendicular to the wall and hence to $\vec{v}$ in the fluid.

The only thing left is the ***pressure*** that acts on the entire block of water between the first surface (lightly shaded) drawn at both the top and the bottom as it moves forward to become the second surface (darkly shaded) drawn at the top and the bottom, effecting this net transfer of mass Δm.

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Work-Mechanical Energy in Fluids: Bernoulli's Equation

The force F_1 exerted to the right on this block of fluid at the bottom is just $F_1 = P_1A_1$; the force F_2 exerted to the left on this block of fluid at the top is similarly $F_2 = P_2A_2$. The work done by the pressure acting over a distance d at the bottom is $W_1 = P_1A_1d$, at the top it is $W_2 = -P_2A_2D$. The total work is equal to the total change in mechanical energy of the chunk Δm:

$$W_1 + W_2 = E_{mech}(final) - E_{mech}(initial) W_{tot} = \Delta E_{mech}$$

$$P_1A_1d - P_2A_2D = \left(\frac{1}{2}mv_2^2 + \Delta mgy_2\right) - \left(\frac{1}{2}mv_1^2 + \Delta mgy_1\right)$$

$$(P_1 - P_2)\Delta V = \left(\frac{1}{2}\rho\Delta V v_2^2 + \rho\Delta V gy_2\right) - \left(\frac{1}{2}\rho\Delta V v_1^2 + \rho\Delta V gy_1\right)$$

$$(P_1 - P_2) = \left(\frac{1}{2}\rho v_2^2 + \rho gy_2\right) - \left(\frac{1}{2}\rho v_1^2 + \rho gy_1\right)$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2 = \text{a constant (units of pressure)}$$

This result is known as ***Bernoulli's Principle***

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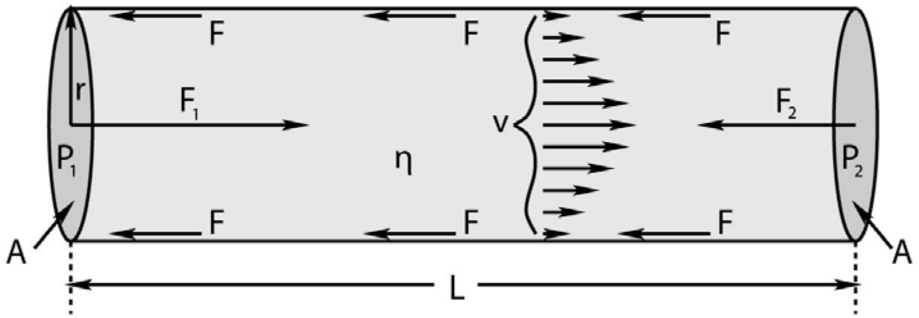
Fluid Viscosity and Resistance

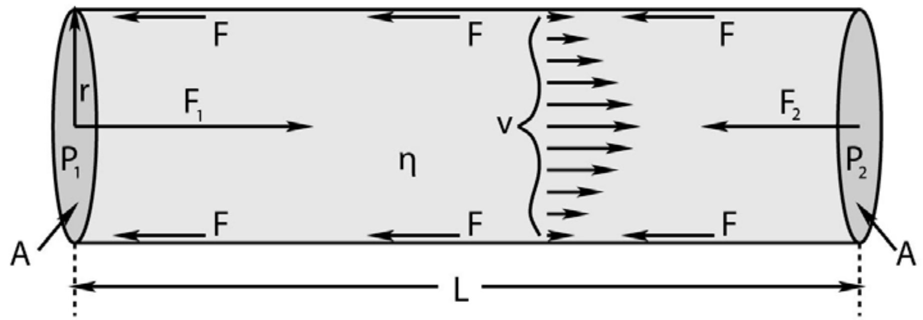
In the discussion above, we have consistently ignored viscosity and drag, which behave like “friction”, exerting a force parallel to the confining walls of the pipe in the opposite direction to the relative motion of fluid and pipe.



In figure a circular pipe is carrying a fluid with viscosity μ from left to right ***at a constant speed***. Once again, this is a sort of dynamic equilibrium; the net force on the fluid in the pipe segment shown must be zero for the speed of the fluid through it to be maintained unabated during the flow.

The fluid is in contact with and interacts with the walls of the pipe, creating a thin layer of fluid at least a few atoms thick that are “at rest”, stuck to the pipe. As fluid is pushed through the pipe, this layer at rest interacts with and exerts an ***opposing force*** on the layer moving just above it via the viscosity of the fluid. This layer in turn interacts with and slows the layer above it and so on right up to the center of the pipe, where the fluid flows most rapidly.

Fluid Viscosity and Resistance



The interaction of the surface layer with the fluid, redistributed to the whole fluid via the viscosity, exerts a net ***opposing force*** on the fluid as it moves through the pipe. In order for the average speed of the fluid to continue, an outside force must act on it with an equal and opposite force. The only available source of this force in the figure is obviously the ***fluid pressure***; if it is larger on the left than on the right (as shown) it will exert a net force on the fluid in between that can balance the drag force exerted by the walls.

The forces at the ends are $F_1 = P_1A$, $F_2 = P_2A$. The net force acting on the fluid mass is thus:

$$\Delta F = F_1 - F_2 = (P_1 - P_2)A$$

All things being equal, we expect the flow rate to increase linearly with v, and for *laminar* flow, the drag force is proportional to v. Therefore we expect that:

$$\Delta F = F_d \propto v \propto I \text{ (the flow)}$$

Fluid Viscosity and Resistance

We can then divide out the area and write:

$$\Delta P \propto \frac{I}{A}$$

We cannot derive the constant of proportionality in this expression, and we will omit some math and just write following result:

$$\Delta P = I\left(\frac{8L\mu}{\pi r^4}\right) = IR$$

where *I* have introduced the resistance of the pipe to flow:

$$R = \frac{8L\mu}{\pi r^4}$$

This equation is know as ***Poiseuille's Law*** and is a key relation for physicians and plumbers to know because it describes both flow of water in pipes and the flow of blood in blood vessels wherever the flow is slow enough that it is laminar and not turbulent

A Brief Note on Turbulence

The velocity of the flow in a circular pipe (and other parameters such as μ and r) can be transformed into a general dimensionless parameter called the ***Reynolds Number (Re)***.

The Reynolds number for a circular pipe is:

$$Re = \frac{\rho v D}{\mu} = \frac{\rho v 2r}{\mu}$$

where $D = 2r$ is the ***hydraulic diameter***, which in the case of a circular pipe is the actual diameter.

The one thing the Reynolds number does for us is that it serves as a ***marker for the transition to turbulent flow***.

For $Re < 2300$ flow in a circular pipe is laminar and all of the relations above hold.

Turbulent flow occurs for Re > 4000. In between is the region known as the ***onset of turbulence***, where the ***resistance of the pipe depends on flow in a very nonlinear fashion***, and among other things *dramatically increases* with the Reynolds number.

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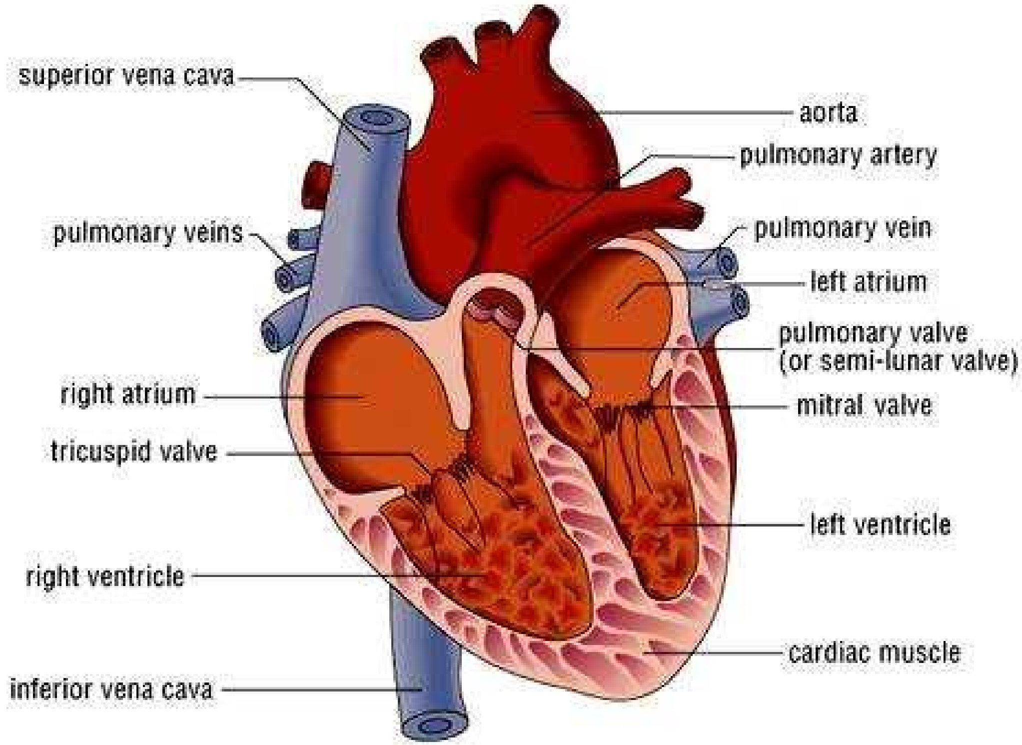
The Human Circulatory System



Here is a list of True Facts about the human cardiovascular system:

- The heart, illustrated in the schematic in figure is the "***pump***" that drives blood through your blood vessels.

The Human Circulatory System

- **The blood vessels are differentiated into three distinct types:**

  - ***Arteries***, which lead strictly ***away from the heart*** and which contain a muscular layer that elastically dilates and contracts the arteries in a synchronous way to help carry the surging waves of blood. This acts as a "shock absorber" and hence reduces the peak systolic blood pressure. Arteries split up the farther one is from the heart, eventually becoming ***arterioles***, the very small arteries that actually split off into capillaries.

  - ***Capillaries***, which are a dense network of very fine vessels (often only a single cell thick) that ***deliver oxygenated blood throughout all living tissue*** so that the oxygen can disassociate from the carrying hemoglobin molecules and diffuse into the surrounding cells in systemic circulation, or ***permit the oxygenation of blood*** in pulmonary circulation.

  - **Veins**, which lead strictly ***back to the heart*** from the capillaries. Veins also have a muscle layer that expand or contract to aid in thermoregulation and regulation of blood pressure as one lies down or stands up. Veins also provide "capacitance" to the circulatory system and store the body's "spare" blood; 60% of the body's total blood supply is usually in the veins at any one time. Most of the veins, especially long vertical veins, are equipped with one-way ***venous valves*** every 4-9 cm that prevent backflow and pooling in the lower body during e.g. diastoli.

    Blood from the capillaries is collected first in ***venules*** (the return-side equivalent of arterioles) and then into veins proper.

The Human Circulatory System

- There are two distinct circulatory systems in humans (and in the rest of the mammals and birds):
  - ***Systemic circulation***, where oxygenated blood enters the heart via pulmonary veins *from* the lungs and is pumped at high pressure *into* systemic arteries that deliver it through the capillaries and (deoxygenated) back via systemic veins to the heart.
  - ***Pulmonary circulation***, where deoxgenated blood that has returned *from* the system circulation is pumped *into* pulmonary arteries that deliver it to the lungs, where it is oxygenated and returned to the heart by means of pulmonary veins. These two distinct circulations ***do not mix*** and together, ***form a closed double circulation loop***.

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The Human Circulatory System

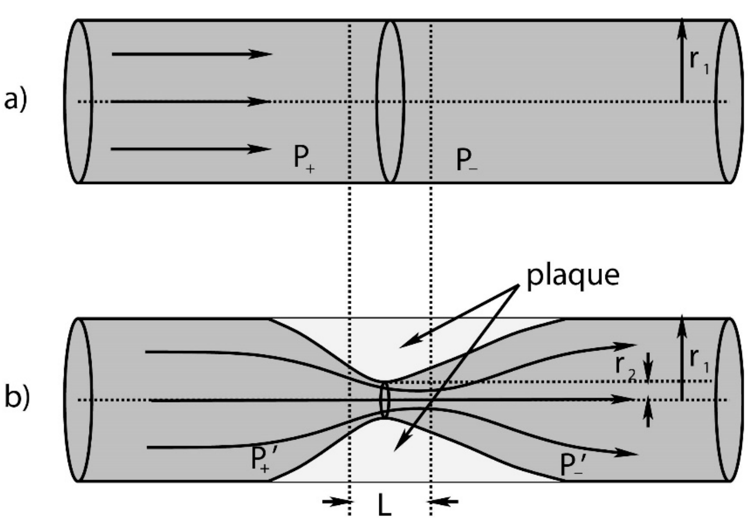
- ***Blood pressure*** is generally measured and reported in terms of two numbers:
  - ***Systolic*** blood pressure. This is the ***peak/maximum arterial pressure*** in the wave pulse generated that drives ***systemic circulation***. It is measured in the (brachial artery of the) arm, where it is supposed to be a reasonably accurate reflection of peak aortic pressure just outside of the heart, where, sadly, it cannot easily be directly measured without resorting to invasive methods that are, in fact, used e.g. during surgery.
  - ***Diastolic*** blood pressure. This is the ***trough/minimum arterial pressure*** in the wave pulse of systemic circulation.

"Normal" Systolic systemic blood pressure can fairly accurately be estimated on the basis of the distance between the heart and the feet; a distance on the order of 1.5 meters leads to a pressure difference of around 0.15 atm or 120 mmHg.

Blood is driven through the relatively high resistance of the capillaries by the ***difference*** in arterial pressure and venous pressure. The venous system is entirely a ***low pressure return***; its peak pressure is typically order of 0.008 bar (6 mmHg). This can be understood and predicted by the mean distance between valves in the venous system – the pressure difference between one valve and another (say) 8 cm higher is approximately $\rho_b g \times 0.08 \approx= 0.008$ bar. However, this pressure is not really static – it varies with the delayed pressure wave that causes blood to surge its way up, down, or sideways through the veins on its way back to the atria of the heart.

Atherosclerotic Plaque Partially Occludes a Blood Vessel

Atherosclerosis – granular deposits of fatty material called ***plaques*** that attach to the walls of e.g. arteries and gradually thicken over time, generally associated with high blood cholesterol and lipidemia. The risk factors for atherosclerosis form a list as long as your arm and its fundamental causes are not well understood, although they are currently believed to form as an inflammatory response to surplus low density lipoproteins (one kind of cholesterol) in the blood.



In figure two arteries are illustrated.

Artery a) is "clean", has a radius of r_1, and (from the Poiseuille Equation above) has a very low resistance to any given flow of blood. Because R_a over the length L is low, there is very little pressure drop between P_+ and P_- on the two sides of any given stretch of length L. The velocity profile of the fluid is also more or less uniform in the artery, slowing a bit near the walls but generally moving smoothly throughout the entire cross-section.

Artery b) has a significant deposit of atherosclerotic plaques that have coated the walls and reduced the effective radius of the vessel to $\sim r_2$ over an extended length L. The vessel is perhaps 90% occluded – only 10% of its normal cross-sectional area is available to carry fluid.

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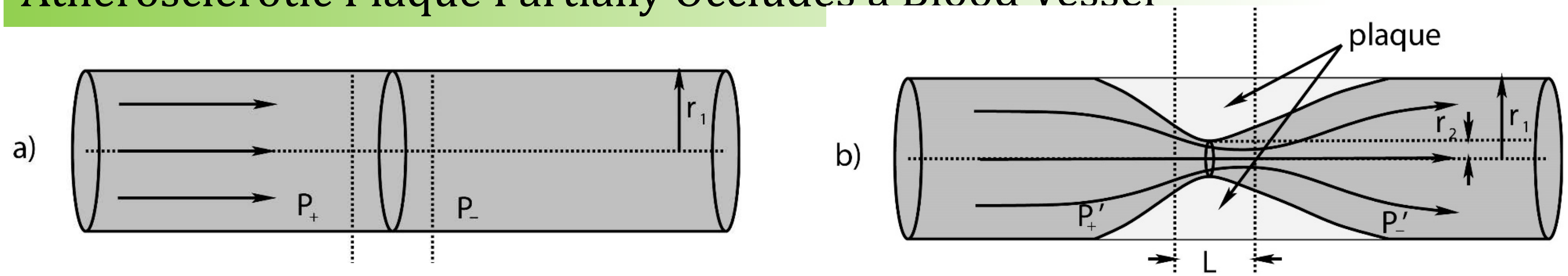
Atherosclerotic Plaque Partially Occludes a Blood Vessel



We can now easily understand several things about this situation. First, if the total *flow* in artery b) is still being maintained at close to the levels of the flow in artery a) (so that tissue being oxygenated by blood delivered by this artery is not being critically starved for oxygen yet) the ***fluid velocity in the narrowed region is ten times higher than normal!*** Since the Reynolds number for blood flowing in primary arteries is normally around 1000 to 2000, increasing v by a factor of 10 increases the Reynolds number by a factor of 10, causing the flow to become *turbulent* in the obstruction. This tendency is even more pronounced than this figure suggests – I've drawn a nice symmetric occlusion, but the atheroma (lesion) is more likely to grow predominantly on one side and irregular lesions are more likely to disturb laminar flow even for smaller Reynolds numbers.

This turbulence provides the basis for one method of possible detection and diagnosis – you can *hear* the turbulence (with luck) through the stethoscope during a physical exam. Physicians get a lot of practice listening for turbulence since turbulence produced by *artificially* restricting blood flow in the brachial artery by means of a constricting cuff is basically what one listens for when taking a patient's blood pressure. It really shouldn't be there, especially during diastole, the rest of the time.