Hindawi Publishing Corporation
Journal of Complex Analysis
Volume 2016, Article ID 4367205, 8 pages
http://dx.doi.org/10.1155/2016/4367205

Research Article

Approximate Conformal Mappings and Elasticity Theory

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Received 4 June 2016; Revised 25 July 2016; Accepted 3 August 2016

Academic Editor: Vladislav Kravchenko

Here, we present the new method of approximate conformal mapping of the unit disk to a one-connected domain with smooth boundary without auxiliary constructions and iterations. The mapping function is a Taylor polynomial. The method is applicable to elasticity problems solution.

1. Introduction

Conformal mappings play an important part in solution of elasticity theory problems if we apply to them complex variable theory. These investigations were started by Muskhelishvili [1]. Muskhelishvili considered multiple solution methods of certain plane problems in the cited monograph. In particular, he gave the solution technique for basic plane elasticity theory problems and that for the rods torsion. In order to do this, the author made an extensive use of conformal mappings from the unit disk to a given domain. The easiest solution appears in the case of the polynomial mapping.

Computer progress stimulated appearance of many numerical conformal mapping construction methods. Many of these methods were connected with the integral equation solutions. If we want to map a given simply connected domain to the disk, then we solve a linear integral equation either analytically or numerically (see, e.g., [2–5]). Note that if we want to map the disk to the given simply connected domain, then this problem solution turns out to be significantly harder and is traditionally reduced to nonlinear Theodorsen equation solution. The effective Wegmann method of this equation solution is based on iteration processes [6, 7].

The approximate conformal mapping of the unit disk to the given domain construction method presented here has the following advantages: it does not use any auxiliary constructions (triangulation, circle packing) or additional conformal mappings (zipper algorithm), it does not use accessory solutions of boundary value problems (conjugate function method, Wegmann method), and it even does not use iterations as Wegmann method or Fornberg method where the preimages of the given-on-the-unit circle points move along the given curve [8]. We solve the integral Fredholm equation. This equation is well known and allows us to construct the conformal mapping of the given domain onto the unit disk [9]. But we use this equation in order to find the monotone function that determines the necessary reparametrization of the given boundary. Moreover, our method provides us with the smooth solution in the form of Taylor polynomial. So it is possible to find the derivatives of the solutions. Also the method is connected with the domain boundary curve reparametrization described in [10].

The auxiliary function involved in the function inverse to the reparametrizing one can be found by integral equation solution. This solution is reduced to solution of the infinite system whose truncated form is regulated by two different polynomial coefficients calculation methods. If the system size is reasonable, then both formulas lead to close values of the desired coefficients. We present the example of nonconvex domain and construct the approximate conformal mapping of the unit disk onto this domain with the help of the boundary curve reparametrization.

The example of the unit disk mapping onto the hypotrochoid interior leads to solution of plane elasticity problems. With the help of the constructed approximate mapping functions, we analytically find the boundary shear stresses and draw the corresponding graphs.