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Representation of tripotents and representations via tripotents

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ABSTRACT

Let $\mathcal{A}$ be an algebra. An element $A \in \mathcal{A}$ is called tripotent if $A^3 = A$. We study the questions: if both A and B are tripotents, then: Under what conditions are $A + B$ and AB tripotent? Under what conditions do A and B commute? We extend the partial order from the Hilbert space idempotents to the set of all tripotents and show that every normal tripotent is self-adjoint. For $\mathcal{A} = \mathbb{M}_n(\mathbb{C})$ we describe the set of all finite sums of tripotents, the convex hull of tripotents and the set of all tripotents averages. We also give the new proof of rational trace matrix representations by Choi and Wu [2].

1. Introduction

Let $\mathcal{A}$, $\mathcal{D}$ be algebras. An element $A \in \mathcal{A}$ is called *idempotent* if $A^2 = A$; and *tripotent* if $A^3 = A$. Let

$$\mathcal{A}^{\mathrm{id}} = \{A \in \mathcal{A} : A^2 = A\}, \quad \mathcal{A}^{\mathrm{tr}} = \{A \in \mathcal{A} : A^3 = A\}.$$

Tripotent matrices have values in applications to digital image encryption [17].

We study the following questions: if both A and B are tripotents, then: Under what conditions are $A + B$ and AB tripotent? Under what conditions do A and B commute? We decompose any tripotent

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