Born out of the need to formalize common sense in Artificial Intelligence (AI), Argumentation Logic (AL) brings together the syllogistic roots of logic with recent argumentation theory [1] from AI to propose a new logic based on argumentation.

Argumentation Logic is purely proof theoretic defined via a criterium of *acceptability of arguments* [3]. Arguments in AL are sets of propositional formulae with the acceptability of an argument ensuring that the argument can defend against any other argument that is inconsistent with it, under a given propositional theory. AL can be linked to Natural Deduction allowing us to reformulate Propositional Logic (PL) in terms of argumentation and to show that, under certain conditions, AL and PL are equivalent. AL separates proofs into direct and indirect ones, the latter being through the use of a restricted form of Reductio ad Absurdum (RAA) where the (direct) derivation of the inconsistency must depend on the hypothesis posed when we apply the RAA rule [4].

As such AL is able to isolate inconsistencies in the given theory and to behave agnostically to them. This gives AL as a conservative paraconsistent [5] extension of PL that does not trivialize in the presence of inconsistency. The logic then captures in a single framework defeasible reasoning and its synthesis with the strict form of reasoning in classical logic. The interpretation of implication in AL is different from that of material implication, closer to that of default rules but where proof by contradiction can be applied with them. AL has recently formed the basis to formalize psychological theories of story comprehension [2].

[1] T. J. M. Bench-Capon and P. E. Dunne, editors, *Argumentation in AI. **Special issue of the Journal of Artificial Intelligence***, vol. 171 (2007), no. 10–11.

[2] I. Diakidou, A. Kakas, L. Michael, and R. Miller, *A psychology-inspired approach to automated narrative text comprehension*, ***International Conference on Principles of Knowledge Representation and Reasoning*** (C. Barral and G. De Giacomo, editors), Vienna, Austria, 2014, to appear.

[3] Antonis Kakas and Paolo Mancarella, *On the semantics of abstract argumentation*, ***Journal of Logic and Computation***, vol. 23 (2013), pp. 991–1015.

[4] Antonis Kakas and Francesca Toni and Paolo Mancarella, *On Reductio ad Absurdum in Propositional Logic*, this Bulletin, submitted.

[5] G. Priest, B. Routley and J. Norman, ***Paraconsistent logic: Essays on the incosistent***, Philosophia Verlag, 1989.

▸ ISKANDER KALIMULLIN AND DAMIR ZAINETDINOV, *On limitwise monotonic reducibility of Σ^0_2-sets.*
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One of the directions of research in modern computability theory focus on studying properties of limitwise monotonic functions and limitwise monotonic sets.

I. Kalimullin and V. Puzarenko [2] introduced the concept of reducibility on families of subsets of natural numbers, which is consistent with Σ-definability on admissible sets. Let $\mathcal{F}_A$ denote the families of initial segments $\{\{x \mid x < n\} \mid n \in A\}$. Accordingly to [2], we define the notion of *limitwise monotonic reducibility* of sets as a Σ-reducibility of the corresponding initial segments, namely $A \leqslant_{lm} B \iff \mathcal{F}_A \leqslant_\Sigma \mathcal{F}_B$.

Let $A \equiv_{lm} B$ if $A \leqslant_{lm} B$ and $B \leqslant_{lm} A$. The *limitwise monotonic degree* (also called lm-degree) of A is $\deg(A) = \{B : B \equiv_{lm} A\}$. Let $\mathbf{S}_{lm}$ denote the class of all lm-degrees of Σ^0_2 sets. The degrees $\mathbf{S}_{lm}$ form a partially ordered set under the relation $\deg(A) \leqslant \deg(B)$ iff $A \leqslant_{lm} B$.

We prove the following theorems.

Theorem 1. *There exist infinite Σ^0_2-sets A and B such that $A \nleqslant_{lm} B$ and $B \nleqslant_{lm} A$.*

Theorem 2. *Every countable partial order can be embedded into $\mathbf{S}_{lm}$.*

Theorem 3 (jointly with M. Faizrahmanov). *There is no maximal element in $\mathbf{S}_{lm}$.*

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[1] I. Kalimullin, B. Khoussainov, and A. Melnikov, *Limitwise monotonic sequences and degree spectra of structures*, ***Proceedings of the American Mathematical Society***, vol. 141, no. 9, American Mathematical Society, 2013, pp. 3275–3289.

[2] I. Kalimullin and V. Puzarenko, *Reducibility on families*. ***Algebra and Logic***, vol. 48 (2009), no. 1, pp. 20–32.

[3] B. Khoussainov, A. Nies, and R. Shore, *Computable models of theories with few models*. ***Notre Dame Journal of Formal Logic***, vol. 38 (1997), no. 2, pp. 165–178.

► AHMAD KARIMI AND SAEED SALEHI, *A universal diagonal schema by fixed-points and Yablo's paradox*.

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In 1906, Russell [5] showed that all the known set-theoretic paradoxes (till then) had a common form. In 1969, Lawvere [3] used the language of category theory to achieve a deeper unification, embracing not only the set-theoretic paradoxes but incompleteness phenomena as well. To be precise, Lawvere gave a common form to Cantor's theorem about power sets, Russell's paradox, Tarski's theorem on the undefinability of truth, and Gödel's first incompleteness theorem. In 2003, Yanofsky [7] extended Lawvere's ideas using straightforward set-theoretic language and proposed a universal schema for diagonalization based on Cantor's theorem. In this universal schema for diagonalization, the existence of a certain (diagonalized-out and contradictory) object implies the existence of a fixed-point for a certain function. He showed how self-referential paradoxes, incompleteness, and fixed-point theorems all emerge from the single generalized form of Cantor's theorem. Yanofsky extended Lawvere's analysis to include the Liar paradox, the paradoxes of Grelling and Richard, Turing's halting problem, an oracle version of the P=?NP problem, time travel paradoxes, Parikh sentences, Löb's Paradox and Rice's theorem. In this talk, we fit more theorems in the universal schema of diagonalization, such as Euclid's theorem on the infinitude of the primes, and new proofs of Boolos [1] for Cantor's theorem on the nonequinumerosity of a set with its powerset. We also show the existence of Ackermann-like functions (which dominate a given set of functions such as primitive recursive functions) using the schema. Furthermore, we formalize a reading of Yablo's paradox [6], the most challenging paradox in the recent years, in the framework of Linear Temporal Logic (LTL [2]) and the diagonal schema, and show how Yablo's paradox involves circularity by presenting it in the framework of LTL. All in all, we turn Yablo's paradox into a genuine mathematico-logical theorem. This is the first time that Yablo's paradox becomes a (new) theorem in mathematics and logic. We also show that Priest's [4] inclosure schema can fit in our universal diagonal/fixed-point schema. The inclosure schema was used by Priest for arguing for the self-referentiality of Yablo's sequence of sentences, in which no sentence directly refers to itself but the whole sequence does so.

[1] George Boolos, *Constructing Cantorian counterexamples*. ***Journal of Philosophical Logic***, vol. 26 (1997), no. 3, pp. 237–239.

[2] Fred Kröger and Stephan Merz, ***Temporal logic and state systems***, EATCS Texts in Theoretical Computer Science, Springer, 2008.

[3] F. William Lawvere, *Diagonal Arguments and Cartesian Closed Categories*, ***Category theory, homology theory and their applications II*** (Seattle Research Center, Battelle Memorial Institute), Lecture Notes in Mathematics 92, Springer, Berlin, 1969, pp. 134–145.

[4] Graham Priest, *Yablo's paradox*. ***Analysis***, vol. 57 (1997), no. 4, pp. 236–242.

[5] Bertrand Russell, *On some difficulties in the theory of transfinite numbers and order types*, ***Proceedings of the London Mathematical Society***, Second Series, vol. 4 (1907), no. 1, pp. 29–53.