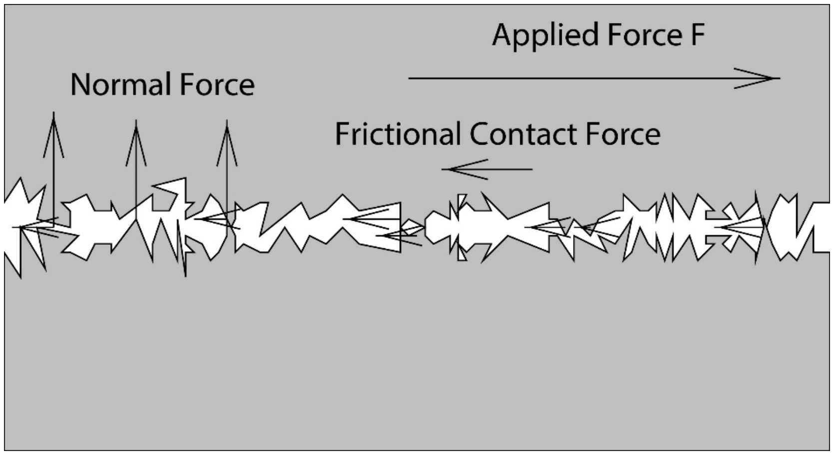
Lecture 2. Newton's Laws

Friction



Static Friction is the force exerted by one surface on another that acts *parallel* to the surfaces to *prevent the two surfaces from sliding*.

Static friction is as large as it needs to be to prevent any sliding motion, up to a maximum value, at which point the surfaces begin to slide.

The maximum force static friction can exert is proportional to both the pressure between the surfaces and the area in contact. This makes it proportional to the product of the pressure and the area, which equals the normal force. We write this as:

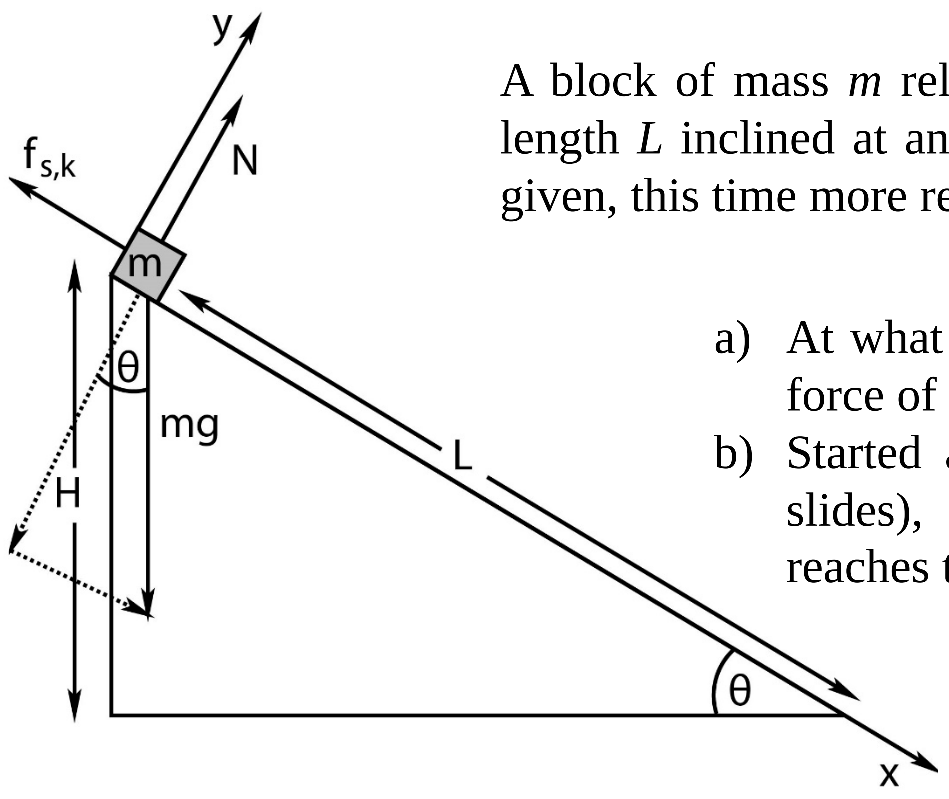
$$f_s \leq f_s^{\ max} = \mu_s N$$

where μ_s is *the coefficient of static friction*, a dimensionless constant characteristic of the two surfaces in contact, and N is the normal force.

The frictional force will depend only on the total force, not the area or pressure separately:

$$f_k = \mu_k P * A = \mu_k \frac{N}{A} * A = \mu_k N$$

Inclined Plane of Length L with Friction



A block of mass m released from rest at time t = 0 on a plane of length L inclined at an angle θ relative to horizontal is once again given, this time more realistically, including the effects of *friction*.

a) At what angle θ_c does the block *barely* overcome the force of static friction and slide down the incline?

b) Started at rest from an angle $\theta > \theta_c$ (so it definitely slides), how fast will the block be going when it reaches the bottom?

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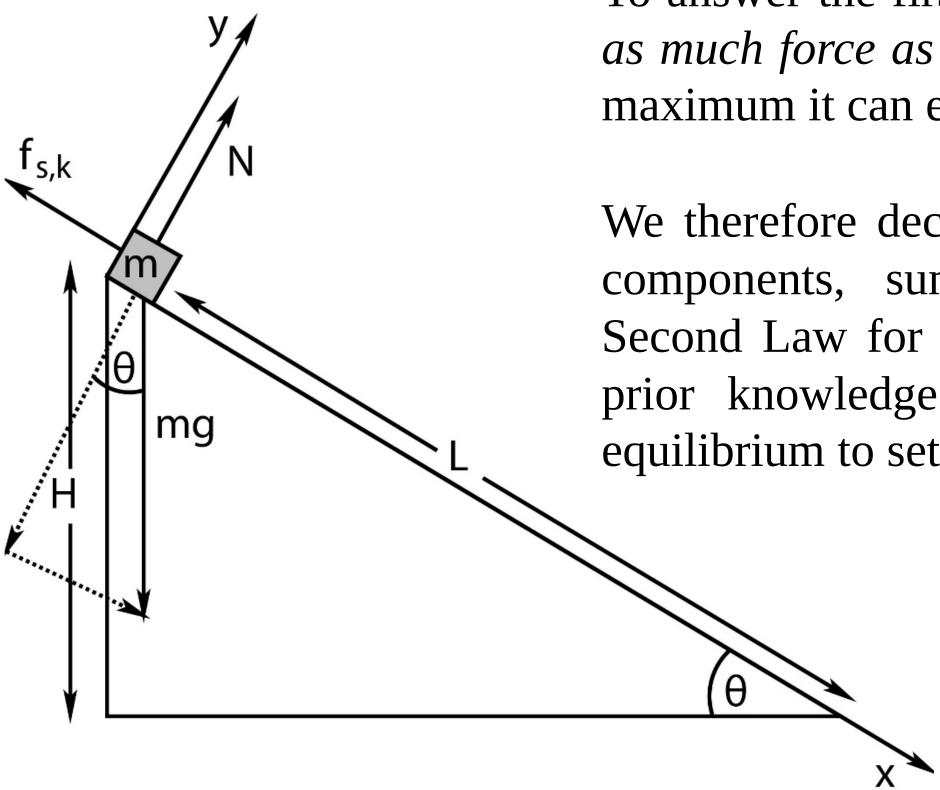
Inclined Plane of Length L with Friction



To answer the first question, we note that static friction exerts *as much force as necessary* to keep the block at rest up to the maximum it can exert, $f_s^{max} = \mu_s N$.

We therefore decompose the known force rules into x and y components, sum them componentwise, write Newton's Second Law for both vector components and finally use our prior knowledge that the system remains in static force equilibrium to set $a_x = a_y = 0$. We get:

$$\sum F_x = mg \sin\theta - f_s = 0$$

(for $\theta \leq \theta_c$ and $v(0) = 0$) and

$$\sum F_y = N - mg \cos\theta = 0$$

So far, f_s is precisely what it needs to be to prevent motion: $f_s = mg \sin\theta$
while N $= mg \cos\theta$. It is true at any angle, moving or not moving, from the F_y equation

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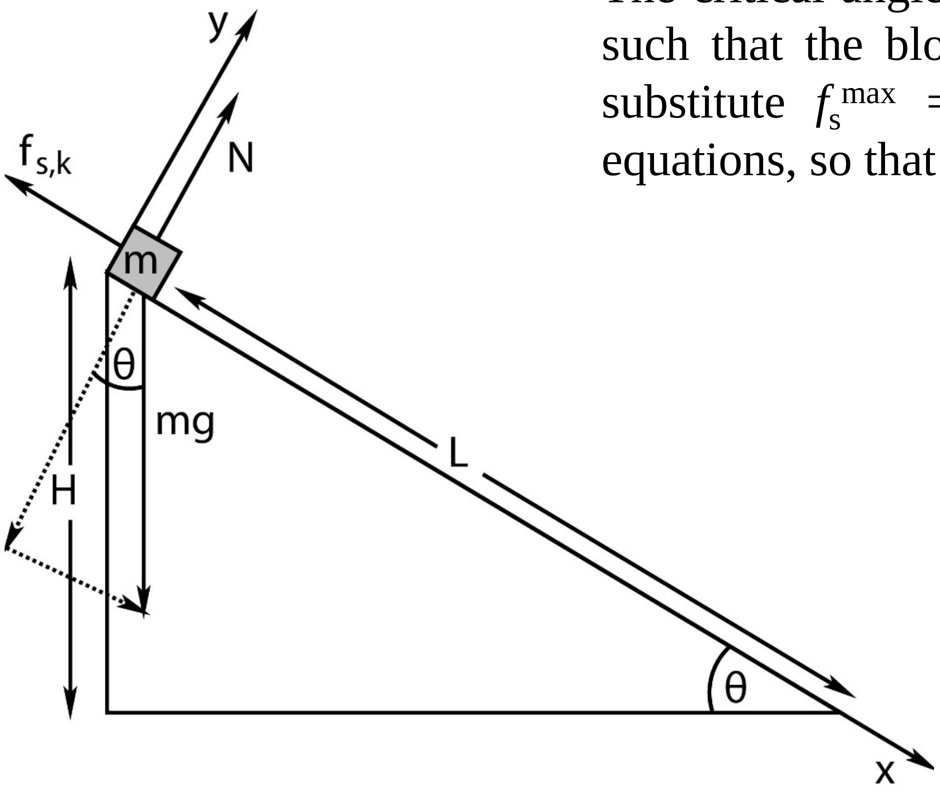
Inclined Plane of Length L with Friction



The critical angle is the angle where f_s is as large as it can be such that the block barely doesn't slide. To find it, we can substitute $f_s^{max} = \mu_s N_c$ where $N_c = mg\cos(\theta_c)$ into both equations, so that the first equation becomes:

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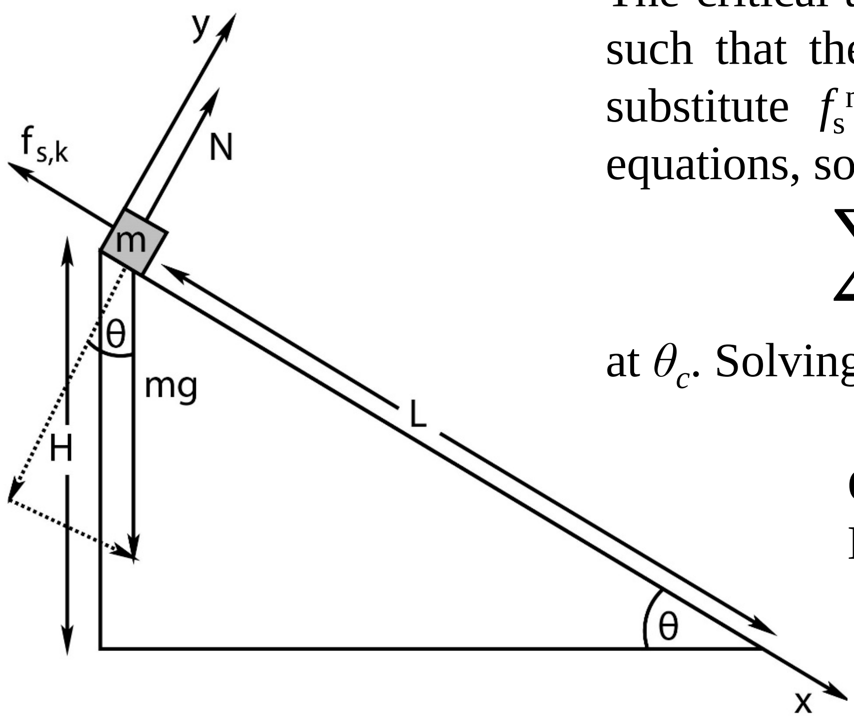
Inclined Plane of Length L with Friction



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$$\sum F_x = mg\sin\theta_c - \mu_s mg\cos\theta_c = 0$$

at θ_c. Solving for θ_c, we get: $\theta_c = \tan^{-1}(\mu_s)$

Once it is moving then the block will accelerate and Newton's Second Law becomes:

$$\sum F_x = mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

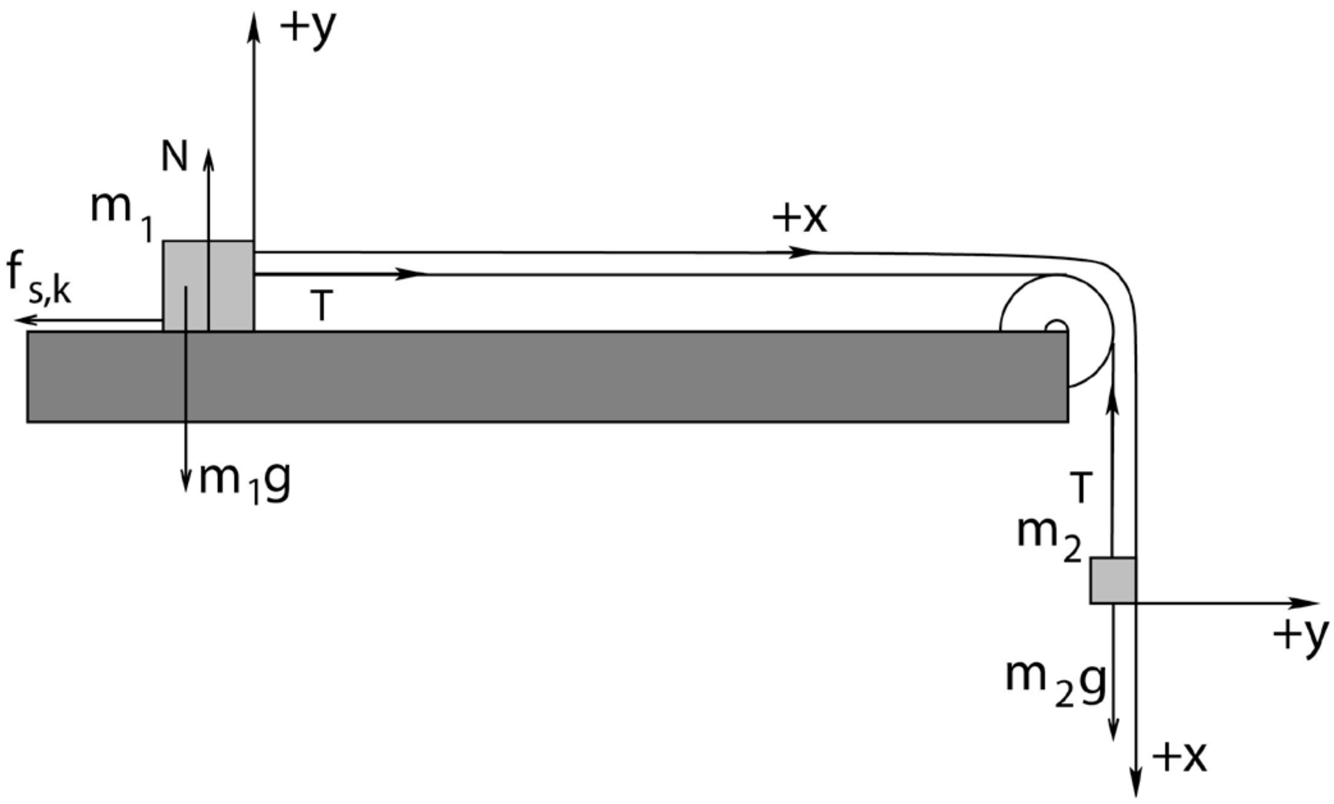
which we can solve for the constant acceleration of the block down the incline:

$$a_x = g\sin\theta - \mu_k g\cos\theta = g(\sin\theta) - \mu_k\cos\theta)$$

Given a_x, it is now straightforward to answer the second question above. For example, we can integrate twice and find $v_x(t)$ and x(t), use the latter to find the time it takes to reach the bottom, and substitute that time into the former to find the speed at the bottom of the incline.

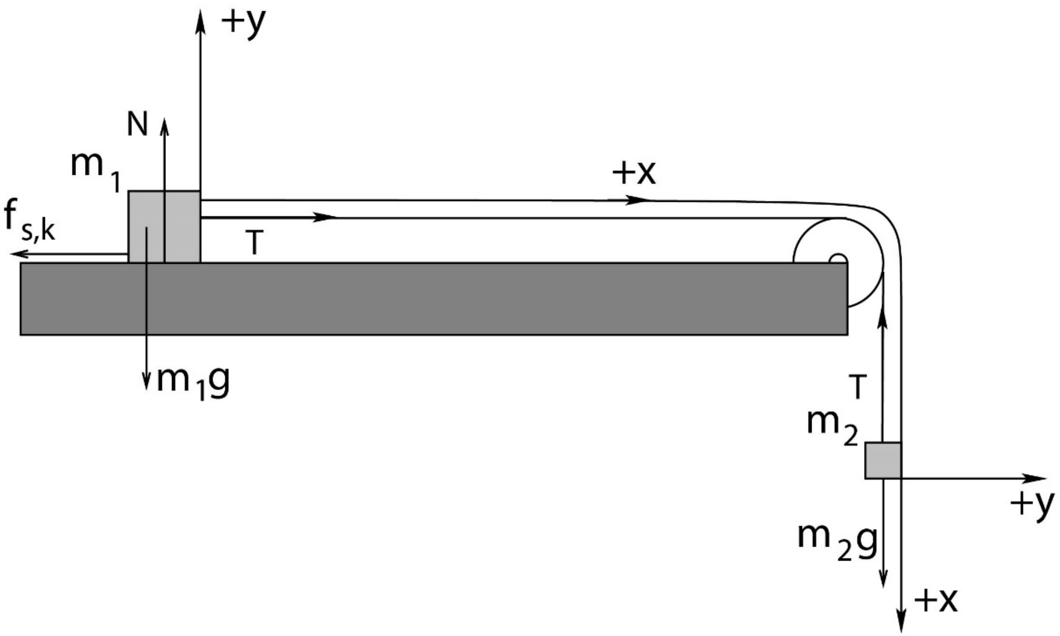
Lecture 2. Newton's Laws

Block Hanging off of a Table



Atwood's machine, sort of, with one block resting on a table with friction and the other dangling over the side being pulled down by gravity near the Earth's surface. Note that we should use an "around the corner" coordinate system as shown, since a1 = a2 = a if the string is unstretchable.

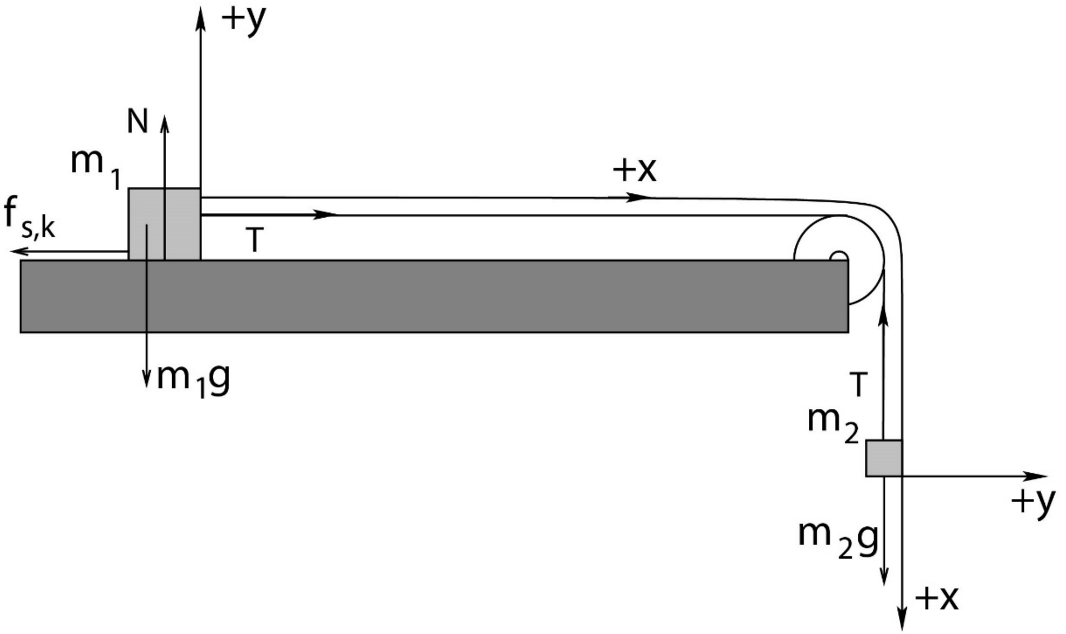
Block Hanging off of a Table



Suppose a block of mass m_1 sits on a table. The coefficients of static and kinetic friction between the block and the table are $\mu_s > \mu_k$ and μ_k respectively. This block is attached by an “ideal” massless unstretchable string running over an “ideal” massless frictionless pulley to a block of mass m_2 hanging off of the table. The blocks are released from rest at time $t = 0$.

What is the largest that m_2 can be before the system starts to move, in terms of the givens and knowns (m_1, g, μ_k, μ_s...)?

Block Hanging off of a Table



Static force equilibrium ($a_x = a_y = 0$):

$$\sum F_{x1} = T - f_s = 0$$

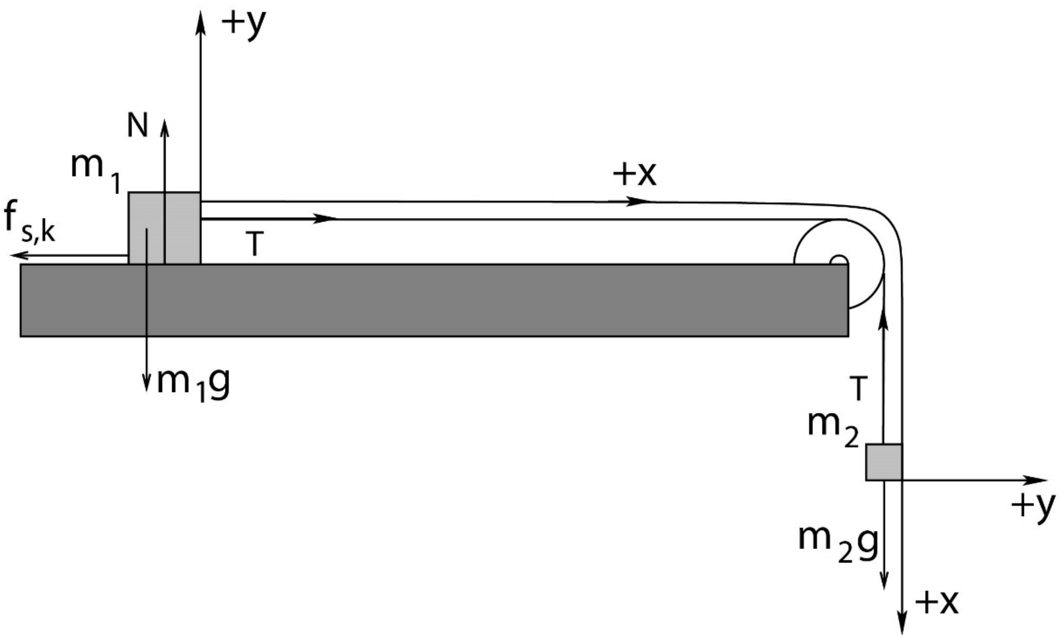
$$\sum F_{y1} = N - m_1 g = 0$$

$$\sum F_{x2} = m_2 g - T = 0$$

$$\sum F_{y2} = 0$$

From the second equation, $N = m_1 g$. At the point where m_2 is the largest it can be (given m_1 and so on) $f_s = f_s^{max} = \mu_s N = \mu_s m_1 g$. If we substitute this in and add the two x equations, the T cancels and we get: $m_2^{max} g - \mu_s m_1 g = 0$ Thus: $m_2^{max} = \mu_s m_1$

Block Hanging off of a Table



If m_2 is larger than this minimum, so m_1 will slide to the right as m_2 falls. We will have to solve Newton's Second Law for both masses in order to obtain the non-zero acceleration to the right and down, respectively:

$$\sum F_{x1} = T - f_k = m_1 a$$

$$\sum F_{y1} = N - m_1 g = 0$$

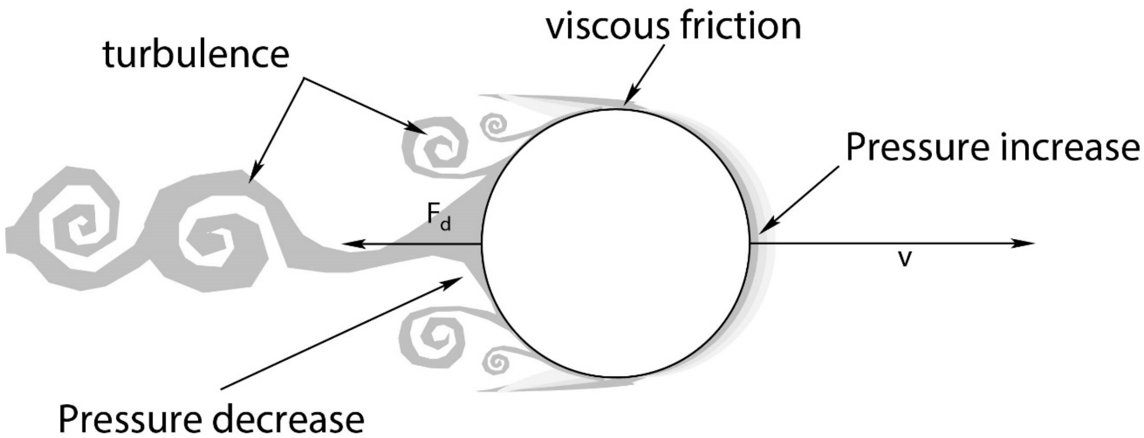
$$\sum F_{x2} = m_2 g - T = m_2 a$$

$$\sum F_{y2} = 0$$

If we substitute the fixed value for $f_k = \mu_k N = \mu_k m_1 g$ and then add the two x equations once again (using the fact that both masses have the same acceleration because the string is unstretchable as noted in our original construction of round-the-corner coordinates), the tension T cancels and we get:

$m_2 g - \mu_s m_1 g = (m_1 + m_2)a$ or $a = \frac{m_2 g - \mu_s m_1 g}{(m_1 + m_2)}$

Drag Forces



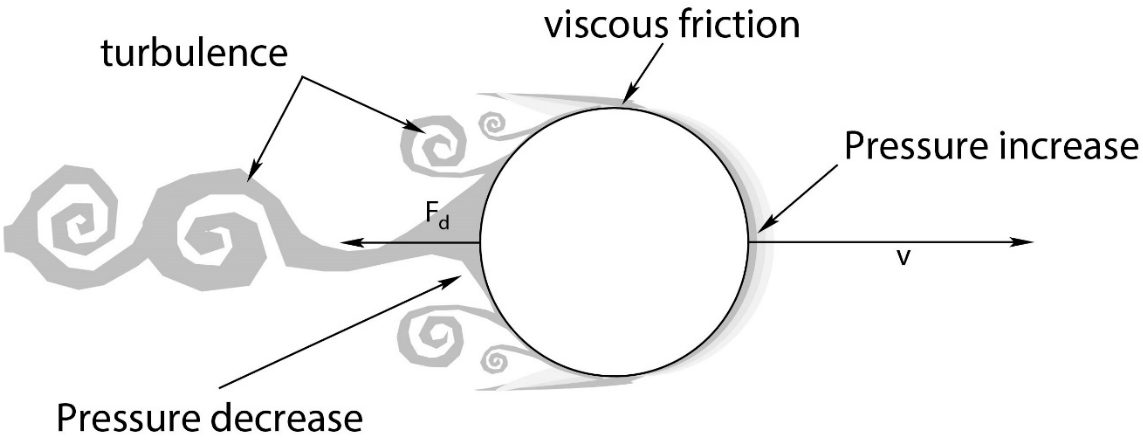
A "cartoon" illustrating the differential force on an object moving through a fluid.

When the object is moving with respect to the fluid then we empirically observe that a friction-like force is exerted on the object called **drag.**

Drag Force is the "frictional" force exerted by a fluid (liquid or gas) on an object that moves through it. Like kinetic friction, it always opposes the direction of *relative* motion of the object and the medium

Note well: When an object is enlongated and passes through a fluid parallel to its long axis with a comparatively small forward-facing cross section compared to its total area, we say that it is a streamlined object as the fluid tends to pass over it in laminar flow. A streamlined object will often have its total drag dominated by skin friction. A bluff object, in contrast has a comparatively large cross-sectional surface facing forward and will usually have the total drag dominated by form drag.

Drag Forces



Note well:

When an object is enlongated and passes through a fluid parallel to its long axis with a comparatively small forward-facing cross section compared to its total area, we say that it is a ***streamlined object*** as the fluid tends to pass over it in laminar flow. A streamlined object will often have its total drag dominated by skin friction.

A ***bluff object***, in contrast has a comparatively large cross-sectional surface facing forward and will usually have the total drag dominated by form drag.

Drag Forces

Drag is an extremely complicated force. It depends on a vast array of things including but not limited to:

- The size of the object.
- The shape of the object.
- The relative velocity of the object through the fluid.
- The state of the fluid (e.g. its velocity field including any internal turbulence).
- The density of the fluid.
- The viscosity of the fluid (we will learn what this is later).
- The properties and chemistry of the surface of the object (smooth versus rough, strong or weak chemical interaction with the fluid at the molecular level).
- The orientation of the object as it moves through the fluid, which may be fixed in time (streamlined versus bluff motion) or varying in time (as, for example, an irregularly shaped object tumbles).

To eliminate most of this complexity and end up with “force rules” that will often be quantitatively predictive we will use a number of idealizations. We will only consider smooth, uniform, nonreactive surfaces of convex bluff objects (like spheres) or streamlined objects (like rockets or arrows) moving through uniform, stationary fluids where we can ignore or treat separately the other non-drag (e.g. buoyant) forces acting on the object.

Drag Forces

There are two dominant contributions to drag for objects of this sort.

The first, as noted above, is ***form drag*** – the difference in pressure times projective area between the front of an object and the rear of an object. It is strongly dependent on both the shape and orientation of an object and requires at least some turbulence in the trailing wake in order to occur.

The second is ***skin friction***, the friction-like force resulting from the fluid rubbing across the skin at right angles in laminar flow.

Stokes, or Laminar Drag

The first is when the object is moving through the fluid relatively slowly and/or is arrow-shaped or rocket-ship-shaped so that streamlined **laminar** drag (skin friction) is dominant. In this case there is relatively little form drag, and in particular, there is little or no **turbulence** – eddies of fluid spinning around an axis – in the wake of the object as the presence of turbulence (which we will discuss in more detail later when we consider fluid dynamics) breaks up laminar flow.

This "low-velocity, streamlined" skin friction drag is technically named **Stokes' drag** or laminar drag and has the idealized force rule:

$$\vec{F}_d = -b\vec{v}$$

This is the simplest sort of drag – a drag force ***directly proportional to the velocity of relative motion of the object through the fluid and oppositely directed***.

Stokes derived the following relation for the dimensioned number b_l (the laminar drag coefficient) that appears in this equation for a sphere of radius R:

$$b_l = -6\pi\mu R$$

where $\boldsymbol{\mu}$ is the dynamical viscosity.

Rayleigh, or Turbulent Drag

On the other hand, if one moves an object through a fluid *too* fast – where the actual speed depends in detail on the actual size and shape of the object, how bluff or streamlined it is – pressure builds up on the leading surface and ***turbulence*** appears in its trailing wake in the fluid.

This high velocity, ***turbulent drag*** exerts a force described by a quadratic dependence on the relative velocity due to Lord Rayleigh:

$$\vec{F}_d = -\frac{1}{2}\rho C_d A|v|\vec{v} = -b_t|v|\vec{v}$$

It is still ***directed opposite to the relative velocity of the object and the fluid*** but now is proportional to that velocity ***squared***. In this formula ρ is the density of the fluid through which the object moves (so denser fluids exert more drag as one would expect) and A is the cross-sectional area of the object perpendicular to the direction of motion, also known as the orthographic projection of the object on any plane perpendicular to the motion. For example, for a sphere of radius R, the orthographic projection is a circle of radius R and the area $A = \pi R^2$.

The number C_d is called the drag coefficient and is a dimensionless number that depends on relative speed, flow direction, object position, object size, fluid viscosity and fluid density.

Example: Falling From a Plane and Surviving

Suppose you fall from a large height (long enough to reach terminal velocity) to hit a haystack of height H that exerts a nice, uniform force to slow you down all the way to the ground, smoothly compressing under you as you fall. In that case, your initial velocity at the top is v_t, down. In order to stop you before $y = 0$ (the ground) you have to have a net acceleration $-a$ such that:

$$v(t_g) = 0 = v_t - at_g$$

$$y(t_g) = 0 = H - v_t t_g - \frac{1}{2}a{t_g}^2$$

If we solve the first equation for t_g and substitute it into the second and solve for the magnitude of a, we will get:

$$-{v_t}^2 = -2aH \quad \text{or } a = \frac{{v_t}^2}{2H}$$

We know also that $F_{haystack} - mg = ma$ or

$$F_{haystack} = ma + mg = m(a+g) = mg' = m\left(\frac{{v_t}^2}{2H} + g\right)$$

Example: Falling From a Plane and Surviving

$$F_{haystack} = ma + mg = m(a+g) = mg' = m\left(\frac{{v_t}^2}{2H} + g\right)$$

Let's suppose the haystack was H = 1.25 meter high and, because you cleverly landed on it in a "bluff" position to keep v_t as small as possible, you start at the top moving at only v_t = 50 meters per second. Then $g' = a + g$ is approximately 1009.8 meters/second^2, 103 'gees', and the force the haystack must exert on you is 103 times your normal weight. You actually have a small chance of surviving this stopping force, but it isn't a very large one.

To have a better chance of surviving, one needs to keep the g-force under 100, ideally well under 100. Since the "haystack" portion of the acceleration needed is inversely proportional to H we can see that a 10 meter haystack would lead to 13.5 gees