

## Subspace Structures in Inner Product Spaces and von Neumann Algebras

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**Abstract** We study subspaces of inner product spaces that are invariant with respect to a given von Neumann algebra. The interplay between order properties of the poset of affiliated subspaces and the structure of a von Neumann algebra is investigated. We extend results on nonexistence of measures on incomplete structures to invariant subspaces. Results on inner product spaces as well as on the structure of affiliated subspaces are reviewed.

**Keywords** Affiliated subspaces · Von Neumann algebras · Measures on subspace structures

### 1 Introduction and Preliminaries

The aim of this paper is to review and deepen recent results on the interplay between structure of a von Neumann algebra  $M$  on one side and order and measure theoretic properties of subspaces which are invariant with respect to operators from the commutant  $M'$  of  $M$  on the other side. We shall be interested in the question of what new phenomena can the structure of invariant subspaces bring in the light of long development of the theory of subspace structures in inner product spaces. The paper is organized as follows. In the first part we summarize briefly main results and history of the discipline. In part two we generalize recent results on the existence of nontrivial completely additive measures on orthogonally closed subspaces to the structure of invariant subspaces. Main part of the present paper is the third section which is an extended form of our lecture at the conference Quantum Structures 2010 held in Boston.

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