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Classes of Invariant Subspaces for Some Operator Algebras

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Abstract New results showing connections between structural properties of von Neumann algebras and order theoretic properties of structures of invariant subspaces given by them are proved. We show that for any properly infinite von Neumann algebra M there is an affiliated subspace $\mathcal{L}$ such that all important subspace classes living on $\mathcal{L}$ are different. Moreover, we show that $\mathcal{L}$ can be chosen such that the set of σ-additive measures on subspace classes of $\mathcal{L}$ are empty. We generalize measure theoretic criterion on completeness of inner product spaces to affiliated subspaces corresponding to Type I factor with finite dimensional commutant. We summarize hitherto known results in this area, discuss their importance for mathematical foundations of quantum theory, and outline perspectives of further research.

Keywords affiliated subspaces · von Neumann algebras · measures on subspace structures

1 Introduction and Preliminaries

The aim of this paper is to study classes of subspaces that are invariant with respect to important types of operator algebras acting on a Hilbert space. We would like to deepen and review recent results showing interplay between the following structures relevant to mathematical foundations of quantum theory: (1) theory of inner product spaces and Hilbert spaces (2) structure theory of algebras of operators acting on a Hilbert space (3) measure and order theoretic properties of quantum structures built on Hilbert space formalism. The paper has grown out of our lecture at the conference Quantum Structures 2012 held in Cagliari.

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