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Advances in Space Research xxx (2015) xxx–xxx

ADVANCES IN SPACE RESEARCH (a COSPAR publication)

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# Dynamics of IGW and traveling ionospheric disturbances in regions with sharp gradients of the ionospheric parameters ☆

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Received 12 November 2014; received in revised form 31 March 2015; accepted 1 April 2015

**Abstract**

The dynamics of the 2D solitary nonlinear internal gravity waves (IGW), as well as traveling ionospheric disturbances (TID) of the electron density excited by them at heights of the ionosphere F-region, for conditions close to those of the F-layer assuming that the source of initial perturbation has the pulse character is studied analytically and numerically. On a level with general case the rather interesting applications when the sharp gradients of the ionospheric parameters are the functions of space coordinates and time, namely the IGW and TID dynamics in the frontal regions of the solar terminator and solar eclipse are considered. The results obtained describe the dynamical structure, evolution and transformation of the IGW and TID at heights of the ionosphere F-layer including its strongly heterogeneous regions.
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*Keywords:* Internal gravity waves; Solitons; TID; Ionosphere; Solar terminator; Solar eclipse

## 1. Introduction

Structure and dynamics of internal gravity waves (IGW) and associated traveling ionospheric disturbances (TID) are extensively studied for more than forty years (Heisler, 1959; Hunsucker, 1982, 1987; Hocke and Schlegel, 1996). Despite extensive observations involving numerous various technics such as, e.g., vertical and slanted ionospheric as well as satellite sounding (Belashova et al., 1990) and recently developed imaging technique using multipoint GPS networks (Tsugawa et al., 2006), the associated theory is less developed.

To solve the wide range of problems associated with wave perturbations at the ionospheric F-layer heights, it is necessary to take into account essential factors such as the middle- and large-scale traveling ionospheric disturbances (TID). TID directly affect variability of the ionospheric parameters as well as those of the Earth's ionosphere waveguide. One of the most convenient approaches to these problems is to study TID dynamics in terms of the internal gravity waves (IGW) (Belashov and Vladimirov, 2005). Of special interest are the IGW solitons as traveling in the F-layer stable large-scale wave formations, caused by various reasons such as the isolated magnetic substorms, solar terminator and solar eclipse (Belashova and Belashov, 2006), seismo-volcanic processes, and high-power artificial explosions (Belashov and Vladimirov, 2005; Belashova and Belashov, 2006). Here we first investigate the dynamics of the solitary nonlinear IGW (as well as TID excited by them at the heights of

☆ This paper has been presented at 40th COSPAR Scientific Assembly 2014, August 3, 2014, Moscow, Russia, C2.2-4-14.

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http://dx.doi.org/10.1016/j.asr.2015.04.008
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Please cite this article in press as: Belashov, V.Y., Belashova, E.S. Dynamics of IGW and traveling ionospheric disturbances in regions with sharp gradients of the ionospheric parameters. Adv. Space Res. (2015), http://dx.doi.org/10.1016/j.asr.2015.04.008

the ionosphere's F-region) for conditions close to those of the F-layer, by omitting the physical nature of the sources, but assuming that it has the pulse character (more details about excitation of the pulse disturbances by various physical sources are given below as well as in the references listed above). Then we consider applications of the obtained results to the problems of the generation of IGW in the regions with sharp gradients of the ionospheric parameters such as electron density, temperature, scale heights for the ions and neutral particles etc. As particular cases we consider the frontal regions of the solar terminator and solar eclipse. To confirm our conclusions we result some results of natural radiophysical experiments in the end.

## 2. IGW solitons and TID of electron density

For the isothermal model of Earth's atmosphere, we take into account $k_\perp^2 \ll k_y^2$, and $|Hk_x| \ll 1$ in the linear approximation, and expanding in $k$ up to the fifth order, write the dispersion law as (Belashov and Vladimirov, 2005)

$$\omega = Vk_x\left\{1+\frac{k_y^2}{2k_x^2}\pm\frac{(\gamma-2)^2}{\gamma^2}H^2k_x^2\left[2+\frac{(\gamma-2)^2}{\gamma^2}\varepsilon H^2k_x^2\right]\pm H^2k_z^2\right\} \quad (1)$$

where the second term in the right-hand side describes the diffraction divergence in the transverse direction of the wave propagation, the third and the fourth terms describe the dispersive effects of corresponding order, and the last term is the same in vertical direction; $V = 2\omega_g H$, $\omega_g = [(\gamma-1)g/\gamma H]^{1/2}$ is the Brunt–Väisälä frequency, $H$ is the scale height of the neutral atmosphere, and $\varepsilon = -V/V_{\min}^{ph}$, where $V_{\min}^{ph}$ is the minimum phase velocity of the linear oscillations. In this case, taking into account the weak nonlinearity of the dimensionless function $u = u_z/ac$ which has a sense of vertical velocity of the neutral particles, $a = \exp(z/2H)$, $c = \sqrt{gH}$ and neglecting dissipative effects, from the hydrodynamic equations for the neutral gas we obtain the equation

$$\partial_t u + ac\frac{2\gamma-1}{\gamma^2}u\partial_\xi u \pm\frac{(\gamma-2)^2}{\gamma^2}VH^2\partial_\xi^3\left[2u+\frac{(\gamma-2)^2}{\gamma^2}\varepsilon H^2\partial_\xi^2 u\right] = \frac{V}{2}\int_{-\infty}^{\xi}\partial_y^2 u d\xi \quad (2)$$

which is written in the reference frame moving along the $x$-axis with the velocity $V$ ($\xi = x - Vt$). The upper signs in (1) and (2) correspond to the positive wave dispersion, and the lower signs correspond to the negative one (without loss of generality we further assume that $V < 0$ and, as can be easily seen from (1), $\varepsilon < -1$). The obtained equation is the generalization of the Kadomtsev–Petviashvili equation (so-called Belashov–Karpman (BK) equation), for the first time it has been obtained in Belashov (1990), Karpman and Belashov (1991) and investigated in detail in a number of works (see Belashov and Vladimirov, 2005). It is written here for the velocity of the neutral component at the heights of the F-region with $\partial_z = 0$ without dissipation and describes the nonlinear IGW solitons and nonlinear wave packets, with the structure determined by both the coefficients and the function $u(0, \xi, y)$ corresponding to the initial condition, i.e., it depends on the sort of perturbation and accordingly the type of the source as well.

The structure of the solutions for the initial disturbance of the wave pulse type corresponding to various physical sources as, for example, the terrestrial and anthropogenic factors [as well as the "quasi-one-dimensional" sources of the global character, such as the solar terminator (ST) and solar eclipse (SE)], is described in detail in Belashov and Vladimirov (2005) and depends on ε. Indeed, the 2D solitons with the algebraic (for $\varepsilon \ll -1$) or the oscillating (in the direction of propagation, for $\varepsilon \leqslant -1$) asymptotics correspond to the upper sign in (2), whereas the dispersing wave packets and/or the 1D solitons which are stable in the case of the negative dispersion (Belashov and Vladimirov, 2005) correspond to the lower sign.

Let us consider the case of the upper sign in (1) and (2) and study the excitation by the IGW solitons of the middle- and large-scale TID for the conditions close to those in the F-layer. Considering the solitary IGW traveling at the near-to-horizontal angles, the continuity equation for the electron density in the F-layer is given by (Belashova and Belashov, 2006)

$$\partial_t N = \partial_z\left[(\partial_z N + N/2H_i)D_0 e^{z/H_i} - u_z(1-e^{-\nu t'})N\sin I\cos I\right] - \beta N + Q \quad (3)$$

where $N(t,z)$ is the total electron density, $D_0\exp(z/H_i) = D_\alpha \sin^2 I$, $D_\alpha$ is the ambipolar diffusion coefficient, $H_i$ is the scale height for ions, $I$ is the magnetic inclination, $\beta = \beta_0\exp(-Pz/H_i)$ and $Q$ are, respectively, the recombination rate and the ion production rate, the exponent $0 \leqslant P \leqslant 2$ characterizes the gas intermixing, $u_z = acu$ is the vertical component of the velocity of neutral particles, and $t' = t - t_0$, $t_0$ is the moment of the start of the neutral component's perturbation. Now we approximate the profile of the electron density at the height $z$ for fixed time moment by $N = N_1\exp(z/H_i)$, $N_1 = N|_{z=0}$, and obtain that solution of (3) is given by (Belashov and Vladimirov, 2005)

$$N(u,t) = N(u,t_0)\exp[G(u,t)], \qquad G(u,t) = \int_{t_0}^{t} g(u,t)\mathrm{d}t \quad (4)$$

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where

$$g(u,t) = C - (1/H_i + 1/2H)f(u,t),$$
$$C = 3a_1/H_i^2 - \beta(1-q),$$
$$f(u,t) = acu[1-\exp(-\nu t')]\sin I\cos I,$$
$$a_1 = D_\alpha \sin^2 I, \quad q = Q/\beta N.$$

Here, the function $u$ satisfies (2). When $\varepsilon \ll -1$ and the solution of (2) is the 2D soliton with the algebraic asymptotics, the solution (4) for the quasi-pulse source of IGW is shown in Fig. 1. In Fig. 2 the results of numerical simulation for initial IGW disturbances normalized on 1, $\bar{u} = u\left(0,\xi,y\right)/\max\limits_{\xi,y}\left[u\left(0,\xi,y\right)\right]$, are presented. If $\varepsilon \leqslant -1$ then the perturbation of the electron density $N$ as well as the IGW soliton has the oscillating asymptotics shown in Fig. 2a.

The solution of (2) and (4) for the conditions typical for the F-layer gives us the following results. The solitary IGW excite in the F-region the solitary TID of the electron density, their structure depends on the form of IGW and the ionospheric parameters determined by the photo-chemical and dynamic processes at the height considered. The amplitude of TID increases in the direction of the geomagnetic latitude $\varphi_m = 45°$, the wave front steepens, and at the latitude $\varphi_m = 45°$ the wave becomes similar to the shock wave.

The twofold increase of the IGW amplitude results in the increase of the TID amplitude: 35% for $\varepsilon \ll -1$; close to 100–105% for $\varepsilon \leqslant -1$. For all the studied cases we note the phase shift of TID relative to the phase of IGW ($\Delta t \sim 0.5 - 5$ min) and the effect of the relaxation of the electron density perturbations which increase with the decreasing $\varepsilon$ characterizing essentially the medium's dispersion. Fig. 2 shows the simulation results for IGW solitons with a velocity on the order of 200 m/s at $z = 0$ and $I = 63.4°$.

Thus such ionospheric characteristics as the height of the maximum and the critical frequency of the F-layer increase proportionally to the TID amplitude when the 2D nonlinear IGW propagates, as well as experience relaxation similar to the relaxation of the electron density $N'$.

## 3. Dynamics of IGW solitons in regions of fronts of ST and SE

In addition to the general study of the dynamics of solitary waves in the F-region of the Earth's ionosphere, the middle-scale and large-scale wave effects associated with motions of the fronts of ST and SE were investigated numerically within the framework of the above developed weakly nonlinear approximation neglecting the dissipation effects. Following (Belashova and Belashov, 2006) let us consider, at first, the wave effects caused by ST motion. Let's define ST as the area separating space, shined by a full disk of the Sun, from area of the full shadow rejected by the Earth.

At heights of the F-layer of ionosphere where concentration of the charged particles is a lot of above, than in underlaying areas, at the ST movement, in connection with infringement of balance of ionization and the dynamic balance by it, caused by fast change of a degree of light exposure in the frontal zone of ST, the area of sharp gradients of the basic ionospheric parameters (electron density, electron and ion temperature, recombination rate, ion production rate, etc.) moving with a ST speed (linear speed of rotation of the Earth at height of F-region) is formed.

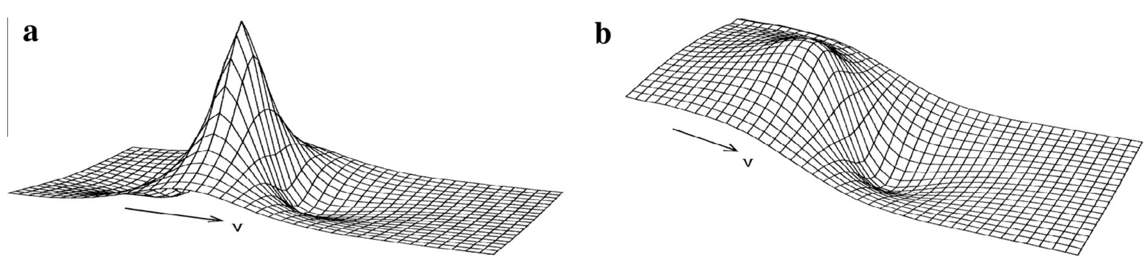


Fig. 1. IGW soliton $u(\xi, y)$ for $\varepsilon = -12$ (a) and associated perturbation of the normalized electron density $N' = \{[N(u,t) - N(0,t)]/N(0,t)\} \times 100\%$ (b).

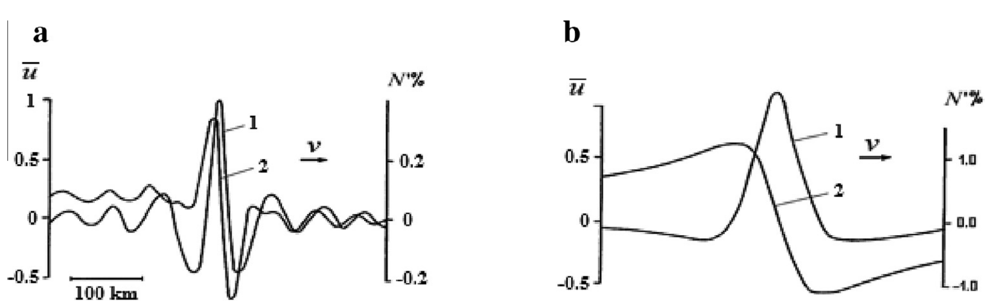


Fig. 2. Profiles of the perturbations (in the relative values of $u$ where the amplitude of the initial condition $u(0, \xi, y)$ is normalized on 1) at $y = 0$: IGW (the curve 1) and TID ($N'$) (the curve 2): $\varepsilon = -1.2$ (a) и $\varepsilon = -12$ (b).

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Let us consider the dynamical model of the F-layer (Belashova and Belashov, 2006) considering time (of corresponding periods) dependences of ionospheric characteristics, defining processes of diffusion, ionization and recombination at heights of F-region, i.e., effects of influence of ST on a plasma, associated with sunrise–sunset processes. This dynamical model is the following set of the equations:

$$\begin{aligned} &N_0 = N_{0m}\exp\left[-\gamma_N(t-t_m)^2/t_{ch}^2\right],\\ &Q(z,\chi) = Q(h_0,0)\exp(1-\sec\chi e^{-\zeta})\exp(-\zeta),\\ &\beta_0 = Q(h_0,0)/N_{0m}, D_0 = D_{0m}T_i,\quad H=(kT_e)/(m_e g),\\ &H_i = k(T_e+T_i)/(m_i g), T_e = T_{em}\exp\left[-\gamma_e(t-t_m)^2/t_{ch}^2\right],\\ &T_i = T_{im}\exp\left[-\gamma_i(t-t_m)^2/t_{ch}^2\right],\quad P = 1+\exp\left[-(t-t_m)^2/t_{ch}^2\right] \end{aligned} \tag{5}$$

where the functions describe background (in relation to time scales of investigated disturbances) changes at heights of the F-region and are defined by the expressions

$$N(t) = N_0(t)N_1(z),\quad \beta = \beta_0\exp(-P\zeta),\quad D_0 = D_\alpha\sin^2 I\exp(-\zeta) \tag{6}$$

where $\zeta = z/H_i$, $z = h-h_0$, $h_0 = H_i\ln\alpha$ corresponds to the electron density maximum; $\alpha = 2H_i\sqrt{\beta_0/D_0}$; $\chi$ is the zenith angle of Sun; $t_{ch}$ characterizes the time scale, the index $m$ corresponds the maximal value of function; $\gamma_k$ are some functions defining effect of influence on corresponding component; other designations are standard in physics of ionosphere.

Let's note following important circumstance. At a choice of corresponding scales and a kind of functions $\gamma_k$ the generalized dynamic model (5) and (6) will describe a time course of the basic ionospheric parameters for corresponding "source", that is equivalent to complement of the problem by initial and boundary conditions.

If, for example, to assume that $t_{ch} = 24$ hours and choose the characteristic for the F-layer values of the amplitudes of $N_{0m}$, $Q(h_0,0)$, $D_{0m}$, $T_{em}$, $T_{im}$ and time moments $t_m$ corresponding to the maxima of corresponding functions, we obtain that the model (5) and (6) describes in some approach (the degree of approach is defined by a choice of functions $\gamma_k$, at $\gamma_k = \mathrm{const}_k$ we have zero approach) a daily course of the basic ionospheric parameters (Belashova and Belashov, 2006). In this case, besides other, the model can be used for investigation of dynamics of middle- and large-scale wave structure of the F-layer in areas with sharply expressed gradients of the ionospheric characteristics (morning and evening sectors) on a background of slow (daily) changes.

From the continuity equation for the electron density in the F-layer (3), considering the wave disturbances propagating under corners, close to a horizontal, at $N_1(z) = \exp\zeta$ with due account of the change $t \to \xi - Vt$ it is easy to obtain (where $V$ is the ST velocity at the corresponding height $h$, and $\xi$ is the spatial coordinate along V) in the reference frame related to the source the expression for vertical component of the neutral particles' velocity (Belashova and Belashov, 2006):

$$\begin{aligned} u_z = \left[ac(1-e^{-\nu t'})\sin I\cos I\right]^{-1}\Bigg\{&V\frac{H_i}{N_0}\partial_\xi N_0(1-e^{-\zeta})\\ &+\left[\beta\frac{H_i}{1-P}\left(1-e^{(1-P)\zeta}\right)+\frac{Qz}{N_0}\right]e^{-\zeta}+\frac{3}{2}\frac{D_0}{H_i}e^{\zeta}\Bigg\} \end{aligned} \tag{7}$$

where $t' = \xi/V$, and the values of parameters are defined by the dynamical model of the F-layer (5) and (6) with due account of dependences of basic parameters on time.

In approach of an isothermal ionosphere, taking into account the weak nonlinearity of the function $u = u_z/ac$, $a = \exp(z/2H)$, $c = \sqrt{gH}$ and weak dispersion ($|Hk_x| \ll 1$), the BK equation (2) and corresponding dispersive equation (1) are valid. For such global phenomenon as ST it is possible with sufficient accuracy to suppose that $\partial/\partial y = 0$. Making the changes $u \to Vu/\alpha$, $\xi \to (-\delta/V)^{1/4}\xi$, $t \to (-\delta/V^5)^{1/4}t$, where $\alpha = ac(2\gamma-1)/\gamma^2$, $\delta = [(\gamma-2)/\gamma]^4\varepsilon VH^4$, $\varepsilon = -V/V_{\min}^{ph}$, $\gamma = c_p/c_v$ and omitting strokes, write Eq. (2) for $\partial/\partial y = 0$ in more convenient form:

$$\partial_t u + u\partial_\xi u + 2(-\varepsilon)^{-1/2}\partial_\xi^3 u - \partial_\xi^5 u = 0. \tag{8}$$

Eq. (8) is the 1D analog of the BK equation and it is written in the reference frame connected with ST. For Eq. (8) with the initial condition (7) and zero boundary conditions (the last are valid for the characteristic periods of a problem, sufficiently small in comparison with length of day), using the model (5) and algorithms developed in (Belashov et al., 2003), we solved the Cauchy (initial) problem at $z = 0$, $H$, $2H$. The values of the ionospheric parameters of model were chosen close to characteristic ones for F-layer in conditions of a daily cycle of winter and summer seasons.

Fig. 3 shows the examples of simulation results obtained in (Belashova and Belashov, 2006) for geomagnetic latitude $\varphi_m = 45°$. As one can see from figures the obtained solutions testify to generation by the ST front both in morning, and in evening sectors some kind of solitons-like wave "forerunners" in the neutral particles velocity $u$ with the periods ~40–60 min which scales are essentially various for summer and winter seasons and are defined by a lot of factors: height $z$, geomagnetic latitude, value of dispersion $\varepsilon$ depending, in its turn, from values of some ionospheric characteristics, and also features of change of ionospheric parameters in a concrete daily cycle. Simulation for $\partial/\partial y \neq 0$ (Eq. (2)) shows that, generally, the picture for $z = 0$ is the train of 2D solitons-like waves (with $k_x \gg k_y$) similar to the multisoliton solutions of the KdV equation (the case $y = 0$). For $z = H$, $2H$ the qualitative form of the solution is maintained, although they are less regular and (on the average) the larger amplitude waves. The characteristics of such soliton-like formations

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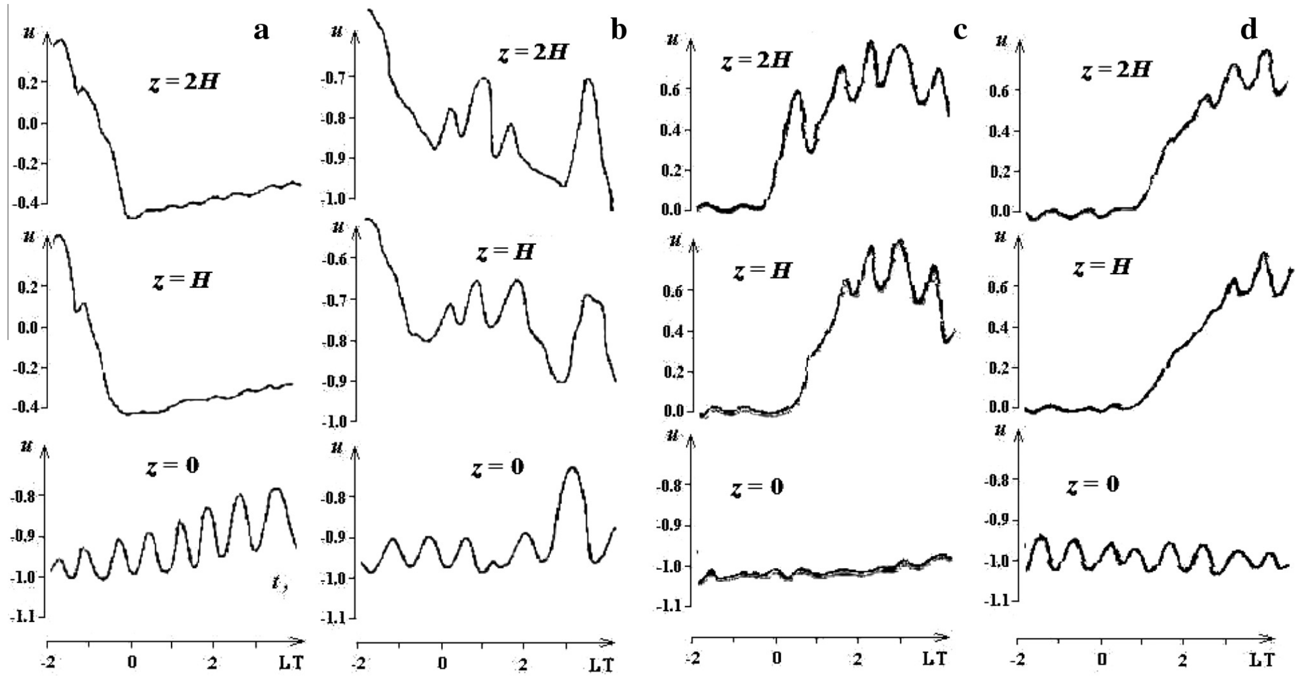


Fig. 3. Perturbations of the neutral particles velocity $u$ in the F-layer of ionosphere caused by ST (a, b – morning sector; c, d – evening sector): (a, c) winter, (b, d) summer; $t = 0$ corresponds to the moment of sunrise (a, b) and sunset (c, d) at height $z = 0$.

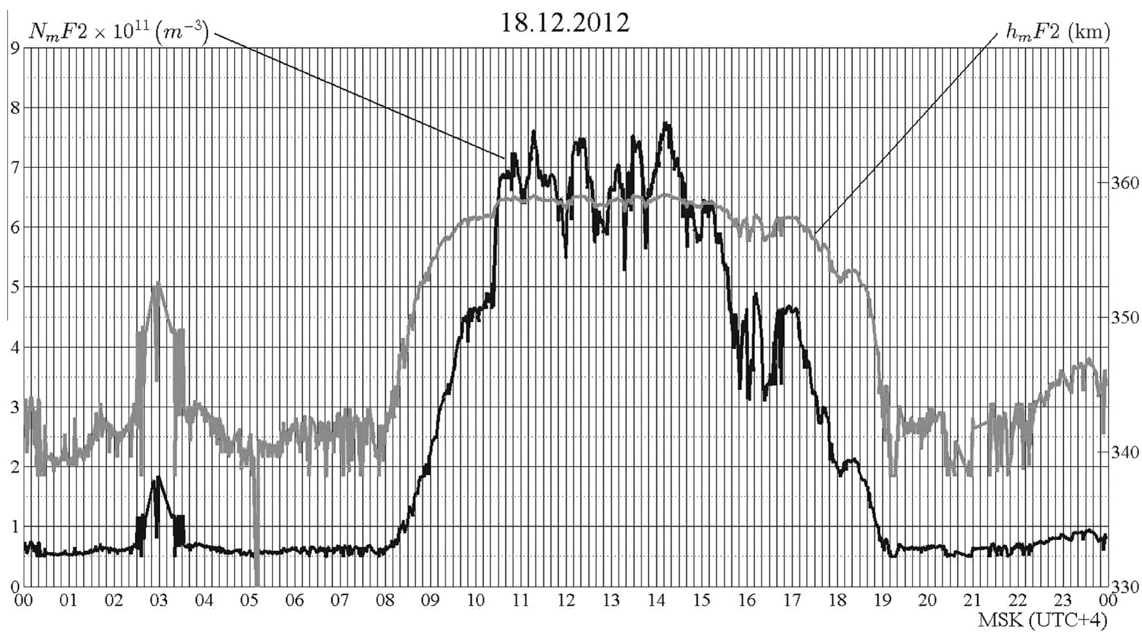


Fig. 4. Daily course of parameters $h_mF2$ and $N_mF2$ (winter, 1-min vertical sounding). Nasyrov, Kazan Federal University, Personal communication, 2014.

strongly depend on the season and the ionospheric parameters.

Figs. 4 and 5 show the results, obtained in the experiments of 1-min vertical sounding of the ionosphere fulfilled in the radio observatory of Kazan Federal University (Nasyrov, Personal communication, 2014). In both figures one can see also the solitons-like wave "forerunners" in the electron density in the maximum of the F-layer $N_mF2$ with $T \sim 50$–60 min. Fig. 5 shows also daily course of $N_mF2$ calculated with use of model IRI-2012 (curve 2) for the same time. One can note that this ST effect is not account in any way by IRI-2012 model.

If in Eqs. (5) and (6) to choose $t_{ch}$ corresponding such source as a SE spot, that the model will describe quite adequately the wave effects in the F-layer of the ionospheres associated with passage of such disturbing factor as SE (Belashova and Belashov, 2006). Such investigations showed (see example in Fig. 6) that the characteristic periods

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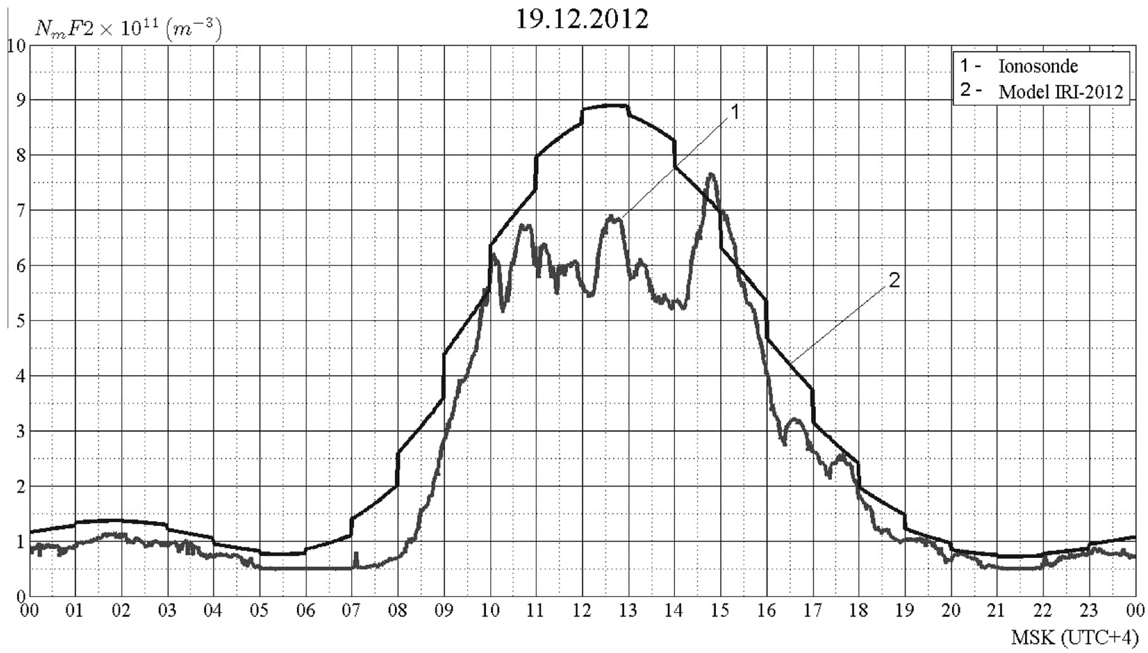


Fig. 5. Daily course of $N_mF2$ (winter, 1-min vertical sounding) and model IRI-2012. Nasyrov, Kazan Federal University, Personal communication, 2014.

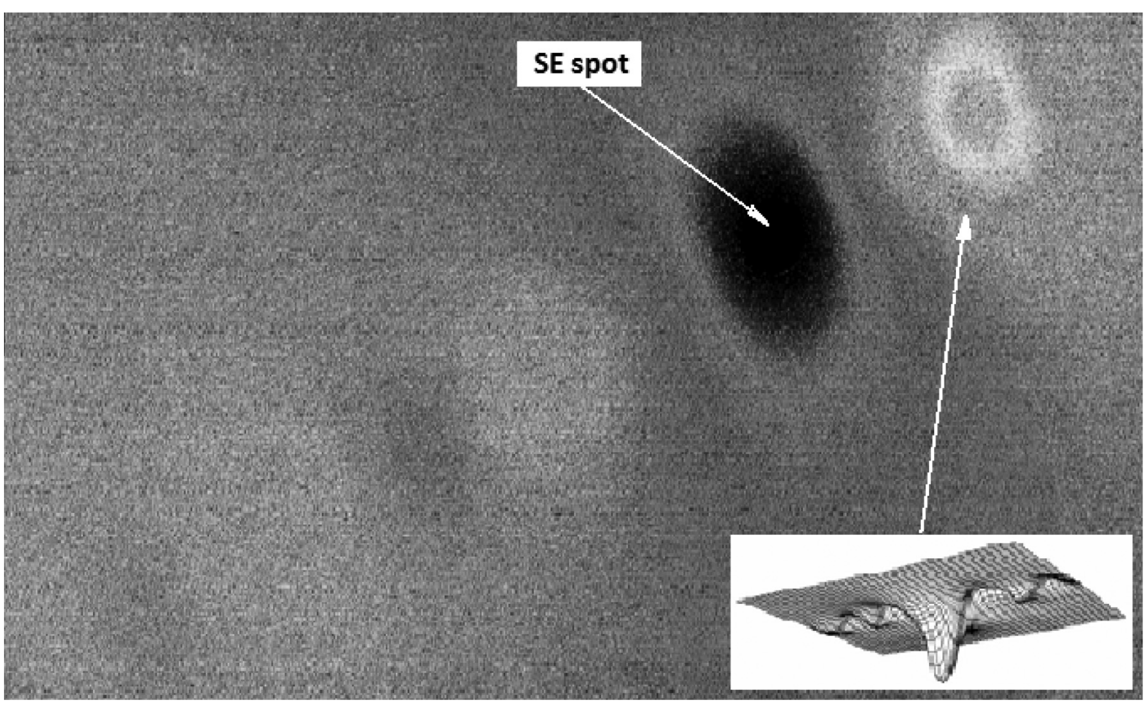


Fig. 6. General view of numerical solution of the GKP equation [function $u(\xi, y)$] for the pulse source of type of the SE spot [$(x, y)$-plane] $V \approx 1667$ km/h = 463 m/c (linear velocity of the Earth rotation at height of the F-layer maximum).

of “forerunners” of the SE front are ~3–10 min, and its spatial scales are defined by the parameters of the F-layer.

Simulations for the conditions corresponding to the partial solar eclipse observed on March 18, 1988, and the sunrise and sunset periods on March 1–10, 1990, (an interval of the International Geophysical Calendar) agree well with the results of special targeted experiments on the passive slanted sounding of the ionosphere done in these periods in the Far-Eastern region of Russia (see Refs. in Belashov and Vladimirov (2005)). Thus we conclude that despite some idealization of the problem, the approach based on the generalized KP equation allows us to predict the effects of the TID dynamics in the F-region of the Earth’s ionosphere reasonably well.

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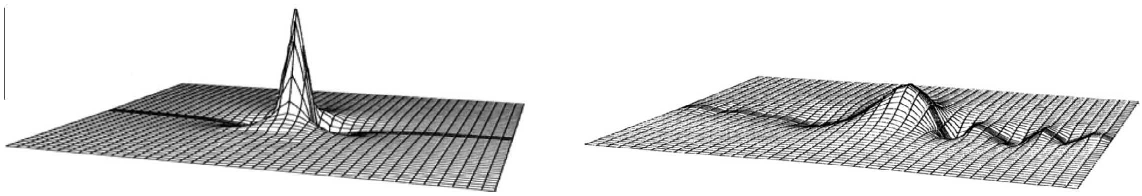

Fig. 7. Evolution of 2D IGW soliton for $\nu = 1$, $\varepsilon = -1.2$: $t = 0$, 0.2.

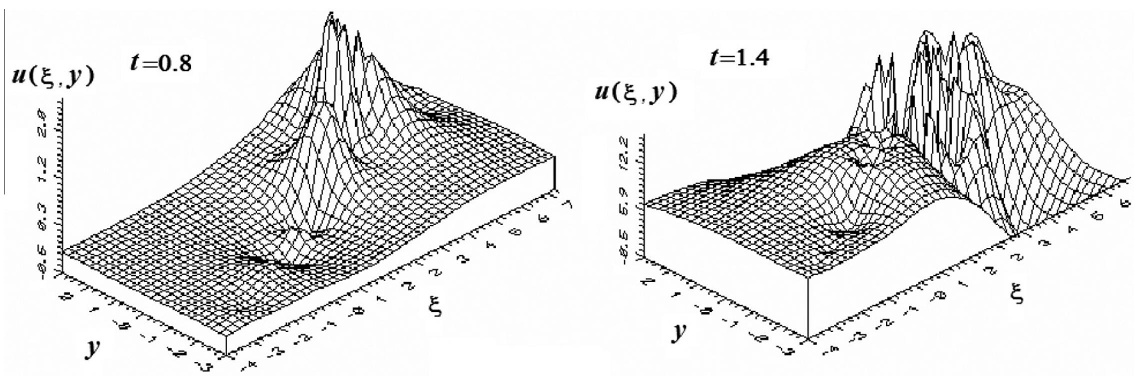


Fig. 8. Evolution of 2D IGW soliton with presence of stochastic fluctuations of the wave field: the Gaussian noise $\chi = \chi(t)$ for the standard deviation $\sigma = 0.02$ ($\varepsilon \ll -1$).

In conclusion, we note that in real conditions in the ionosphere it is necessary to take into account dissipative processes which really lead to decreasing of the perturbations. In this case Eq. (2) should be complemented by the dissipative term of form in the left-hand side (Belashov and Vladimirov, 2005) with factor

$$\nu = (\rho_0/2\rho)\left(c_\infty^2 - c_0^2\right)\tau \int_0^\infty \mu\varphi(\mu)d\mu$$
$$= (2\rho_0)^{-1}\left[4\eta/3 + \varsigma + \gamma(1/c_v - 1/c_p)\right]$$

where $c_\infty$ and $c_0$ have a sense of the "highfrequency" and "lowfrequency" sound. The case $\nu \neq 0$ was investigated in detail in (Belashov and Vladimirov, 2005), where it was shown that the presence of dissipative term leads to both the exponential decrease of the amplitude with the rate $\Gamma(t) \sim \nu$ and effects of destruction of the structure and the symmetry of the IGW soliton (see example in Fig. 7) accompanied by the relaxation in the recovery of the electron density after the wave passes (Belashova and Belashov, 2006).

The effects of stochastic fluctuations of the wave field $u(t, x, y)$ on the evolution of the ionospheric perturbation can also be accounted for in the basic equations. Thus, accordingly, (2) should be complemented by the term like $\chi(t, x, y)$. In the case of the lowfrequency fluctuations when $\chi = \chi(t)$, Eq. (2) for $\varepsilon = 0$ was investigated analytically in (Belashov, 1995).

The obtained results can be easily applied to (2) with $\chi = \chi(t)$ on the left-hand side. Thus the interpretation of the results (Belashov, 1995) in terms of the problem (2) and (3) enables us to conclude that even small stochastic fluctuations of the wave field lead to the damping of the solitary IGW (with its propagation) accompanied by the transform of the wave to an oscillatory structure. Fig. 8 shows an example of the results of numerical simulation of the IGW soliton evolution in medium with presence of stochastic fluctuations of the wave field in form of Gaussian noise. One can see that soliton with evolution acquires a short wave structure and it is destroyed. In the case $\chi(t, x, y)$, however, the analytical study of the process becomes too complicated, and in (Belashov and Vladimirov, 2005) numerical integration of (2) and (3) with the stochastic term was done. The obtained results appear to be qualitatively similar to the case $\chi = \chi(t)$, namely, the decrease of the amplitude of the solitary oscillating IGW is observed, with the subsequent destruction of the wave.

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