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Manifestation of Seismic Impacts in the Ionosphere Far from the Earthquake Epicenter

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Abstract—On the basis of the results of theoretical studies of the manifestation of seismic activity it was shown that as a result of the impact on the atmosphere and the neutral component of the ionosphere of the acoustic pulse caused by a Rayleigh surface wave, both in ground-based experiments and in satellite observations at heights of the *F*-region of the ionosphere the perturbations of the vertical velocity of the neutral component and electron density can be recorded, which leads further to the formation and evolution in the far zone of a solitary internal gravitational wave (IGW) and to the excitation by this wave of a traveling ionospheric disturbance (TID) with corresponding characteristic spatial scales, which propagate radially from the epicenter at angles close to the horizontal, with velocities of ~200 ms^{-1}. Consideration 3-dimensional case taking into account all significant factors (weak nonlinearity and dispersion, dissipation and stochastic fluctuations of the wave field) enabled us to refine the results previously obtained by other authors and showed that in the far zone from the epicenter of the earthquake, the form of the ionosphere response to a seismic event depends significantly on the values of the main parameters of the ionosphere, determining its dispersive characteristics, the fluctuation and dissipative processes in the region of propagation of the IGW and the excited by it TID: it can be a solitary wave disturbance, and a wave packet with characteristic IGW scales. It was found that that there is both a phase shift of the TID relative to the IGW phase (within 0.5–5 min), and the relaxation effect in the recovery of the electron density after the passage of the IGW soliton. The obtained results of the analysis of seismic-ionospheric post-effects, displaying in the formation of soliton-like IGW and TID disturbances in the far zone and representing a great interest, in particular, for a better understanding of causal relationships in the “solid earth–atmosphere–ionosphere” system and can be used for finding direction to the earthquake epicenters and distinguishing the seismic-induced oscillations in the spectrum of ionospheric fluctuations.

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1. INTRODUCTION

The problems of study of the effects of seismic phenomena in ionospheric plasma attract the close attention of researchers due to the importance of this problem, not only for fundamental science but also for the safety of the population of earthquake-prone regions and on the planet as a whole. At the same time, along with the study of seismic–ionospheric phenomena, which are predictors of seismic activity, the study of seismic–ionospheric post-effects plays an important role for a number of reasons, e.g., a better understanding of the cause-and-effect relationships in the solid earth–atmosphere–ionosphere system, the direction-finding of earthquake epicenters, the identification of seismically induced oscillations in the spectrum of ionospheric fluctuations, etc.

As known, one of the first attempts of a sufficiently rigorous theoretical study of seismic–ionospheric effects caused by a Rayleigh surface wave was undertaken by Golitsyn and Klyatskin (1967), who solved this problem in a one-dimensional linear non-dissipative approximation. In a similar approach, Pavlov (1979) took into account the losses due to the weak electrical conductivity of the medium. The influence of weak nonlinearity on the propagation of an earthquake-induced acoustic pulse in the ionosphere was studied by Pavlov in (1986), but there too, only the one-dimensional case was considered. The three-dimensional nonstationary problem of seismic influence on ionospheric plasma was considered by Doilnitsyna et al. (1981); however, in this study, the authors disregarded the nonlinear effects. Thus, all studies known to us (see a fairly complete review in (Belashov, 1990; Belashov and Vladimirov, 2005)) have some drawback and, therefore, the estimates of influence of seismic events on the dynamics of ionospheric plasma obtained in them turned out to be very approximate. More accurate results can be obtained when all significant factors are taken into account. A good example, in this case, can be the theoretical estimates (Belashov, 1997a) for the internal gravity waves (IGWs) excited by the Rayleigh wave, in comparison with the results of Pavlov (1979).

This paper presents the results of a theoretical study of the problem taking into account weak nonlinearity and dispersion and the influence of dissipation and stochastic fluctuations of the electron density occurring in the ionosphere, in three-dimensional geometry.

2. BASIC EQUATIONS AND ANALYSIS

Let us consider the influence of a wave disturbance (impulse) excited by a Rayleigh surface wave caused by an earthquake on the neutral atmosphere's component and its further propagation at ionospheric heights close to the horizontal plane in the region far from the earthquake nidus. Here, "high frequency" acoustic oscillations are damped due to the influence of dispersion effects, which results in a shift of the spectral maximum to a lower frequency region: the oscillations related to the IGW branch start prevail. Next, we will discuss the effect of such IGWs on the ionized component of the ionospheric plasma, as well as the effect of stochastic fluctuations of the wave field on the propagating wave.

To describe the motions of the neutral component, let us consider the following set of the gas dynamics equations (Belashov and Vladimirov, 2005):

$$\begin{gathered}\partial_t \rho' + \operatorname{div}[\rho_0(z)\mathbf{V}] = 0,\\ \rho_0(z)[\partial_t \mathbf{V} + (\mathbf{V}\nabla)\mathbf{V}]\\ = -\nabla p' - \rho' g\mathbf{e}_z + \eta(z)\left[\frac{1}{3}\nabla \operatorname{div}\mathbf{V} + \nabla^2\mathbf{V}\right]\\ + \varsigma(z)\nabla(\nabla\mathbf{V}),\\ \partial_t p' + (\mathbf{V}, \nabla p_0(z)) = -c^2\rho_0(z)\operatorname{div}\mathbf{V},\end{gathered} \tag{1}$$

where the unperturbed values of the variables determining the wave field are marked with the subscript "0" and dashes denote the perturbed values of the functions if their unperturbed values are zero. Functions without subscripts describe fields with zero unperturbed values. In equations (1), ρ is the density; p is the pressure at which the unperturbed value $p_0(z) = \rho_0(z)c^2/\gamma = gH\rho_0(z)$, where $\gamma = C_p/C_v$; H is the scale height of the neutral atmosphere; $\eta(z) \approx 3c^2\rho_0(z)/2\gamma\nu_{nn}(z)$ is the dynamic viscosity coefficient; ν_{nn} is the frequency of collisions of neutral particles; and $\varsigma(z)$ is the kinematic viscosity coefficient. We further assume that the density of the unperturbed atmosphere and ionosphere is uniform with height z: $\rho_0(z) = \rho_0(0)\exp(-z/H)$.

Equations (1) do not take into account stochastic fluctuations of fields, and we will introduce the corresponding terms further.

The first boundary condition, which approximates the Rayleigh surface wave at large distances from the epicenter, is chosen in the following form:

$$V_z|_{z=0} = d_t Z(r', t), \quad Z(r', t) = h(t)\exp\left[-(r')^2/L^2\right], \tag{2}$$

where $(r')^2 = \xi^2 + y^2$, $\xi = x - v_R t$; v_R is the velocity of the Rayleigh wave. Thus, we will consider the problem in the coordinate system associated with the Rayleigh wave. Another boundary condition must correspond to the asymptotics $z \to +\infty$, and we can therefore require that the function $V_z(t, z, \xi, y)$ was a superposition of waves $V_z(\omega, z, k, y)$ (Belashov, 1997a) ($V_z(\omega, z, k, y) \to 0$ at $z \to +\infty$ for $\eta \neq 0$), i.e., Fourier transform functions $V_z(t, z, \xi, y)$ by t and by ξ[1]. The formulation of such a boundary condition ensures energy leakage into the region $z = +\infty$. Rayleigh wave (2) leads to the formation of an upward wave with an amplitude that increases with height, which is associated with an exponential decrease in density with height: $\rho_0(z) = \rho_0(0)\exp(-z/H)$ (Belashov, 1990). Nonlinear effects begin to manifest themselves at heights of the ionospheric F region, when a nonlinear solitary IGW forms under the action of an upward going wave excited by the Rayleigh surface wave (Belashov and Vladimirov, 2005).

Taking into account the geometry of the problem, we will assume that $k_x^2 \gg k_\perp^2$, $|Hk_x| \ll 1$, i.e., for nonlinear solitary waves propagating at angles to the horizon close to 90°, the Boussinesq approximation is valid. The set of equations (1) with a due account of weak nonlinearity for the velocity of neutral particles $u(t, r', z) = V(t, r, z)|_{x=\xi+vt}$ at $\partial_z = 0$ can then be reduced to one fifth-order equation (Belashov, 1990) with due account of the term describing dissipative effects of the viscous type (Karpman and Belashov, 1991; Belashov and Vladimirov, 2005; Belashov and Belashova, 2015):

$$\begin{gathered}\partial_t u + \frac{2\gamma - 1}{\gamma^2}u_z\partial_\xi u - \sigma\partial_\xi^2 u + 2\frac{(\gamma-2)^2}{\gamma^2}vH\partial_\xi^3\\ \times\left[u + \frac{(\gamma-2)^2}{2\gamma^2}\varepsilon H^2\partial_\xi^2 u\right] = \frac{v}{2}\int_{-\infty}^{\xi}\partial_y^2 u\, d\xi,\end{gathered} \tag{3}$$

where $\gamma = C_p/C_v$; $\varepsilon = -v/v_{\min}^{ph}$; $v_{\min}^{ph}$ is the minimum phase velocity of linear oscillations and coefficient σ describes the viscosity (Belashov and Belashova, 2015):

[1] We do not present detailed expressions here due to their cumbersomeness; they can be found in (Belashov, 1997a).

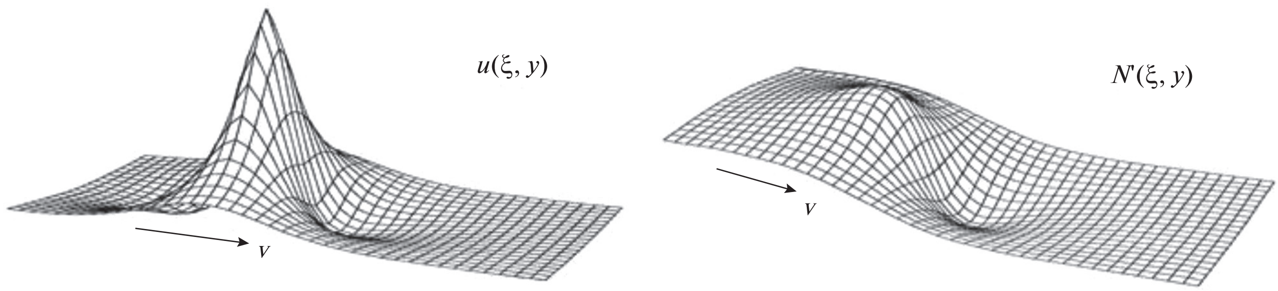

Fig. 1. Seismic-induced IGW disturbance in the F region of ionosphere of the soliton type (a) at $\varepsilon = -12$ and the corresponding disturbance $N' = \{[N_e(u,t) - N_e(0,t)]/N_e(0,t)\} \times 100\%$.

$$\sigma = (\rho_0/2\rho)\left(c_\infty^2 - c_0^2\right)\tau\int_0^\infty \mu\varphi(\mu)d\mu$$
$$= (2\rho_0)^{-1}\left[\frac{4}{3}\eta + \varsigma + \gamma\left(\frac{1}{C_v} - \frac{1}{C_p}\right)\right],$$

where c_∞ and c_0 are the velocity of "high-frequency" and "low-frequency" sound (Karpman, 1973), respectively.

Taking into account the solitary waves moving at angles close to the horizontal plane, we can write the continuity equation for the electron density N_e in the F region in the following form (Belashov, 1990; Belashov and Vladimirov, 2005):

$$\partial_t N_e = \partial_z\left[\left(\partial_z N_e + N_e/2H_i\right)D_0\,\mathrm{e}^{z/H_i} - u_z\left(1 - \mathrm{e}^{-\upsilon t'}\right)\sin I\cos I\right] - \beta N_e + Q, \quad (4)$$

where $D_0\exp(z/H_i) = D_\alpha \sin^2 I$; D_α is the ambipolar diffusion coefficient, and $\beta = \beta_0(-Pz/H_i)$, and Q are the recombination rate and the ion production rate, respectively; $t' = t - t_0$, where t_0 is the moment of the start of the neutral component's perturbation; H_i is the scale height for ions. Approximating the electron density profile in the range of heights z by the exponent $N_e = N_{e0}\exp(z/H_i)$, $N_{e0} = N_e|_{z=0}$, it is possible to obtain the solution of equation (4) in form (Belashov, 1989; Belashov and Belashova, 2015)

$$N_e(u,t) = N_e(u,t_0)\exp[\Im(u,t)],$$
$$\Im(u,t) = \int_{t_0}^{t} g(u,t)dt, \quad (5)$$

where $g(u,t) = C - \left(1/H_i + 1/2H\right)f(u,t)$; $C = 3a/H_i^2 - \beta(1-q)$; $q = Q/\beta N_e$; $a = D_\alpha \sin^2 I$; $f(u,t) = uc\exp(z/2H)\left(1 - \mathrm{e}^{-\upsilon t'}\right)\sin I\cos I$. Function u in solution (5) satisfies equation (3).

As shown in the works (Belashov, 1990; Karpman and Belashov, 1991; Belashov and Vladimirov, 2005; Belashov and Belashova, 2015), the solutions of equation (3) at $\sigma = 0$ in the case $\varepsilon \ll -1$ have the form of classical algebraic solitons of the Kadomtsev–Petviashvili equation; therefore, in this case, the integral in the right-hand side of solution (5) can be calculated analytically (due to its cumbersomeness, we do not present here the analytical form of the functional $\Im(u,t)$). Solution (5), which describes a traveling ionospheric disturbance (TID), will also have the form of an algebraic soliton (Belashov, 1989; Belashov, 1990) (see Fig. 1). If, however, $\varepsilon \to -1$ or $\varepsilon \ll -1$ at $\sigma \neq 0$, one must resort to the numerical integration of Eq. (3), since its analytical solutions are unknown for these cases. The results of numerical integration of equations (3), (5) at $\sigma = 0$ for typical values of parameters of F region and soliton solutions of Eq. (3) describing disturbances that movie with velocities of the order of 200 ms^{-1} were presented in our works (Belashov, 1989; Belashov, 1990; Belashov and Vladimirov, 2005) (see an example of these results in Fig. 2). These works showed that the function $N'(u,t)$ in this case has a wave character with an increasing steepness of the leading front, similar to a shock wave. In this case, it is noted as a phase shift of the TID relative to the IGW phase (within 0.5–5 min) and the relaxation effect in recovery N' after the passage of the IGW soliton. These effects increase with decreasing ε which determines the value of spatial dispersion. As one can see in Figure 2, the form of the ionospheric response to a seismic event significantly depends on the values of the ionospheric parameters determined by the value of the parameter $\varepsilon = -v/v_{\min}^{ph}$ in the region of the propagation of the IGW and the TID excited by it: this can be either a solitary wave disturbance or a wave packet.

The case of $\sigma \neq 0$ was studied in detail by Karpman and Belashov (1991), who showed that allowance for the dissipative term leads to exponential decay of the perturbation with a decrease in its amplitude with decrement $\Gamma(t) \sim \sigma$. In this case, the effects of rupture of the structure and symmetry of the IGW soliton are also observed, along with the relaxation effect in the recovery N'.

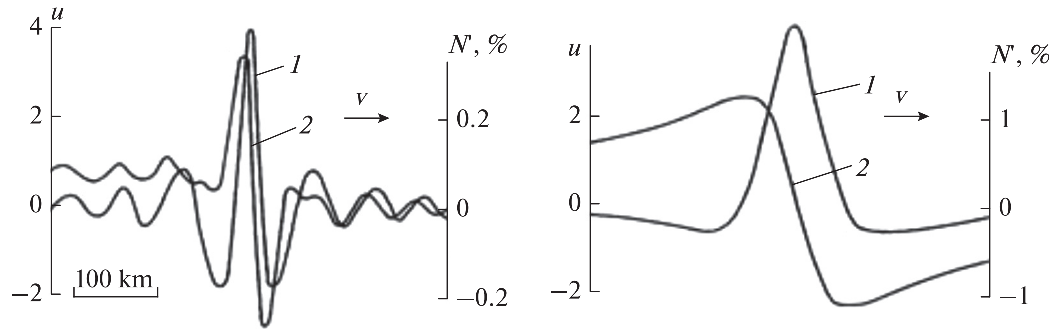

Fig. 2. IGW profiles in the far zone from the nidus and the electron density TIDs excited by them in the ionosphere at $y = 0$: *1*, IGW; *2*, TID (N') at $\varepsilon = -1.2$ (left) and $\varepsilon = -12$ (right).

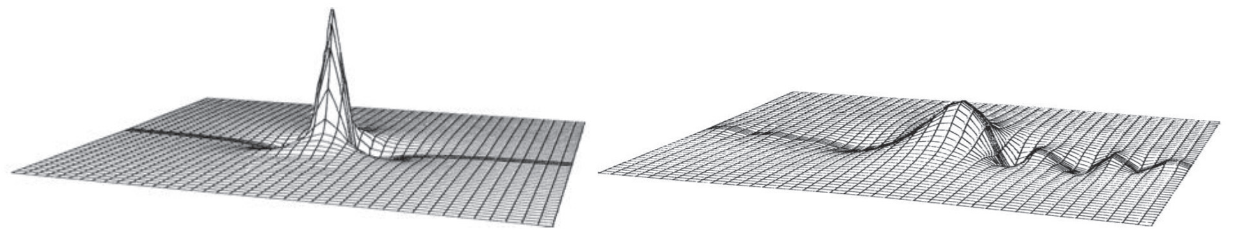

Fig. 3. Evolution in the ionospheric *F* region of the IGW soliton excited by the Rayleigh wave with allowance for dissipative effects ($\sigma = 1$) for $\varepsilon = -1.2$: $t = 0, 0.2$.

Due to the influence of stochastic fluctuations of the wave field, which nearly always occur at ionospheric heights, on the character of the evolution of the TID excited by the Rayleigh wave, Eq. (3) should be supplemented with the term $\kappa(t, r', z)$, which describes such fluctuations. The case of low-frequency, stochastic fluctuations, when it is permissible to take $\kappa = \kappa(t)$ and to count $\varepsilon = 0$ in equation (3), was analyzed in detail analytically in paper (Belashov, 1995). All of the results obtained there can be easily transferred into our equation (3) with the term $\kappa = \kappa(t)$. Thus, even small stochastic fluctuations of the wave field lead to scattering of the disturbance during its propagation, and the scattered IGW soliton of Eq. (3) acquires a wave structure.

However, in the case $\kappa = \kappa(t, r', z)$, the analytical study of this process becomes extremely difficult (Belashov et al., 2018), and we used numerical simulation for this purpose. The results obtained in this case turned out to be qualitatively similar to the case for $\kappa = \kappa(t)$ (Belashov and Vladimirov, 2005; Belashov and Belashova, 2015): here the decay of oscillating solutions and their destruction with time are also observed. Our estimates show that it is almost impossible in the *F* region to distinguish the ionospheric response excited by the Rayleigh surface wave for the distances from the epicenter of $r \gg 12{-}13\ L$.

3. CONCLUSIONS

The paper presents the results of theoretical studies of the manifestation of seismic activity in variations of the vertical velocity of the neutral component and electron density at heights of the ionosphere's *F* region. The considered three-dimensional case takes into account the weak nonlinearity and dispersion, dissipation, and stochastic fluctuations of the wave field in the ionosphere, that allows to obtain more accurate results for the far zone from the epicenter of an earthquake in comparison with the results of earlier studies by other authors. Seismic–ionospheric post-effects are of great interest, in particular, for a better understanding of the cause-and-effect relationships in the solid earth–atmosphere–ionosphere system, the direction-finding of earthquake epicenters, the distinguishing the seismically induced oscillations in the spectrum of ionospheric fluctuations, etc. The effect of a pulsed atmospheric disturbance caused by a Rayleigh surface wave on the ionosphere and the subsequent formation and evolution of a solitary IGW and the electron density TID excited by it at heights of the ionosphere's *F* region in the far zone from the epicenter were considered.

The presented results are in good agreement with the results obtained in complex radiophysical experi- 1
ments during seismic events at the network of stations in the Far East region of Russia (Belashov, 1997b). It is also important to note that the disturbances caused

by seismic sources in the ionosphere's *F* region can be effectively recorded in experiments on Doppler sounding of the ionosphere, in light of earlier results (Belashov, 2018). This type of disturbance can be also observed as a result of the impact of other impulse-type sources, e.g., such as volcanic eruptions, launches of spacecraft and ballistic missiles, and powerful artificial (underground, ground and atmospheric) explosions (Row, 1967; Belashov, 1989; Drobzheva and Krasnov, 2003).

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