

## Integral Equation Methods in Optical Waveguide Theory

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**Abstract**— Optical waveguides are regular dielectric rods having various cross sectional shapes where generally the permittivity may vary in the waveguide's cross section. The permittivity of the surrounding medium may be a step-index function of coordinates. The eigenvalue problems for natural modes (surface, leaky, and complex eigenmodes) of inhomogeneous optical waveguides and for step-index optical waveguides with the smooth boundary of the cross-section are formulated as problems for the set of time-harmonic Maxwell equations with partial radiation conditions at infinity in the cross-sectional plane.

The original problems are reduced with the aid of the integral equation method (using appropriate Green functions) to nonlinear spectral problems with Fredholm integral operators. Theorems on the spectrum localization are proved. It is shown that the sets of all eigenvalues of the original problems may consist of isolated points on the Riemann surface and each eigenvalue depends continuously on the frequency and permittivity and can appear or disappear only at the boundary of the Riemann surface.

The original problems for surface waves are reduced to linear eigenvalue problems for integral operators with real-valued symmetric polar kernels. The existence, localization, and dependence on parameters of the spectrum are investigated.

Collocation and Galerkin methods for numerical calculations of the natural modes are proposed, the convergence of the methods is proved, and some results of numerical experiments are discussed.