One Nonlinear Tri-step Magnetometer Calibration algorithm

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Abstract. This paper describes one numerical procedure for calibrating three-axis magnetometers (accelerometers). Three-step techniques of magnetometer calibration is developed. The first calibration step is solved by using the standard least squares linear estimation techniques and calculate the deviation and the combined scale factors values of the sensor. Second step is based on Newton discrete scheme and is solved algebraically. Error parameters of rotation circle to become an ellipse are derived. Third step is based on the functional minimization by numerical discrete Newton scheme. Sensors misalignment error is defined as an angles between the magnetic sensor sensing axes and the device body axes.

Key words. Three-step techniques of 3 - axis sensors calibration, Newton discrete scheme, functional minimization

Introduction.

Magnetometers measure the intensity of magnetic fields and used in many scientific and engineering applications. Up-to-date miniaturized 3-axis magnetometers and accelerometers well suited for portable navigation application and are important built-in units of many inertial navigation systems. Conventional navigation systems comprised tri-axis angular rate sensors, accelerometers and magnetometers and is combined with sattelits navigation devices. Angular rate sensors and accelerometers is used to collect the 3-axis raw data for the pitch and roll calculation, magnetic sensor raw data for the heading calculation. Calibration is needed to obtain parameters to convert sensor raw data to normalized values for the further navigation calculation.

Reliability degree of 3-axis data source can be defined by certain geometry clause. For example 3 values x, y, z of ideal 3-axis magnetometer must be satisfied of sphere equation

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2} = 1, \qquad (1)$$

If three full round rotations along with device body axis $Z -$ down, $Y -$ down, and $X -$ down respectively at a leveled smooth surface without a nearby interference magnetic field can be performed as depicted on fig.1 we have got a data that is shone on fig.2.

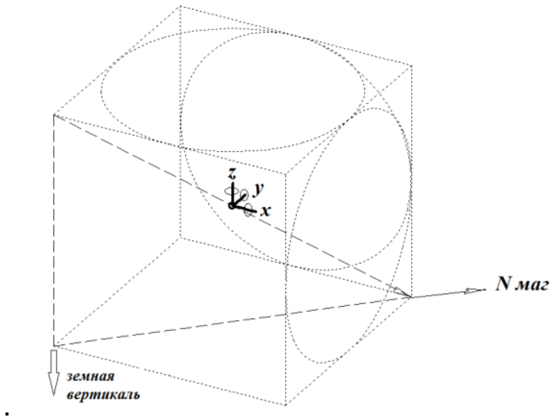
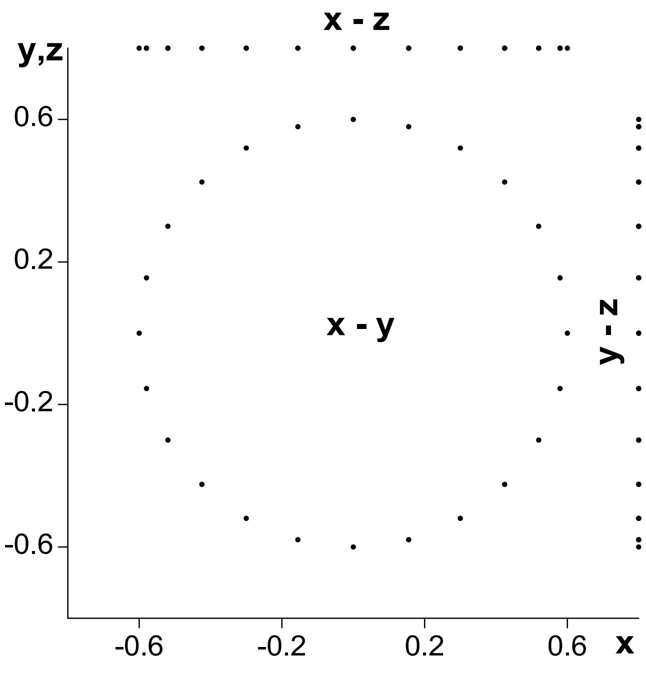


Fig.1

Similar data for example can be obtained from Inertial Measurement Units IMU ADIS16480. Manufacturer provide fully calibrated frame-aligned set of inertial MEMS sensors. On fig.3 we can see that the factory calibrating characteristics is not remained in customer environment.



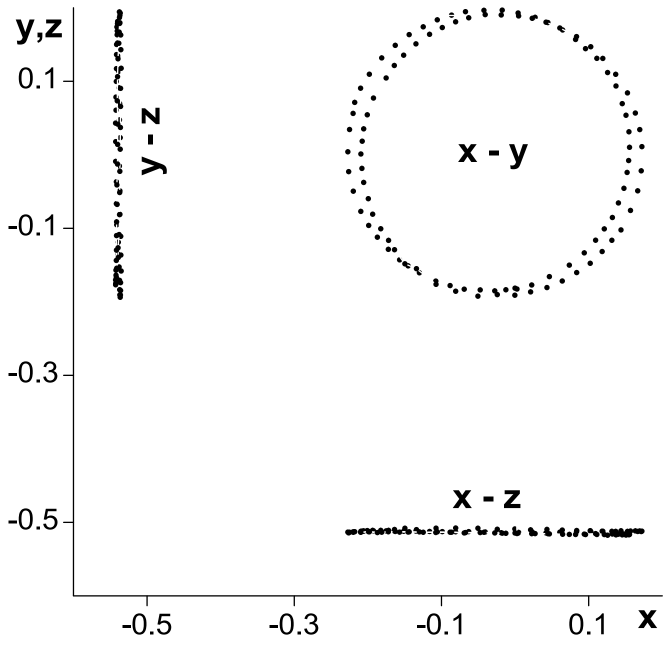


Fig. 2 Raw data of ideal 3-axis magnetometer

Fig.3 Raw data of 3-axis magnetometer with factory-built calibration

The ADIS16480 provides user write control for accelerometer and magnetometer covariance values in user-accessible registers in accordance of following scheme

$$\begin{bmatrix} 1+s_{11} & s_{12} & s_{13} \\ s_{21} & 1+s_{22} & s_{23} \\ s_{31} & s_{32} & 1+s_{33} \end{bmatrix}$$

In order to improve the accuracy of raw sensor data, especially when dealing with low-cost sensors, mathematical models must be built to take into account the various sources of errors. Some, such as scale-factors, misalignment and the resulting cross-coupling of axes, apply to all kinds of sensors (gyroscopes, accelerometers).

There exist various methods for three-axis sensors calibration. Some procedures and algorithm have been proposed for magnetometers calibration [1 - 8]. Our modified algorithm comprise three step calculations.

Lets consider base state of magnetic field distortions. Hard-iron interference magnetic field is normally generated by ferromagnetic materials. The effect of this superposition is to bias the magnetic sensor outputs. A soft-iron interference magnetic field is generated by magnetically soft materials inside the handheld device. The effect of the soft-iron distortion is to make a full round rotation circle become a tilted ellipse. Scale factor error is defined as the mismatch of the sensitivity of the magnetic sensor sensing axes. The effect of the scale factor error causes the full round rotation circle to become an ellipse. Misalignment error is defined as the angles between the magnetic sensor sensing axes and the device body axes. All these factors are gathered into a general matrix $[\mathrm{C}]$, and a zero-bias vector $\boldsymbol{x_0}$, [3].

$$\boldsymbol{x} = [\mathrm{C}]\boldsymbol{x}_c + \boldsymbol{x_0} \qquad (2)$$

were $\boldsymbol{x}_c = (x_c, y_c, z_c)^T$ and $\boldsymbol{x} = (x, y, z)^T$ - are calibrated and raw sensor data, $\boldsymbol{x_0} = (x_0, y_0, z_0)^T$ – is zero-bias vector, $[\mathrm{C}]$ –is resultant matrix defined by diagonal matrix of scale factors r_x, r_y, r_z, 3 - axis rotation matrix $[R] = [R(\theta_1, \theta_2, \theta_3)]$ and misalignment error matrix $[Q]$

$$[\mathrm{C}] = \begin{bmatrix} r_x & 0 & 0 \\ 0 & r_y & 0 \\ 0 & 0 & r_z \end{bmatrix} [R][Q],$$

Fifteen parameters $\boldsymbol{s} = [x_0, y_0, z_0, r_x, r_y, r_z, \theta_1, \theta_2, \theta_3, q_{12}, q_{13}, q_{21}, q_{23}, q_{31}, q_{32}]^T$ have to be defined by calibration procedure.

Equation (2) can be rewritten the following form

$$\boldsymbol{x}_c = [A](\boldsymbol{x} - \boldsymbol{x_0}), \qquad (3)$$

where $[A] = [C]^{-1}$. Substitution of equation (3) into equation (1) yields

$$(\boldsymbol{x} - \boldsymbol{x_0})^T [A]^T [A](\boldsymbol{x} - \boldsymbol{x_0}) = 1. \qquad (4)$$

Matrix form of resulting equation (4) can be written as follower

$$\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}\right)^T \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}\right) = 1. \qquad (5)$$

Matrix elements a_{ij} of equation (5) can be defined by least square method (LSM). More convenient approach for a_{ij} calculation we found in [3] and then slightly modified it.

In accordance of this approach equation (5) can be rearranged to following explicate form

$$\frac{1}{3}(x^2 + y^2 - 2z^2)(\hat{a}_{11} - \hat{a}_{33}) + \frac{1}{3}(x^2 - 2y^2 + z^2)(\hat{a}_{11} - \hat{a}_{22})\ ]$$

$$+2\hat{a}_{12}xy + 2\hat{a}_{13}xz + 2\hat{a}_{23y}yz -$$

$$-(2\hat{a}_{11}x_0 + 2\hat{a}_{12}y_0 + 2\hat{a}_{13}z_0)x - (2\hat{a}_{12}x_0 + 2\hat{a}_{22}y_0 + 2\hat{a}_{23}z_0)y$$
$$-(2\hat{a}_{13}x_0 + 2\hat{a}_{23}y_0 + 2\hat{a}_{33}z_0)z -$$

$$-1 \cdot \left(\frac{1}{u} - \hat{a}_{11}x_0^2 - 2\hat{a}_{12}x_0y_0 - 2\hat{a}_{13}x_0z_0 - \hat{a}_{22}y_0^2 - 2\hat{a}_{23}y_0z_0 - \hat{a}_{33}z_0^2\right)$$
$$= -\frac{1}{3}(x^2 + y^2 + z^2), \qquad (6)$$

where $u = a_{11} + a_{22} + a_{33}$, $\hat{a}_{ij} = a_{ij}/u$.

Then we have to form the LSM equation on batch of magnetometers data $x_i, y_i, z_i, i = 1, m$

$$[\Lambda]\boldsymbol{e}_{LSM} = \boldsymbol{e}, \qquad (7)$$

where ith row of $[\Lambda]_{m\times 9}$ matrix is defined by following expression

$$\left[\frac{1}{3}(x_i^2 + y_i^2 - 2z_i^2),\ \ \frac{1}{3}(\ x_i^2 - 2y_i^2 + z_i^2),\ \ 2x_iy_i,\ \ 2x_iz_i,\ \ 2y_iz_i,\ \ x_i, y_i, y_i,\ \ -1\right],$$

$$i = 1, m$$

ith element $e_{(i)} = \frac{1}{3}\left(x_i^2 + y_i^2 + z_i^2\right)$, vector $\boldsymbol{s}_{LMS}$ is presented by linear combination of $\hat{a}_{ij}$ coefficients.

Required batch of magnetometers data x_i, y_i, z_i we can obtain by three full round rotation along with device body axis Z – approximately down, the Y – down, and X – down respectively. For this we use radio transmitter for sending magnetometer data to PS.

After yielding of $\boldsymbol{e}_{LMS}$ by solving least square equation

$$[A]^T[A]\boldsymbol{e}_{\text{МНК}} = [A]^T\boldsymbol{e} \tag{8}$$

we can define coefficients $\hat{a}_{ij}$ u, and then calculate matrix components a_{ij}, bias $[x_0 \quad y_0 \quad z_0]^T$, from following dependences: $e_{LMS}(1) = \hat{a}_{11} - \hat{a}_{33}$; $e_{LMS}(2) = \hat{a}_{11} - \hat{a}_{22}$; $\hat{a}_{11} + \hat{a}_{22} + \hat{a}_{33} = 1$; $e_{LMS}(3) = \hat{a}_{12}$; $e_{LMS}(4) = \hat{a}_{13}$; $e_{LMS}(5) = \hat{a}_{23}$; $e_{LMS}(6) = 2\hat{a}_{11}x_0 + 2\hat{a}_{12}y_0 + 2\hat{a}_{13}z_0$; $e_{LMS}(7) = 2\hat{a}_{12}x_0 + 2\hat{a}_{22}y_0 + 2\hat{a}_{23}z_0$; $e_{LMS}(8) = 2\hat{a}_{13}x_0 + 2\hat{a}_{23}y_0 + 2\hat{a}_{33}z_0$; $e_{LMS}(9) = \left(\frac{1}{u} - \hat{a}_{11}x_0^2 - 2\hat{a}_{12}x_0y_0 - \hat{a}_{13}x_0z_0 - \hat{a}_{22}y_0^2 - 2\hat{a}_{23}y_0z_0 - \hat{a}_{33}z_0^2\right)$.

The Cholesky factorization provides an upper triangular matrix D, such that $D^TD = B = [A]^T[A]$. We have to take into account that there are uncountable quantity of matrices D such that $D^TD = B$. This is easy to see since any rotation matrix R satisfies $R^TR = I$ and $(RD)^T(RD) = D^TR^TRD = D^TD = B$. Therefore only diagonal elements of triangular matrix D will be taken at the first step calibration.

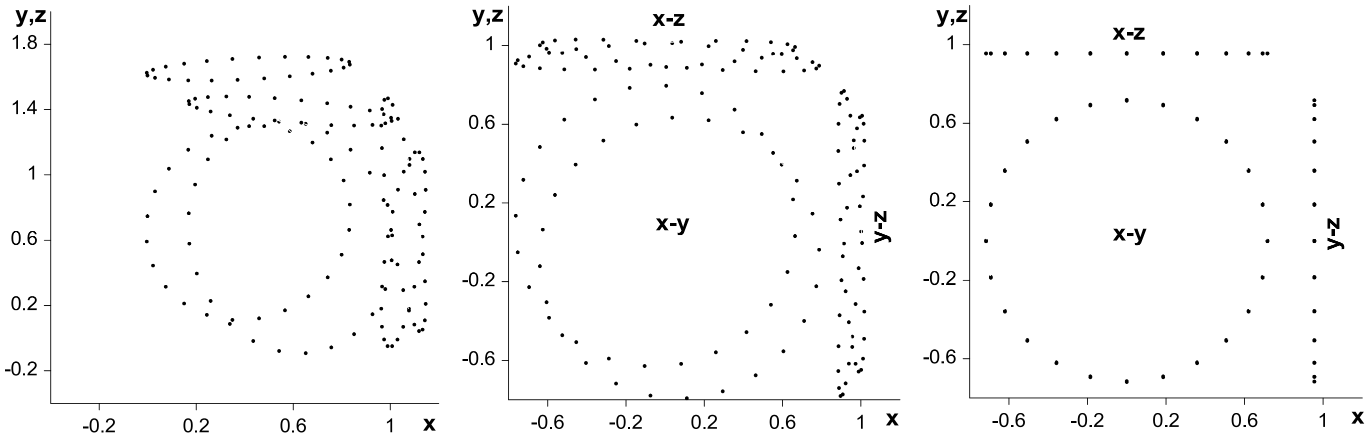
Some approach use eigenvectors decomposition of symmetric matrix. Matrix $[A]^T[A]$ can be expressed as

$$[A]^T[A] = \begin{bmatrix} \sqrt{r_x^2} & & \\ & \sqrt{r_y^2} & \\ & & \sqrt{r_z^2} \end{bmatrix} \left[\begin{vmatrix} s_{11} \\ s_{12} \\ s_{13} \end{vmatrix} \quad \begin{vmatrix} s_{21} \\ s_{22} \\ s_{23} \end{vmatrix} \quad \begin{vmatrix} s_{31} \\ s_{32} \\ s_{33} \end{vmatrix}\right]^T, \tag{10}$$

where s_i –are eigenvalue, (s_{i1}, s_{i2}, s_{i3}) – are correspondent eigen-vectors of $[A]^T[A]$.

All above mentioned approach yields bias value x_0, y_0, z_0 and scale coefficients r_x, r_y, r_z. Rotation angles $\theta_1, \theta_2, \theta_3$ can be obtained only in case of $r_x \neq r_y \neq r_z$.

Therefore we devise following algorithm of vector $\boldsymbol{s}$ components calculation. We develop numerical Newton scheme for rotation angle $\theta_1, \theta_2, \theta_3$ calculation that be write below. At the first lets put throw following numerical test. Distort data of ideal magnetometer by scaling r_x, r_y, r_z, by rotation $\theta_1, \theta_2, \theta_3$ and by shift x_0, y_0, z_0 along $X - Y - Z$ axis and then try to restore its by above mentioned scheme.



a) b) c)

Fig.4 Magnetometer data restore

The distorted of ideal magnetometer data is shone on fig.4a. Calibrated data after the first step is depicted on the fig.4b. Calibrated data after calculation of rotation parameter $\theta_1, \theta_2, \theta_3$ is shone on the fig.4c. We can see that calculation of rotation parameter $\theta_1, \theta_2, \theta_3$ is enough for data restore at the second step calibration in this case.

If compare fig.4b and fig.4c the following calculation scheme can be suggested. Magnetometer data at the second must be obtained by three full round rotations along with device body axis $Z -$ strictly down, $Y -$ strictly down, and $X -$ strictly down respectively at a leveled smooth surface without a nearby interference magnetic field. The following function can be chosen as a clause for $\theta_1, \theta_2, \theta_3$ calculation by numerical discrete Newton scheme

$$J_{(\theta_1, \theta_2, \theta_3)} = (z_{max} - z_{min})^2_{rot\_Z} + (y_{max} - y_{min})^2_{rot\_Y} + (x_{max} - x_{min})^2_{rot\_Z}. \quad (11)$$

Discrete scheme means that we use numerical (not analytical) finite-difference method for derivatives calculation in Newton scheme

$$(\theta_1,\theta_2,\theta_3)^T{}_{(n+1)} = (\theta_1,\theta_2,\theta_3)^T{}_{(n)} - \left[\frac{d^2 J_{(\theta_1,\theta_2,\theta_3)}}{d[(\theta_1,\theta_2,\theta_3)^T]^2}\right]^{-1} \frac{dJ_{(\theta_1,\theta_2,\theta_3)}}{d(\theta_1,\theta_2,\theta_3)^T} . \qquad (12)$$

Solution can be obtained of few iteration (12) if numerical calculations is proper arranged. Final result of data restore is shone on fig.4c. Take into account that after the Cholesky factorization of matrix $B = [A]^T[A]$ addition normalization of r_x, r_y, r_z is done because we do not need absolute value of r_x, r_y, r_z but only ratio of coefficients. Magnetometer data on fig.3 confirm the good factory calibration of ADIS16480 for the first step calculations of $x_0, y_0, z_0, r_x, r_y, r_z$ and poor one for the second step calculations of $\theta_1, \theta_2, \theta_3$. On the fig.5 results of re-calibration are depicted: a - corresponds raw data, b - corresponds one step calibration, c - corresponds two step ones. We can see that the second step calculation yields more conspicuous data correction, fig.4c.

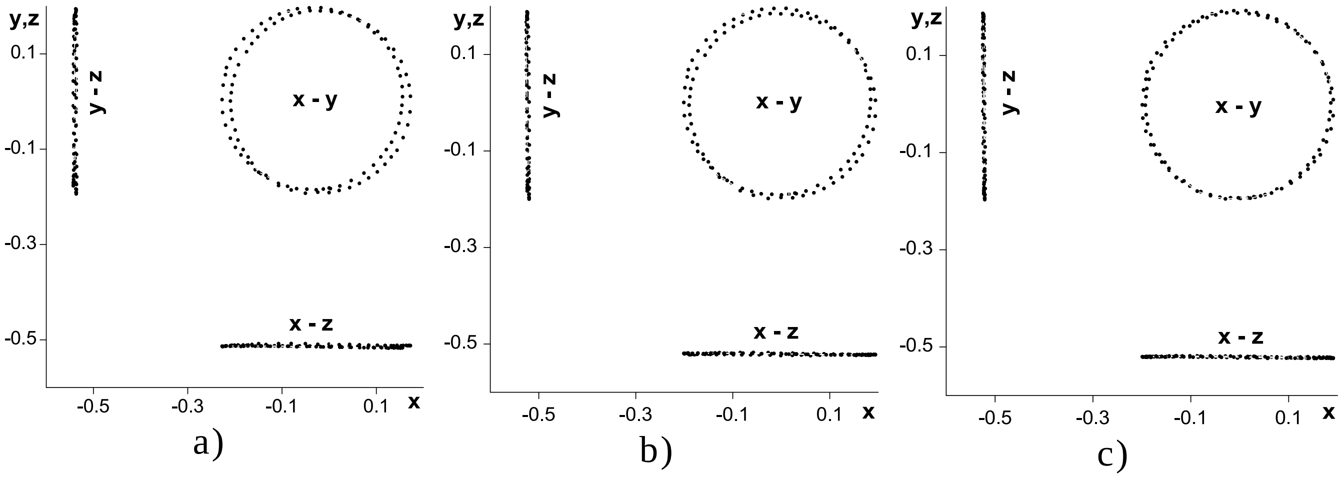


Fig.5 Two sep calibration of IMU ADIS16480

It sould be stated that if the first step requires three full rotation along three arbitrary axis the second step requires three full rotation along three strictly vertical axis.

At the third calibration we have to define misalignment error matrix $[Q]$. Due to expectable small value of misalignment angles we can suppose diagonal elements equal 1 and calculate only six value of $q_{ij}, (i \neq j)$. For this we use clause (11).

$$J_{(q_{ij})} = (z_{max} - z_{min})^2_{rot\_X} + (y_{max} - y_{min})^2_{rot\_Y} + (x_{max} - x_{min})^2_{rot\_Z})$$

and Newton scheme

$$(q_{ij})^T{}_{(n+1)} = (q_{ij})^T{}_{(n)} - \left[\frac{d^2 J_{(q_{ij})}}{d[(q_{ij})^T]^2}\right]^{-1} \frac{dJ_{(q_{ij})}}{d(q_{ij})^T} \; . \qquad (14)$$

Solution can be obtained of few iteration (14) if numerical calculations is proper arranged.

Comparative results of the second and third step calibrations are shone on fig.6 (a and b)

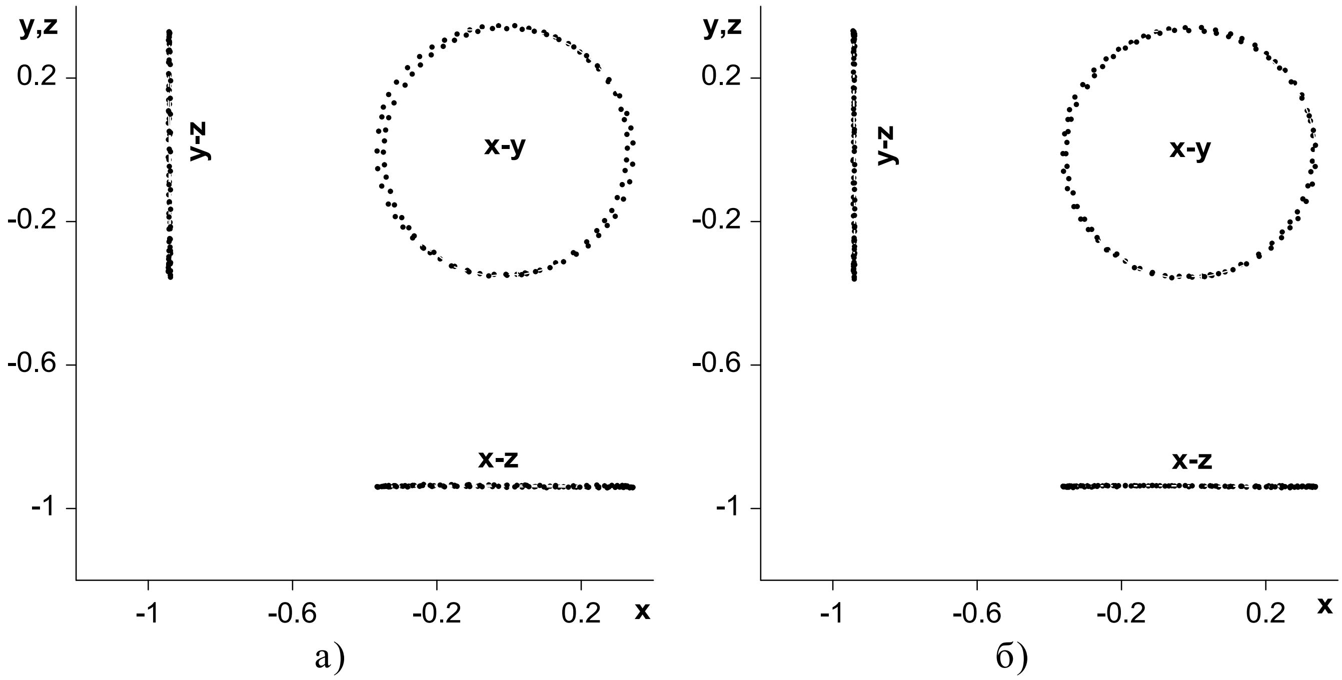


Fig.6 Second and third step calibrations

The third step calibration yields value of q_{ij} that defines misalignment error

$$\begin{bmatrix} 1.0 & 0.0061 & -0.0039 \\ 0.0054 & 1.0 & 0.0055 \\ -0.0032 & 0.0057 & 1.0 \end{bmatrix} \qquad (15)$$

The small value of $q_{ij}, (i \neq j)$ is explained by high quality of IMU.

At the above we can state:

1. Only three step 3-D magnetometer calibration provide calculation of full set correction parameters that comprise hard-iron and soft-iron interference, scale factor error and misalignment error. The first calibration step is solved by using the standard least squares linear estimation techniques. Second and third steps are based on Newton discrete scheme and is solved algebraically. Solution can be obtained of few iteration if numerical calculations is proper arranged.

2. Magnetometer calibration scheme by Cholesky factorization of symmetric matrix $[A]^{T}[A]$ provides good correction scale factor, hard-iron and adequate soft- iron interference, but do not provide misalignment correction.

3. Magnetometer calibration scheme by eigenvectors decomposition of symmetric matrix provide adequate correction parameters only in case of $r_x \neq r_y \neq r_z$ and do not provide misalignment correction.

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