Mechanics of Composite Materials, Vol. 50, No. 2, May, 2014 (Russian Original Vol. 50, No. 2, March-April, 2014)

THEORETICAL-EXPERIMENTAL METHOD FOR DETERMINING THE PARAMETERS OF DAMPING BASED ON THE STUDY OF DAMPED FLEXURAL VIBRATIONS OF TEST SPECIMENS

1. EXPERIMENTAL BASIS

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Keywords: *theoretical-experimental method, internal damping, aerodynamic damping, experimental setup, test specimens, damped flexural vibrations*

A theoretical-experimental method is proposed for determining the logarithmic decrement of materials. The method is based on the measurement of amplitudes of damped vibrations at the free end of cantilever test specimens in the first resonance mode. To determine the damping properties of materials in tension-compression, three-layer test specimens with a steel core and external layers of a soft damping material is used, whereas for the case of shear deformation, steel external layers and a core from a soft damping material layer are employed. A considerable effect of aerodynamic forces on the logarithmic decrement of the vibrations was revealed. This effect can become decisive for test specimens of width exceeding 15 mm.

Introduction

Resonance is one of the most dangerous modes of dynamic deformation, which is realized in a structure when the frequency of its natural vibration coincides with the frequency of an external cyclic action. It is known that, at such a loading mode, the peak values of parameters of the dynamic stress–strain state (SSS) grow manifold. For their correct and reliable theoretical determination, with an accuracy adequate for practical purposes, in calculation relations, it is necessary to take proper account of the damping properties of structural materials governed by internal friction. Presently, methods for their

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Translated from Mekhanika Kompozitnykh Materialov, Vol. 50, No. 2, pp. 185-198 , March-April, 2014. Original article submitted October 11, 2013.

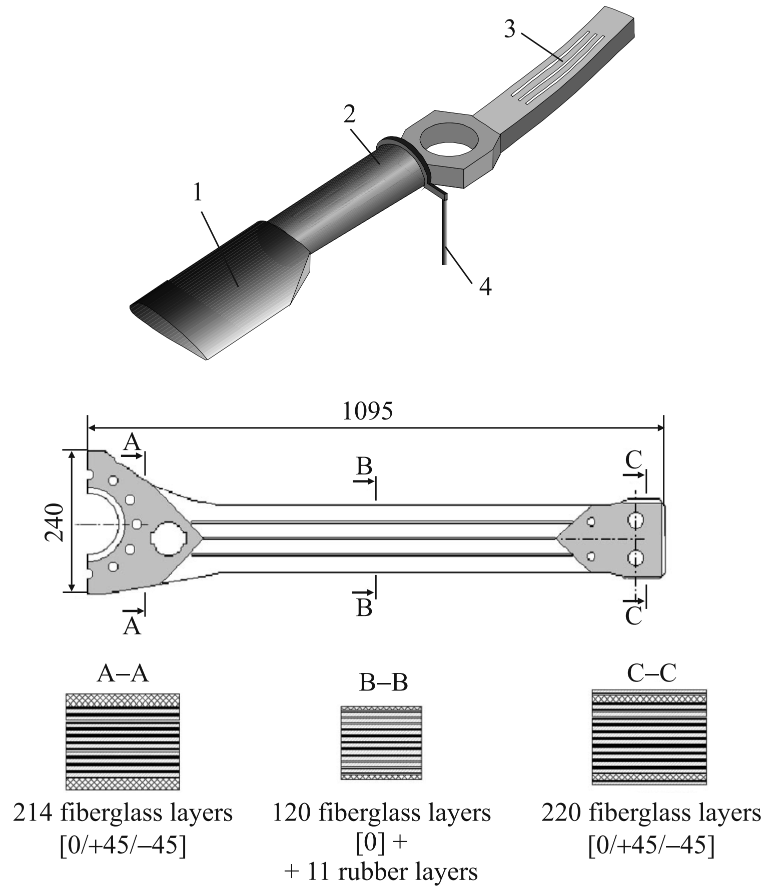

Fig. 1. Structural scheme of the torsion bar of a lightweight helicopter: 1 — blade, 2 — casing of the torsion bar, 3 — torsion bar, and 4 — pull rod of blade pitch control.

determination and the corresponding mathematical models elaborated for their description have been extensively discussed in the literature (see, for example, [1-12]).

The traditional structural materials (metals and their alloys), characterized by great values of elastic and strength parameters, as a rule, have small indices of damping properties. Therefore, to raise the damping parameters of thin-walled structures made of traditional materials, their elements are manufactured in the form of multilayered systems, by alternating rigid layers with rather soft ones having good damping properties across the thickness of the structure. Such elements are now widely employed in structures of aircraft engineering and shipbuilding, in automobiles, in civil and industrial constructions, and others, where various elastomers (rubber), foam plastics, and fillers of a particular structure are used as damping layers. In operation, such layers, as parts of a composition of multilayered elements of the structures, as a rule, are subjected to small tension-compression deformations comparable with linear deformations of the rigid layers, but they can frequently occur in the conditions of moderate transverse shear deformations. Consequently, their damping properties should be investigated only in the range of small deformations of tension–compression and moderate deformations of transverse shear.

In aircraft engineering, a typical example of a multilayered structural element which must possess the damping parameters required is the torsion bar connecting the blade with the shaft of the main rotor in lightweight helicopters of new generation (Fig. 1) [13].

Structurally, the torsion bar is a multilayered lamellar rod construction consisting of layers of fiberglass and soft rubber alternating across its thickness. During operation, the torsion bar is subjected to intense vibrations in the flapping and rotation planes, whose damping is mainly ensured by the soft rubber layers of the torsion bar. Therefore, the problem of determina-

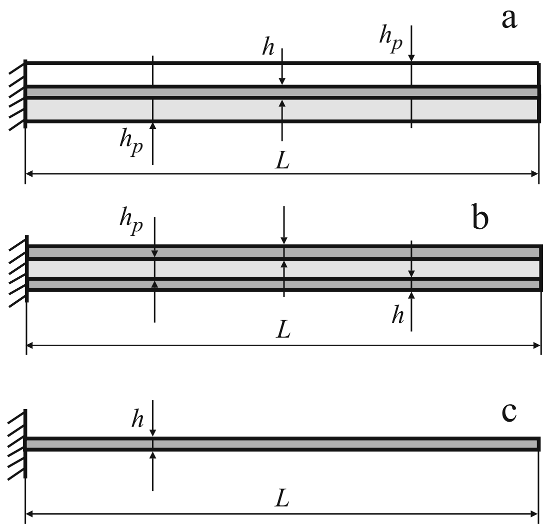

Fig. 2. Specimens for testing the damping properties of materials in tension-compression (a), shear (b), and base (c).

tion of damping properties of the resin is part of the important general problem of prediction and determination of damping properties of the torsion bar and blades of the main rotor.

1. Method of Experimental Investigation of Damping Parameters of Test Specimens during Their Damped Flexural Vibrations

At present, an experimental determination of the damping properties of materials in the range of frequencies from 50 to 5000 Hz is based on the American Standard ASTM E-756 [14], according to which the acoustic method in the resonance mode is used to investigate the dynamic behavior of cantilever test specimens of various structure (Fig. 2).

Test specimens of a symmetric structure in the form of a plane rectangular sandwich plate consisting of an inner layer (core) made of a rigid material (steel) and two outer layers made of the soft material to be tested (see Fig. 2a) make it possible to examine the damping properties of a material in tension-compression. To determine the damping properties of the material in the conditions of transverse shear deformations, it is suggested to use test specimens in the form of a three-layer rod consisting of two thin outer layers made of the rigid base material and the soft inner layer material to be tested (see Fig. 2b). In both cases, realization of the testing standard mentioned to determine the damping properties of the soft material requires a preliminary estimation of damping properties of the base (see Fig. 2c).

In the sphere of application and practical use, it is of importance to estimate the damping properties of different structural materials in the range of frequencies $f =$ 0-100 Hz, which corresponds to the frequency range of dynamic loading of structures most often met in practice in the real conditions of their operation. Investigations in this range of frequencies cannot be realized with a necessary degree of accuracy without modification of the above-mentioned standard [14], which is based on measuring and processing the peak values of pressure of acoustic waves radiated by test specimens upon their vibrations in the resonance mode. Therefore, in the present study, it is suggested to determine the damping properties of materials in a low-frequency range of dynamic deformation by processing the experimental vibrorecords of damping flexural vibrations of test specimens.

It is obvious that reliable results in using this method can be obtained only in the case of a sufficiently accurate description of the entire dynamic process of deformation of test specimens over a time interval great enough. For this purpose, a special experimental setup was created, whose scheme is shown in Fig. 3. It consists of a foundation 1 and a bearing wall 2,

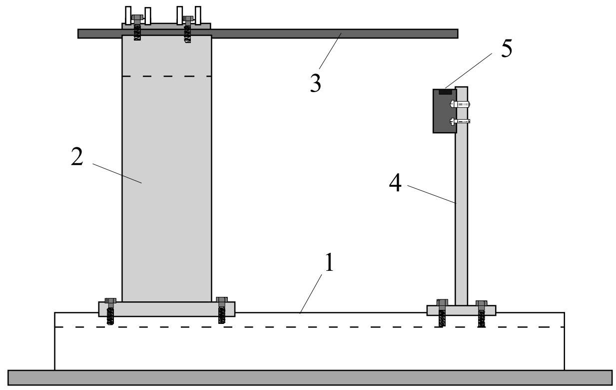

Fig. 3. Scheme of the experimental unit. Explanations in the text.

which are rigidly connected together. On the wall, a cantilever test specimen 3 is fixed. The fastening is realized with the help of separated laths connected to the wall by two rows of bolts, which exclude rotation of the test specimen in the fastening section. Also, a rack 4 for fastening a laser sensor of displacements 5 is erected on the foundation. The position of the sensor can be changed along the foundation to measure the vibration amplitudes of points of the test specimen when its boom is varied. The setup is supplied with a triangulation laser sensor of trademark RIFTEK (RF603-X/100), providing measurements of vibration amplitudes accurate to 0.01mm. Results of the measurements in the digital format are transferred to a PC.

The damping properties of materials were investigated on cantilever test specimens, which, after an initial deflection imparted to them, made free damped flexural vibrations. The measurements began with some delay necessary for transition from the initial (static) flexed state to the first, lowest vibration mode of the specimen. The vibration amplitudes of the end point of the test specimen were registered during the period of a great (up to thousand) number of cycles, while deviations of the plate from equilibrium still allowed sufficiently accurate measurements.

The software developed allows one to carry out up to 1000 amplitude measurements per second and thus to reflect the damping character of vibrations rather accurately. For example, at a vibration frequency of 25 Hz, during one period of vibrations, about 40 amplitude measurements can be realized. A typical vibrorecord of vibrations of the end point of a test specimen is shown in Fig. 4. The results presented were obtained in testing a base made of a St3 sheet steel of thickness $h = 1$ mm having a boom with $L = 300$ mm and width $b = 15$ mm (see Fig. 2c). The upper part of the figure illustrates the general evolution of the process, but the insets show vibration dynamics on various time intervals in detail. The dots are experimental points. The continuous line is a harmonic approximation of actual data. The approximation algorithm is described in the following section. As seen from the insets, the measurement accuracy is sufficient for confident registration of the amplitude and frequency characteristics of vibrations even at vibration amplitudes of order of fractions of a millimeter.

2. Processing of Experimental Results

In processing the results of investigation of a test specimen, the logarithmic decrement δ and the cyclic frequency ω of vibrations of the end of the cantilever beam and the current value of their amplitude A must be determined from the experimental vibrorecord. The resulting relations $\delta(A)$ and $\omega(A)$ serve as a basis for estimating the damping properties of the material (materials) of test specimens.

At the first stage of processing of experimental data, a harmonic approximation of the actual vibrorecord is constructed according to the following algorithm.

(1) The zeros $t_1, t_2, \ldots$ of the experimental relation $A^{\exp}(t)$ and the values of maxima (minima) of $A^{\exp}(t)$ on the intervals $\left[t_k, t_{k+1}\right]$ are found: $A_{3/2}, -A_{5/2}, A_{7/2}, -A_{9/2}, \ldots$

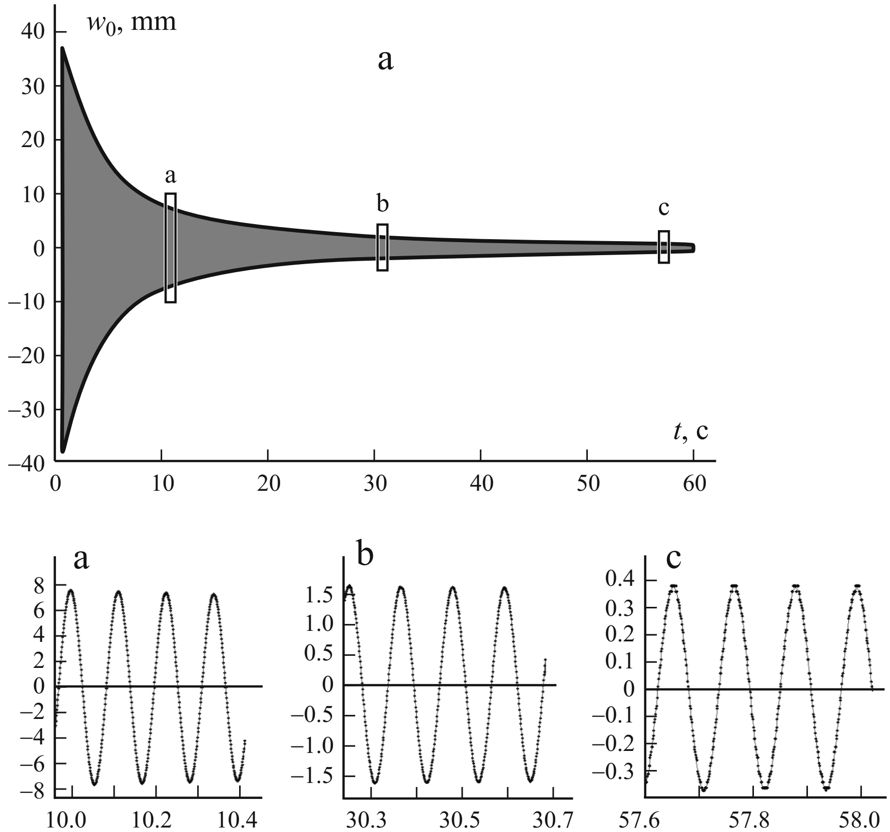

Fig. 4. Vibration dynamics of the end of a test specimen. Explanations in the text.

2. On each interval $\left[t_{k-1}, t_{k+1}\right]$, a harmonic approximation is found in the form

$$A^{\text{appr}} = \varepsilon_k + A_k \sin \omega_k t.$$

Three required parameters $\left(\varepsilon_k, A_k, \omega_k\right)$ are found from the condition of minimum of the root-mean-square deviation A^{appr} from $A^{\exp}$ on the interval considered. As an initial approximation to $\left(\varepsilon_k, A_k, \omega_k\right)$, the expected values $\left(0, 0.5\left(A_{k+1/2} + A_{k-1/2}\right), 2\pi / \left(t_{k+1} - t_{k-1}\right)\right)$ are chosen.

3. The values of ω_k obtained are smoothed out by taking the moving average over several (usually 4-6) neighboring points.

As a result of the procedure, approximate relations for the vibration amplitude and frequency as functions of time, $A(t)$ and $\omega(t)$, are determined.

The need for additional smoothing of frequency (step 3) is explained by the fact that, contrary to the vibration amplitude, it varies within narrow limits close to the basic frequency ω_0, which corresponds to the first vibration mode of a cantilever plate. Therefore, experimental errors affect the determination of the "informative" relation $\omega\left(t\right) - \omega_0$ much stronger than that of $A(t)$. This effect is most pronounced at great values of time (at small vibration amplitudes), when the relative experimental error increases. This is illustrated in Fig. 5a, where dots and the thin continuous line show frequencies for a steel test specimen found by a harmonic approximation without and with smoothing, respectively. It is seen that the smoothing considerably reduces the scatter of experimental data.

For the vibration amplitude, the results of the first stage of processing experimental data are depicted by dots in Fig. 5b as a relation between $\ln A$ and time. It is seen that $\ln A(t)$ is a smoothly varying function. Therefore, it is convenient to approximate it by a smoothing spline with a small (5-8) number of nodes. The relationship $\omega(t)$ is also approximated by a

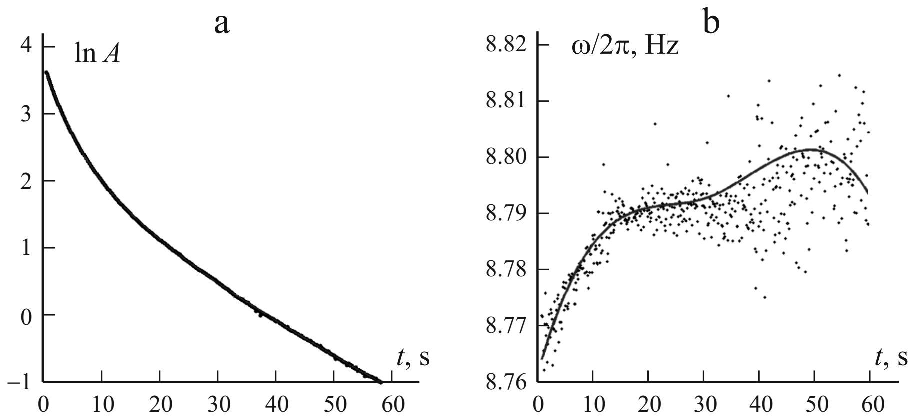

Fig. 5. Logarithmic amplitude and vibration frequency vs time for a plate of length $L = 300$ mm and width $b = 15$ mm made of a St3 sheet steel of thickness $h = 1$ mm. Explanations in the text.

smoothing spline with 4-5 nodes. The second stage of processing of experimental data consists in constructing the spline approximations mentioned. The result is shown by the continuous lines in Fig. 5.

The need for the second stage is associated with the fact that, for determining the logarithmic decrement of vibrations, it is necessary to differentiate the quantity $\ln A$ with respect to time:

$$\delta = \frac{2\pi}{\omega_0} \frac{d \ln A}{dt}.$$

The spline approximation regulates the initially incorrect operation of differentiation of experimental data. Having found the relationship $\delta(t)$ and knowing the spline approximations $A(t)$ and $\omega(t)$, we can express the quantities ω and δ in terms of A, thus achieving the result sought-for.

3. Experimental Results

3.1. Vibration frequency. The typical $\omega(A)$ relations found according to the algorithm described are shown in Fig. 6: the relation between the vibration frequency and the dimensionless vibration amplitude $\kappa_0 = A/b$ for a steel plate ($L = 300$ mm, $b = 15$ mm, and $h = 1$ mm) (a) and for a sandwich plate ($L = 300$ mm, $b =$ 10 mm, and $h + 2h_p =$ 1 + 1.2 = 2.2 mm) (b). The presumed continuation of these relations toward the region of low amplitudes is shown by the dotted lines. As seen, the qualitative behavior of $\omega(A)$ is similar in both cases. With growing amplitude, the vibration frequency decreases. It is obvious that, for a sandwich plate, the amplitude dependence of frequency is pronounced more markedly.

We should point to the wide scatter of experimental data for the quantity ω at great values of time. This forced us to regard the resulting spline approximation $\omega(t)$ at $t > 30$ s, when the vibration amplitude (see Fig. 4) fell below the level of 1.5 mm, with great caution. At the same time, the growth in frequency observed at $t < 10$ s (the vibration amplitude exceeds 7 mm) cannot be explained by experimental errors. Indeed, the variation in vibration frequency in principle does not depend on the internal damping. As will be shown further, the aerodynamic forces can cause only a 0.1% increase in frequency. At the same time, on the interval $t < 30$ s (see Fig. 5), the frequency varies by 0.7%. To our opinion, the regularity observed here should find its explanation in the account of geometrically nonlinear effects in the basic equation of deformation of plates. In prospect, this will allow one to obtain an extensive additional experimental information useful in developing nonlinear theories. However, the discussion of this question is beyond the scope of the given series of studies. Therefore, in what follows, we will analyze only the relation between the logarithmic decrement of vibrations and the current vibration amplitude of the plate. In view of the purposes of present investigations, this relation is more informative.

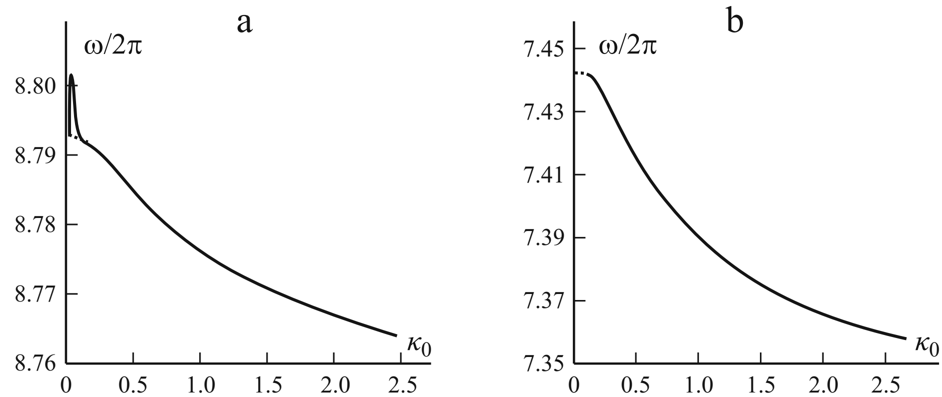

Fig. 6. Vibration frequency vs the dimensionless amplitude $\kappa_0 = A_0/b$ for steel (a) and sandwich (b) plates. Other explanations in the text.

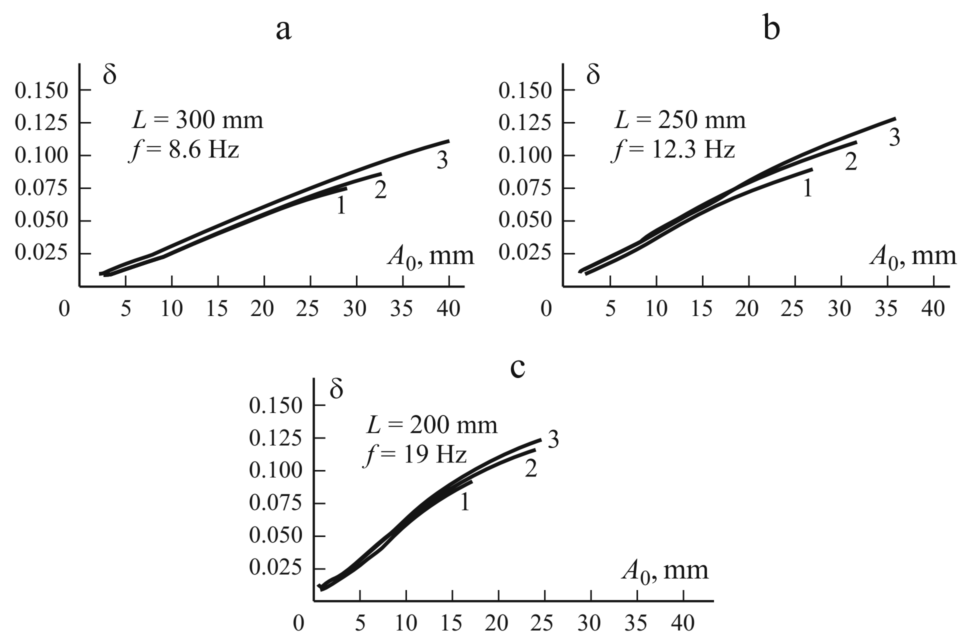

Fig. 7. Logarithmic decrement of vibrations δ vs amplitude A_0 for steel test specimens of width 10 (1), 15 (2), and 20 mm (3).

The investigations into the damping properties of the material of the base (St3 steel) performed at different vibration frequencies confirm the commonly adopted conclusion that the relation between the energy dispersion and frequency of cyclic deformation of the material is weak.

3.2. Relation between the logarithmic decrement of vibrations and the width of plate. Figure 7 illustrates the relationships between the logarithmic decrement of vibrations δ and the vibration amplitude A_0 of the end of a homogeneous steel cantilever beam 1 mm thick, found as a result of the above-described procedure of processing experimental data.

An analysis of the relationships reveals an increase in the damping properties of test specimens of the base, even at an insignificant increase in the width of plate (see Fig. 7), which can be explained only by the effect of external aerodynamic damping. This dependence is most marked in testing specimens with significantly different widths.

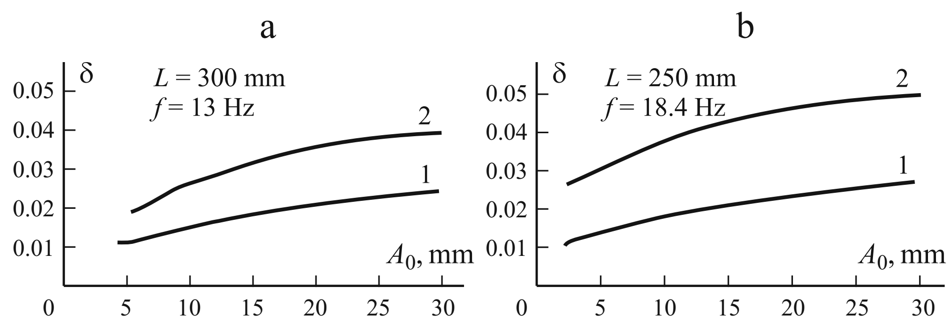

Fig. 8. Logarithmic decrement of vibrations δ vs A_0 for steel test specimens of width 10 (1) and 50 (2) mm.

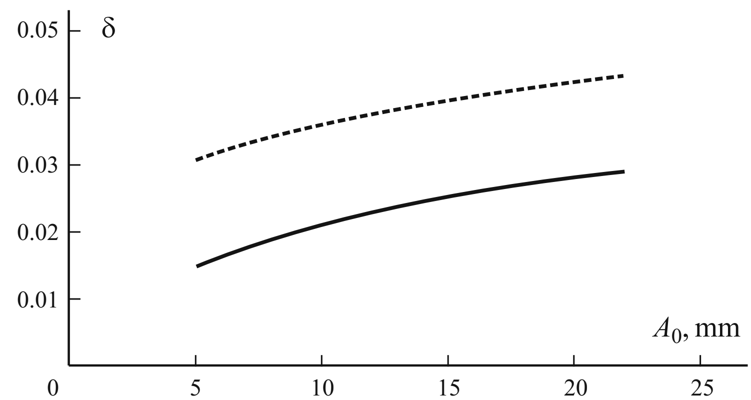

Fig. 9. Logarithmic decrements of vibrations δ vs A_0 for test specimens of the base (——) and for a sandwich test specimen (– – –).

Figure 8 shows the results of processing experimental vibrorecords of test specimens of thickness $h = 1.5$ mm and width $b = 10$ and 50 mm made of a steel with low internal damping. It is seen that the external aerodynamic damping is comparable with the internal damping of the steel.

We should note that, according to the international standard [14], we examined only vibrations with a small amplitude and did not take into account the aerodynamic damping. Therefore, to minimize the influence of external aerodynamic damping on the character of vibrations, it is recommended to investigate the damping properties of materials on specimens of width $b = 10$ mm.

An analysis of results of the experiments (see Figs. 7 and 8) showed also a noticeable increase in the damping properties of specimens of the base with increasing vibration frequency. This fact contradicts the commonly accepted concept [1-12] saying that the forces of internal inelastic resistance do not depend on strain rate (on vibration frequency), which is confirmed by experimental data in a wide range of frequencies and amplitudes of strains [1-12.] Again, the contradiction revealed can only be explained by the external aerodynamic damping, which naturally increases with vibration frequency.

The results of analysis of the experimental vibrorecords of flexural damped vibrations of test specimens of the base revealed an urgent need for account of the aerodynamic interaction of test specimens in estimating the damping properties of a material.

3.3. Tension-compression tests of a material. The problem on determining the damping properties of the soft rubber layers of the torsion bar (see Fig. 1) can be solved by testing cantilever test specimens (see Fig. 2) for damped flexural vibrations and their registration in the form of vibrorecords with subsequent theoretical processing.

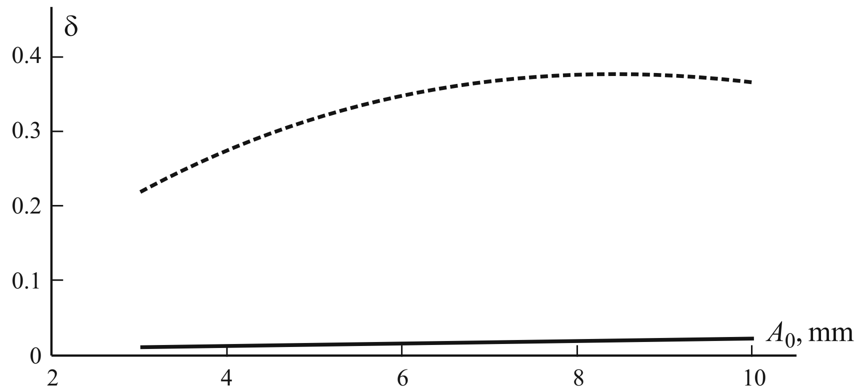

Fig. 10. Logarithmic decrements of vibrations δ vs A_0 for test specimens of the base (——) and for a sandwich test specimen (– – –).

External layers made of a soft sheet rubber of thickness $h = 1.2$ mm (see Fig. 2a) were glued on test specimens of the base of width $b = 10$ mm (see Fig. 2c).

The results obtained for a sandwich test specimen of length $L = 200$ mm are shown in Fig. 9.

As seen from the data in Fig. 9, the resin layers slightly raise the damping properties of the test specimen, although the material examined is characterized by a significant internal friction. This is explained by small arms of the damping forces of tension–compression for thin test specimens under flexural vibrations. The insignificant difference in the logarithmic decrements of vibrations of test specimens imposes more rigorous requirements on the accuracy of determination of damping properties, which cannot be satisfied without account of the external aerodynamic damping.

We should also note that the difference between the logarithmic decrements of vibrations obtained only slightly depends on the vibration amplitude, which points to a poor strain dependence of damping properties of the resin.

3.4. Shear tests of a material. To investigate the damping properties of rubber in the conditions of transverse shear, three-layer test specimens (see Fig. 2b) of width $b = 10$ mm with load-carrying layers (base) made of St3 steel of thickness $h = 0.52$ mm and a soft rubber filler of thickness $h_p = 0.6$ mm were prepared.

The results of processing of the vibrograms of damped vibrations of test specimens of the base and of a three-layer rod of length $L = 300$ mm are presented in Fig. 10. The multiple increase in the damping properties of the sandwich specimen compared with those of the test specimen of base clearly seen in the figure is caused by the intense damping of transverse shear deformations in the resin filler during vibrations. Apparently, it can be assumed that, in this case, the account of external aerodynamic damping will affect the estimate of the damping properties of material to a lesser degree. We should also indicate that the damping properties of resin considerably depend on the vibration amplitude.

The relationships obtained in the theoretical-experimental way (see Figs. 9 and 10) can serve as a basis for identification of the parameters of internal damping of the material examined in conditions of tension-compression and transverse shear.

Conclusions

The experimental results presented show that, in principle, it is possible to determine the damping properties of soft materials. For this purpose, it is necessary (i) to take into account the aerodynamic damping and (ii) to interpret experimental results on the basis of the general nonlinear theory of damping of soft materials so that to derive reliable results. The following two parts of the present study will be dedicated to this problem.

Acknowledgments. This study was carried out within the framework of the Federal Targeted Program "Scientific and Scientific-Pedagogical Personnel of the Innovative Russia" for 2009-2013, created by order of the Russian Ministry of Education and Science, March, 22, 2012, No. 223 and July, 13, 2012, No. 1/22/3, on the subject "Development of a theoretical-experimental method to examine the damped vibrations with account of internal and aerodynamic external damping and to investigate the problems on propagation of sound waves through single-layer and sandwich panels for creating calculation procedures of parameters of sound absorption" (state contract No. 2012-1.2.1-12-000-1002-030)

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