**Study of Propagation of the Current and Voltage Waves Induced by the Lightning Discharge in the Resistive Cable Line with Linear and Nonlinear Elements**

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One of major aspect of investigations, when the problems associated with reliability of activity of electrical power systems (EPS) are considered, is the problem of study of effect of the external in relation to EPS fields on their different structures. The investigations in this direction are represented important for ensure of stabilization of operation of the EPS and their different units, warning of emergencies, connected with outage or abnormal operational mode of the EPS units [1, 2].

On studying of the problems induced by the external EM field in the one-wire line we considered a case when the external source exciting the EM field in the conducting line is a happening on sufficient distance lightning discharge. In this case, main reason of appearance of critical over-voltage in lines, induced by the lightning discharge, is the generation of the current and voltage wave (CVW) owing to spreading of charges, "hitched" by the electrostatic field of the thunderstorm cloud [3]. At fast discharge of a thunderstorm cloud such charges on a conductor located in a weakly conducting medium, spread on the cable shell derivating CVW.

We described the propagation of the CVW by the set of the inhomogeneous equations [4]:

$$\partial_t I + \hat{\mathrm{A}}[I,U,R,L] = 0, \qquad \partial_t U + \hat{\mathrm{B}}[I,U,C,G] = f(t,x); \qquad -\infty < x < \infty,\ t > 0 \tag{1}$$

where $\hat{\mathrm{A}}$ and $\hat{\mathrm{B}}$ are some functionals; *R, C, L, G* are the distributed parameters, namely: resistance, capacitance, inductance and coefficient of creepage calculated per unit length.

In the simplest case when current in the line is absent the initial conditions can be written as

$$I(x,0) = 0, \qquad U(x,0) = 0\,, \tag{2}$$

The source function in (1) can be presented in form of

$$f(t,x) = \psi(x)\left[\,dQ(t)/dt\,\right], \tag{3}$$

where *Q* is the charge of the cloud, and $\psi$ is the function describing the problem geometry [4].

In case, when the line with the distributed sources of type shown in fig. 1 is considered the functionals are defined by equalities [4, 5]

$$\hat{\mathrm{A}} = (\partial_x U + RI)/L, \qquad \hat{\mathrm{B}} = (\partial_x I + GU)/C, \tag{4}$$

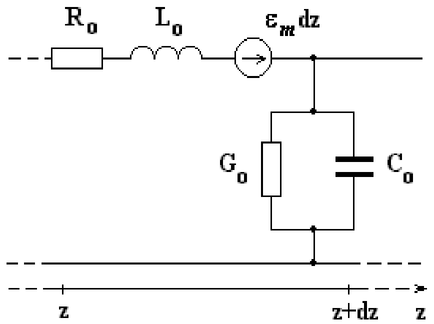


Fig. 1. Elementary site of a line

and set (1) is the set of the inhomogeneous telegraph equations:

$$\partial_x U + L\partial_t I + RI = 0, \quad \partial_x I + C\partial_t U + GU = f(t,x), \quad -\infty < x < \infty, \quad t > 0. \tag{5}$$

At first, consider set (5) with $f(t,x) = 0$ as more simple case. Differentiate the first equation (5) on *x*, and the second one on *t* and exclude derivative $\partial_{xt}^2 I$ from the found expression. We obtain the differential equation of the second order for function *U* [4]:

$$\partial_x^2 U = LC\partial_t^2 U + (RC+GL)\partial_t U + GRU .$$

Obtain the differential equation for current $I$ by analogy: $\partial_x^2 I = LC\partial_t^2 I + (RC+GL)\partial_t I + GRI$ .

One can see that voltage $U$ and current $I$ satisfy to the same equation

$$\partial_x^2 w = a_0\partial_t^2 w + 2b_0\partial_t w + c_0 w, \tag{6}$$

where $a_0 = LC,\quad 2b_0 = RC - GL,\quad c_0 = GR$.

Introduce new function $U(x,t)$ and having put $w = \mathrm{e}^{-(b_0/a_0)t}U$ , result (6) to more simple form:

$$\partial_x^2 U = a^2\partial_x^2 U + b^2 U , \tag{7}$$

where $a = a_0^{-1/2},\quad b = a_0^{-1}\sqrt{b_0^2 - a_0 c_0}$ . Using further a method of Riemann with initial conditions $U\big|_{t=0} = f(x)$ , $\partial_t U\big|_{t=0} = F(x)$ , one can obtain the analytical solution of equation (7) written through an integral of the Bessel functions of zero order [6]:

$$U(x,t) = \frac{f(x-at)+f(x+at)}{2} + \frac{1}{2}\int_{x-at}^{x+at}\Phi(x,t,z)dz ,$$

where

$$\Phi(x,\,t,\,z) = \frac{1}{a}F(z)J_0\left(\frac{b}{a}\sqrt{(z-x)^2 - a^2t^2}\right) + bt\ f(z)\frac{J_0'\left(\frac{b}{a}\sqrt{(z-x)^2 - a^2t^2}\right)}{\sqrt{(z-x)^2 - a^2t^2}} .$$

Operating by analogy and using standard methods of mathematical physics [7], it is possible to obtain the analytical solution of a problem (5) with $f(t,x) \neq 0$:

$$\begin{aligned} I(t,x) &= \frac{a}{2}\int_0^t e^{\alpha(t-\tau)}\left[ f(\xi^-,\tau) - f(\xi^+,\tau) + \kappa\int_{\xi^-}^{\xi^+}\frac{x-\eta}{\xi} f(\eta,\tau)\, I_1(\kappa\xi)\, d\eta\right] d\tau, \\ U(t,x) &= \frac{e^{\alpha t}}{2aC}\left\{\int_{x-at}^{x+at} f(\eta,0)\, I_0(\kappa\beta)\, d\eta + \int_0^t d\tau\, e^{-\alpha t}\int_{\xi^-}^{\xi^+}\left[ f_\tau(\eta,\tau) + \frac{R}{L} f(\eta,\tau)\right] I_0(\kappa\xi)\, d\xi\right\}, \end{aligned} \tag{8}$$

where $I_0(x), I_1(x)$ are the modified Bessel functions [6], and

$$\xi^{\pm} = x \pm a(t-\tau),\ \beta^2 = a^2t^2 - (x-\xi)^2,\ \xi^2 = a^2(t-\tau)^2 - (x-\eta)^2,$$
$$a^2 = 1/LC;\ \alpha = -(RC+LG)/2LC,\quad \kappa = \left| RC - LG \right| / 2\sqrt{LC} .$$

As we can see from (10), the analytical solution is rather unwieldy and does not allow to estimate completely the properties of the investigated system. In this connection we simulated of the CVW evolution for the set (5) on the basis of the finite-difference approach with different initial conditions including zero ones. At this we have obtained the numerical solutions describing the CVW propagation in a one-wire line with due account of the influence of the external (disturbing) sources for a wide spectrum of the values of parameters *R*, *C*, *L*, *G*. The examples of the results for source describing function

$$f(t,x) = \exp\left[-(t-t_0)^2 / l_t\right] \exp\left(-x^2 / l_x\right) \tag{9}$$

for $L = 10^{-7}$ H/m, $C = 10^{-10}$ F/m , $t_{max}$= 50 μs, 2 $t_{max}$= 100 μs are shown in fig. 2.

In the case, when we have a line which includes the distributed nonlinear elements with magnetic coupling (see fig. 3), the functionals in the set (1) can take form [8]

$$\hat{A} = (\alpha_1 I\,\partial_x U + \beta_1 \partial_x^3 U + RI)/L, \qquad \hat{B} = (\alpha_2 U\,\partial_x I + \beta_2 \partial_x^3 I + GU)/C,$$

and we have the set of coupled KdV-type equations with resistive terms [7, 8]:

$$\partial_t I + (\alpha_1 I \partial_x U + \beta_1 \partial_x^3 U + RI)/L = 0, \qquad \partial_t U + (\alpha_2 U \partial_x I + \beta_2 \partial_x^3 I + GU)/C = f(t,x) \tag{10}$$

with initial conditions $I(x,0) = I_0(x)$ and $U(x,0) = U_0(x)$.

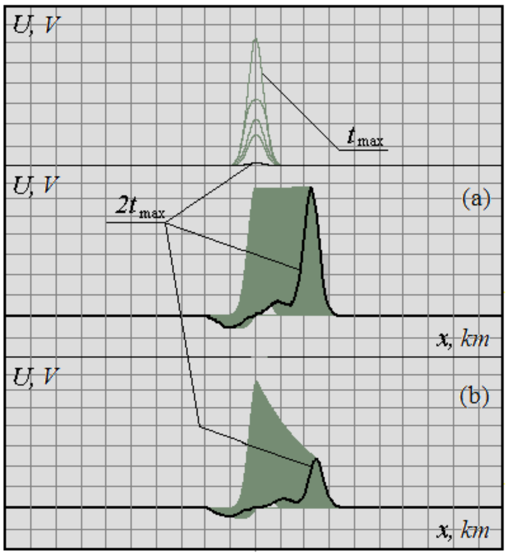


Fig. 2. Evolution of amplitude and shape of signal in a line under influence of the variable in time external impulse for $2t_{max}$= 100 μs: (a) $R = G = 0$, (b) $R = 10^{-3}$ Ω/m $G = 10^{-6}$ 1/Ω·m

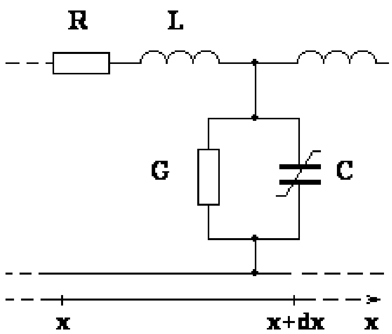


Fig. 3. One element of a line with magnetic coupling

The initial value problem for this set even in case when $f(t, x)$=0 is very complicate and for any for $f(t, x)$ in general is hardly possible for analytical study [7] and we solved it numerically using the finite-difference approach on the basis of both implicit and explicit schemes with high order of approximation [7, 8]. At this, for a numerical integration, from reasons of convenience we rewrite (10) in a dimensionless form:

$$\partial_t I + \tilde{\alpha}_1 I \partial_x U + \tilde{\beta}_1 \partial_x^3 U + \tilde{\gamma}_1 I = 0, \qquad \partial_t U + \tilde{\alpha}_2 U \partial_x I + \tilde{\beta}_2 \partial_x^3 I + \tilde{\gamma}_2 U = \tilde{f}(t,x), \tag{11}$$

where the coefficients and function $\tilde{f}(t,x)$ are dimensionless ones.

We solved problem (11) with function $f(t,x)$ in form (3), (9) including case $f(t,x) = 0$ numerically. In our numerical simulation with the different initial conditions and values of the parameters *R*, *C*, *L*, *G* we observed the following cases (see figs. 4-6). For $R = G = 0$ the CVW soliton with the oscillating tail behind a mail maximum is formed from the initial pulse (figs. 4(a), 5(a)) or, for rather big $\tilde{\alpha}_i / \tilde{\beta}_i$ $(i = 1,2)$, a decay of initial pulse on the sequence of stable CVW solitons is observed (figs. 4(b), 6).

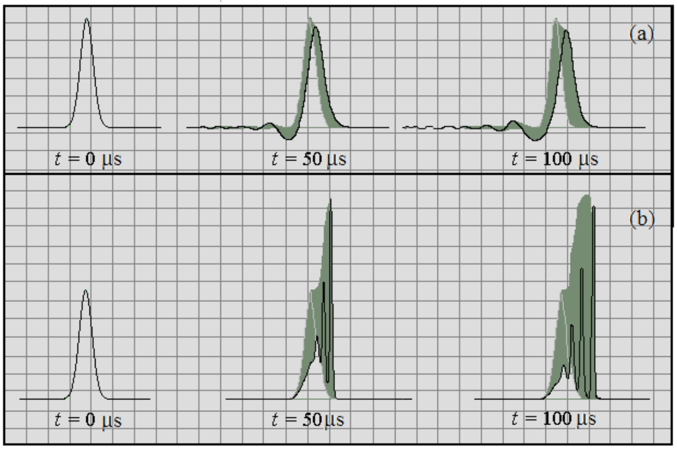


Fig. 4. Evolution of amplitude and shape of signal in a line with $R = G = 0$ at $f(t, x) = 0$, $\tilde{\alpha}_1 = \tilde{\alpha}_2 = 1$ for: (a) $\tilde{\beta}_1 = \tilde{\beta}_2 = 3.47 \times 10^{-3}$, (b) $\tilde{\beta}_1 = \tilde{\beta}_2 = 1.02 \times 10^{-4}$

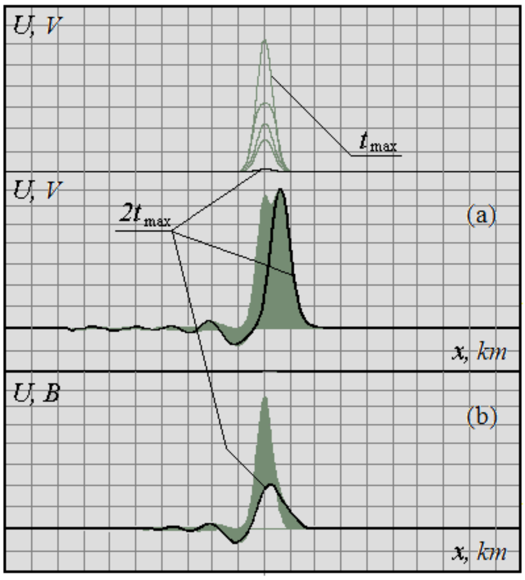


Fig. 5. Evolution of amplitude and shape of signal in a line under influence of the variable in time external impulse (7) for $\tilde{\alpha}_1 = \tilde{\alpha}_2 = 1$, $\tilde{\beta}_1 = \tilde{\beta}_2 = 3.42 \times 10^{-3}$, $t_{max}$= 50 μs: (a) $R = G = 0$, (b) $\tilde{\gamma}_1 = \tilde{\gamma}_2 = 0.15$

At rather small losses in a line (the small values of $\tilde{\gamma}_1$, $\tilde{\gamma}_2$ or $R$, $G$) at the initial stage the CVW soliton-like pulses with steep fronts are formed, and further their amplitudes decrease exponentially with propagation in the line (fig. 6).

In case of big $R$ and $G$ the solitons are not formed. We also obtained that for some special initial conditions and the equations' parameters the parametric amplification phenomenon can be observed [8].

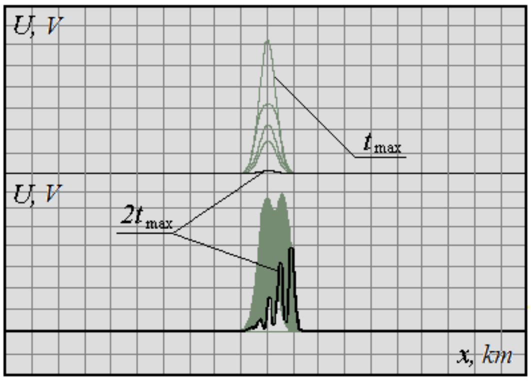


Fig. 6. Evolution of amplitude and shape of signal in a line under influence of the variable in time external impulse (7) for $\tilde{\alpha}_1 = \tilde{\alpha}_2 = 1$, $\tilde{\beta}_1 = \tilde{\beta}_2 = 1.02 \times 10^{-4}$, $\tilde{\gamma}_1 = \tilde{\gamma}_2 = 0.05$ at $t_{max}$= 50 μs

The results obtained for all mentioned above cases were used for study of problem of the effect of the external EM sources of a wide frequency spectrum on the parameters of the initial unperturbed functions of current $I(x,t)$ and voltage $U(x,t)$ in the cable lines of different assignment (including feeding and information coaxial lines – radiofrequency cables). The results of numerical solution of the set of equations (1) for different forms of the functionals $\hat{A}$ and $\hat{B}$, defined by the different elements of the electric circuit, and various model functions of the perturbing sources well illustrating the effects of the excited CVW on the current and voltage parameters in the lines can be found in [4, 5, 7, 8]. The examples considered well illustrate fruitfulness of the explained above approach to an estimation of the influence of the EM fields on different structures of the EPS. The results obtained can be useful on solving the problems of operational reliability and optimum, from the point of view of a noise immunity, designing of the EPS, their different structures, and also study of reasons of change of figure of merits of the electric power.

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