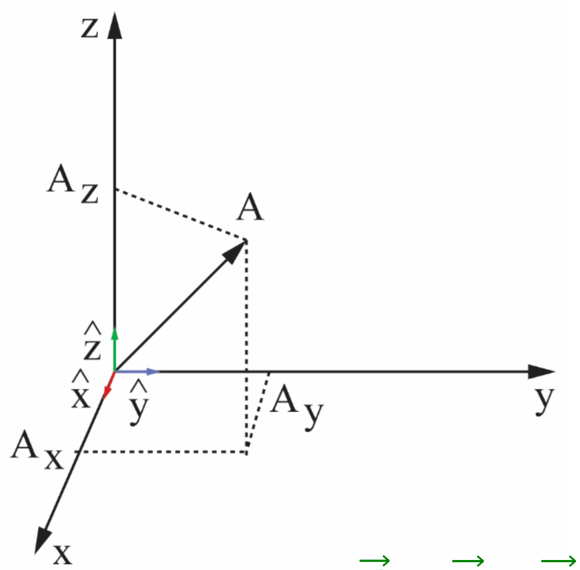
Coordinate Systems, Points, Vectors



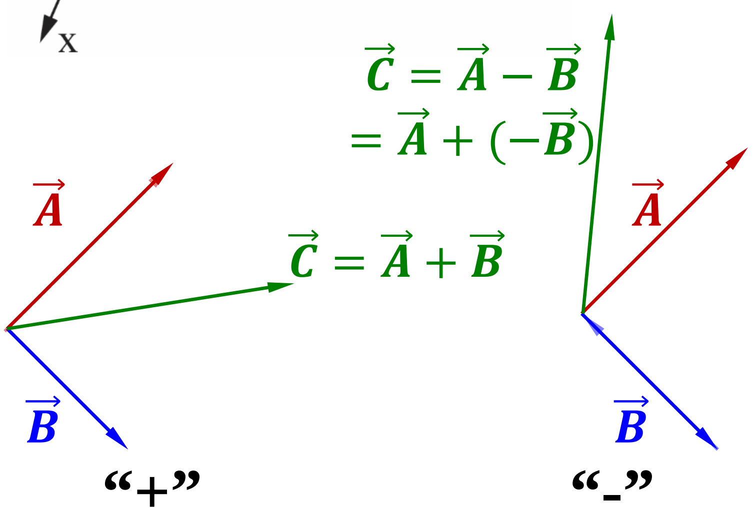
A **vector** in a coordinate system is a directed line between two points. It has **magnitude** and **direction**. Once we define a coordinate origin, each particle in a system has a **position vector** (e.g. – $\vec{A}$) associated with its location in space drawn from the origin to the physical coordinates of the particle (e.g. – (A_x, A_y, A_z)):

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

The position vectors clearly depend on the choice of coordinate origin. However, the **difference vector** or **displacement vector** between two position vectors does **not** depend on the coordinate origin. To see this, let us consider the **addition** of two vectors:

$$\vec{A} + \vec{B} = \vec{C}$$

Note that vector addition proceeds by putting the tail of one at the head of the other, and constructing the vector that completes the triangle.

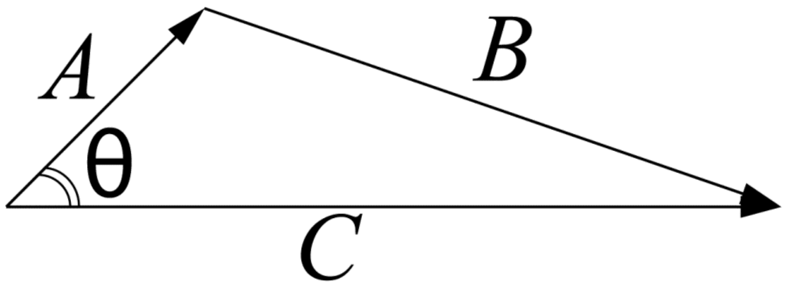


Vector

If we are given a vector in terms of its **length** (magnitude) and **orientation** (direction angle(s)) then we must evaluate its cartesian components before we can add them (for example, in 2D):

$$A_x = |\vec{A}|cos(\theta_A) \qquad B_x = |\vec{B}|cos(\theta_B)$$
$$A_y = |\vec{A}|sin(\theta_A) \qquad B_y = |\vec{B}|sin(\theta_B)$$

This process is called **decomposing** the vector into its cartesian components.



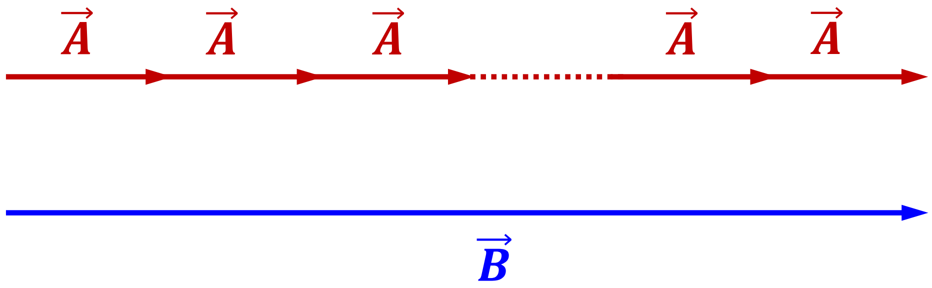
The **difference** between two vectors is defined by the addition law. Subtraction is just adding the negative of the vector in question, that is, the vector with the **same** magnitude but the **opposite** direction. This is consistent with the notion of adding or subtracting its components.

Scalar

When we reconstruct a vector from its components, we are just using the law of vector addition itself, by **scaling** some special vectors called **unit vectors** and then adding them. Unit vectors are (typically perpendicular) vectors that define the essential directions and orientations of a coordinate system and have unit length. Scaling them involves multiplying these unit vectors by a number that represents the magnitude of the vector component. This scaling number has no direction and is called a **scalar**.

$$\vec{B} = C\vec{A}$$

where C is a scalar (number) and $\vec{A}$ is a vector. In this case, $\vec{A} \parallel \vec{B}$ ($\vec{A}$ is parallel to $\vec{B}$).



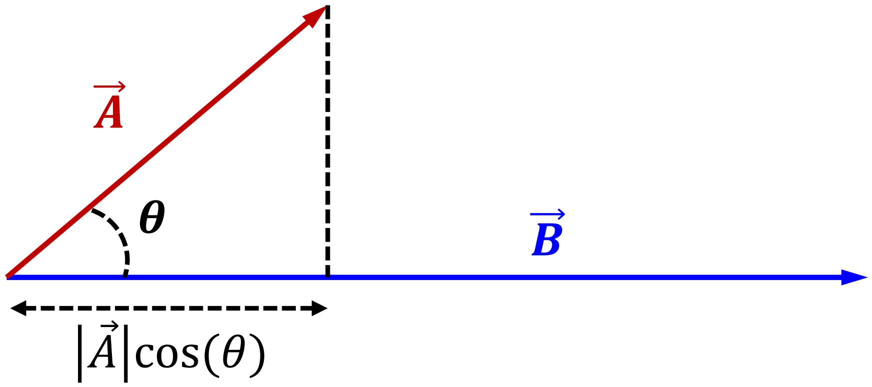
Let's define products that **multiply** two vectors together

The first product creates a scalar (ordinary number with magnitude but no direction) out of two vectors and is therefore called a **scalar product** or (because of the multiplication symbol chosen) a **dot product**.

$$|\vec{A}| = +\sqrt{\vec{A} \cdot \vec{A}}$$

$$\vec{A} \cdot \vec{B} = A_x * B_x + A_y * B_y \dots = |\vec{A}||\vec{B}|\cos(\theta_{AB})$$

A scalar product is the length of one vector (either one, say $|\vec{A}|$) times the component of the other vector ($|\vec{B}|\ cos(\theta_{AB})$) that points in the same direction as the vector $\vec{A}$.
This product is *symmetric* and *commutative* ($\vec{A}$ and $\vec{B}$ can appear in either order or role).



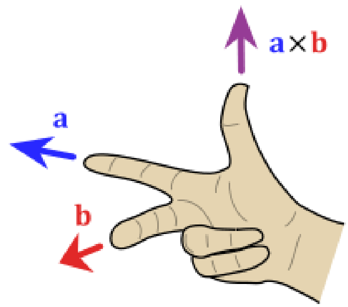
A vector product

The other product multiplies two vectors in a way that creates a third vector. It is called a **vector product** or (because of the multiplication symbol chosen) a **cross** product.

$$\vec{A} \times \vec{B} = \left(A_x * B_y - A_y * B_x\right)\hat{z} + \left(A_y * B_z - A_z * B_y\right)\hat{x} + (A_z * B_x - A_x * B_z)\hat{y}$$

$$\left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right| \sin(\theta_{AB})$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$



Let’s define the direction of the cross product using the **right hand rule**:

Let the fingers of your right hand lie along the direction of the first vector in a cross product (say $\vec{A}$ below). Let them curl naturally through the small angle (observe that there are two, one of which is larger than π and one of which is less than π) into the direction of $\vec{B}$. The erect thumb of your right hand then points in the general direction of the cross product vector – it at least indicates which of the two perpendicular lines should be used as a direction, unless your thumb and fingers are all double jointed or your bones are missing or you used your left-handed right hand or something.