

On Normal τ -Measurable Operators Affiliated with Semifinite Von Neumann Algebras

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Abstract—Let τ be a faithful normal semifinite trace on the von Neumann algebra \mathcal{M} , $1 \geq q > 0$. The following generalizations of problems 163 and 139 from the book [1] to τ -measurable operators are obtained; it is established that: 1) each τ -compact q -hyponormal operator is normal; 2) if a τ -measurable operator A is normal and, for some natural number n , the operator A^n is τ -compact, then the operator A is also τ -compact. It is proved that if a τ -measurable operator A is hyponormal and the operator A^2 is τ -compact, then the operator A is also τ -compact. A new property of a nonincreasing rearrangement of the product of hyponormal and cohyponormal τ -measurable operators is established. For normal τ -measurable operators A and B , it is shown that the nonincreasing rearrangements of the operators AB and BA coincide. Applications of the results obtained to F -normed symmetric spaces on (\mathcal{M}, τ) are considered.

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INTRODUCTION

Let τ be a faithful normal semifinite trace on the von Neumann algebra \mathcal{M} , let $\widetilde{\mathcal{M}}$ be the $*$ -algebra of all τ -measurable operators, and let $1 \geq q > 0$. In this paper, the following generalizations of problems 163 and 139 from the Halmos book [1] to τ -measurable operators are obtained; it is established that:

- (1) each τ -compact q -hyponormal operator is normal (Theorem 2.2);
- (2) if an operator $A \in \widetilde{\mathcal{M}}$ is normal and, for some natural number n , the operator A^n is τ -compact, then the operator A is τ -compact (item (i) of Corollary 3.2).

The proof of Theorem 2.2 is based on a deep result from [2]. It is shown by us that if the operator $A \in \widetilde{\mathcal{M}}$ is hyponormal and the operator A^2 is τ -compact, then the operator A is also τ -compact (item (i) of Corollary 3.4). We establish the new property of the nonincreasing rearrangement of the product of the hyponormal and cohyponormal τ -measurable operators (Theorem 3.5). For normal operators $A, B \in \widetilde{\mathcal{M}}$, it is shown that the nonincreasing rearrangements of the operators AB and BA coincide (Corollary 3.6). A well-known rearrangement property (see item (6) of Lemma 1.1) implies that a nonnegative operator $A \in \widetilde{\mathcal{M}}$ is τ -compact if and only if A^p is τ -compact for all $p > 0$. It is shown in Theorem 4.1 that a similar assertion also holds for the product of nonnegative operators $A, B \in \widetilde{\mathcal{M}}$: the τ -compactness of AB is equivalent to the τ -compactness of the operators $A^p B^r$ for all $p, r > 0$. The results obtained are applied to F -normed symmetric spaces on (\mathcal{M}, τ) .

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